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PARKER INSTABILITY: CONTINUUM VS. DISCRETE SOLUTIONS

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I. INTRODUCTION

Since the pioneering work of Parker (1966), there have been many studies on the stability of a magnetized disk under externally given gravitational fields (Shu 1974; Zweibel & Kulsrud 1975; Foglizzo & Tagger 1994). Most of the studies take a uniform gravitational acceleration for the field, and adopt an exponential distribution of density as an initial equilibrium state. Since the exponential disk has a cusp at the mid-plane $z = 0$, their analyses could not treat the problem of possible interplays between upper and lower halves of the disk. Giz & Shu (1993) considered a non-uniform gravitational acceleration in the Galactic disk, $g(z) \propto \tanh(z/H)$, and showed that a new family of discrete solutions exists. Here, we consider a linear gravitational acceleration, $g(z) = -g'z$ with g' being a constant, and investigate under what conditions one may expect the usual continuum family of the Parker solutions and the new discrete one.

II. LINEAR STABILITY ANALYSIS

Under the influence of the linear acceleration, initial equilibrium distributions of density, gas and cosmic-ray pressures, and magnetic field take all Gaussian functions of z. Linearized perturbation equations of these variables reduce to a type of Schrödinger's equation

$$\frac{d^2\psi}{d\zeta^2} + (E - V_o\zeta^2)\psi = 0, \tag{1}$$

where ζ is proportional to z and $\psi^2(= \rho_o(z)v_z^2)$ is kinetic energy density along the z-direction. E and V_o are given by

$$E \equiv \frac{1}{2} + \frac{\Omega^4 - [s + p(\nu_x^2 + \nu_y^2)]\Omega^2 + q(\nu_x^2 + \nu_y^2)\nu_y^2}{p\Omega^2 - q\nu_y^2}, \tag{2}$$

$$V_o \equiv \frac{1}{4} - \frac{2\alpha s\nu_x^2\Omega^2 + 2\alpha\gamma\nu_{y,P}(\nu_x^2 + \nu_y^2)(\Omega^2 - 2\alpha\nu_y^2)}{(\Omega^2 - 2\alpha\nu_y^2)(p\Omega^2 - q\nu_y^2)} \tag{3}$$

where $p \equiv 2\alpha + \gamma$, $q \equiv 2\alpha\gamma$, $s \equiv 1 + \alpha + \beta$, and $2\alpha\gamma\nu_{y,P} \equiv (1 + \alpha + \beta)(1 + \alpha + \beta - \gamma)$. Here, α is the ratio of magnetic to gas pressure, β the ratio of cosmic-ray to gas pressure, γ the effective adiabatic index, Ω the normalized angular frequency, and ν_x and ν_y are the normalized wavenumbers along the radial and azimuthal directions.

The eigensolutions of equation (1) can be classified into two families. When $V_o < 0$ (regardless of the sign of E), they constitute the continuum family; while when $E > 0$ and $V_o > 0$ they do the discrete family.

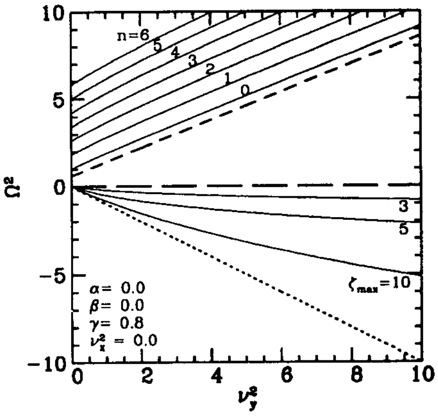

Fig. 1.— Three loci of $V_o = 0$ (dotted line), $E = 0$ (dashed line), and $\Omega^2 = 0$ (long-dashed line) divide the dispersion domain into four regions with different signs of E and V_o. Dispersion relations with n nodal points in the upper region ($E > 0$ and $V_o < 0$) belong to the discrete family. All the pairs of (ν_y^2, Ω^2) in the region $V_o < 0$ enclosed by the loci $\Omega^2 = 0$ and $V_o = 0$ constitute the dispersion relations which belong to the continuum family. As a specific example, three dispersion relations with the first nodal point (ζ_{node}) at 3, 5, and 10 are plotted.

III. MODE IDENTIFICATION

When $\alpha = 0$ and $\beta = 0$, non-magnetized disk has eigensolutions of the gravo-acoustic wave mode and of the convective mode. As was pointed out by Nelson (1976), the former is of the discrete nature; while the latter is of the continuum nature. When $\gamma = 1$, only the gravo-acoustic waves exist. When $\gamma > 1$ the convective mode becomes stabilized, but when $\gamma < 1$ it gets destabilized (Fig. 1). The convective instability is more unstable under the linear acceleration than under the uniform acceleration.

An inclusion of magnetic field and cosmic-ray spawns five modes of eigensolutions: a mode of fast MHD waves modulated by gravity, an Alfv́en wave mode, the original Parker mode of slow MHD waves modulated by gravity, and two additional modes which are also attributed to slow MHD waves modulated by gravity. Presence of magnetic field transforms the Nelson's

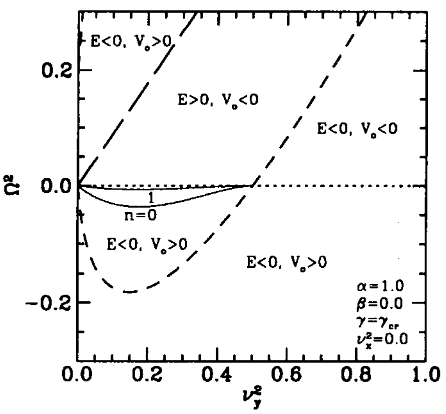

Fig. 2.— The loci of V_o=0 in dotted line, $E = 0$ in two dashed lines, and $(2\alpha + \gamma)\Omega^2 - 2\alpha\gamma\nu_y^2 = 0$ in a long-dashed line divide the dispersion domain into six regions. There exist two regions $V_o < 0$ in which continuum family resides, and one region $E > 0$ and $V_o > 0$ in which discrete family does. Because the medium with γ_{cr} is hard enough to stabilize the continuum modes, unstable solutions exist only in the lower regions where we plotted the dispersion relations with n nodal points in solid lines.

gravo-acoustic waves into the fast MHD waves of the discrete nature. When the wavenumber perpendicular to both directions of the initial magnetic field and the gravitational acceleration is equal to zero, the Alfv́en wave mode gets decoupled from the slow and fast modes of the MHD waves.

In terms of γ the instability criterion of the original Parker mode of the continuum nature takes exactly the same form for both the uniform and linear accelerations, but the growth rate becomes larger under the linear gravity. The additional two modes are of the discrete nature. One of them is stable and always exists. An existence of the other mode depends on the disk conditions, and it can be either stable or unstable. When this mode becomes unstable, it is the discrete mode Giz & Shu (1993) found. But they did not discuss the existence of the always existing stable discrete mode we found.

IV. DISCUSSION

When γ is larger than $\gamma_{cr}= (1+\alpha+\beta)^2/(1+3\alpha/2+\beta)$ for $\nu_x = 0$, the Parker mode becomes stable. So the only unstable eigensolutions are from the discrete unstable mode. Then, density perturbation with an odd function of z grows faster than that with an even function, and the disk may develop corrugated features due to the fastest growing perturbation. For the parameters, $\alpha = 1, \beta = 0$, and $\gamma = \gamma_{cr}$, its minimum growth time is 1.3×10^8 years and corresponding wavelength 2.4 kpc (Fig. 2). We notice that this length scale is consistent with the wavelength of the vertical structures found from the Carina-Sagittarius spiral arm by Alfaro *et al.*(1992).

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