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STUDY ON GRAVOTHERMAL OSCILLATIONS WITH TWO-COMPONENT FOKKER-PLANCK MODELS

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ABSTRACT

Two-component models (normal star and degenerate star components) are the simplest realization of clusters with a mass spectrum because the high mass stars quickly evolve off leaving degenerate stars behind, while low mass stars survive for a long time as main-sequence stars. In the present study we examine the post-collapse evolution of globular clusters using two-component Fokker-Planck models that include three-body binary heating. We confirm that a simple parameter $\epsilon \equiv (E_{\rm tot}/t_{rh})/(E_c/t_{rc})$ well describes the occurrence of gravothermal oscillations of two-component clusters. Also, we find that the degree of instability depends on the steepness of the mass function such that clusters with a steeper mass function are less exposed to instability.

Key Words : celestial mechanics – stellar dynamics – globular clusters : general

I. TWO-COMPONENT MODELS

The course of dynamical evolution of pre- and post-corecollapse globular clusters is determined by many factors such as initial mass function, the nature and efficiency of energy generation mechanisms, tidal cut-off, unisotropy of velocity distribution, primordial and dynamical binaries, and stellar evolution. There have been many efforts in developing more and more complex cluster models including such factors, making analysis and interpretation rather difficult.

However, studying simpler models could be more instructive in identifying important physical processes governing the evolution. Since the main-sequence lifetime increases exponentially as the initial stellar mass decreases, a cluster can be assumed to start its dynamical evolution with a turn-off point massvery similar to that observed nowadays. Most of white dwarfs in a cluster is believed to have masses of near or little less than the present turn-off point mass. For a simple description for the dynamical evolution of globular clusters, we employ only two mass components, one for the main-sequence stars, and the other for the neutron stars.

In the postcollapse expansion phase, if the outer part of a cluster is not able to absorb the energy generated in the core efficiently, the core will expand much faster than the outer part and its temperature will drop. If its temperature drops too much, the core will contract while the heat inflows from the outer part rather than outflows from the core. After being heated enough by the heat inflow from the outer part, the core will expand again and face such a density oscillation recursively. This postcollapse instability, gravothermal oscillation, was first demonstrated by Sugimoto & Bettwieser (1983) and Bettwieser & Sugimoto (1984), and investigated further by few scientists such as Goodman (1987) among others.

II. INSTABILITY PARAMETER ϵ

In equal mass clusters, in which the number of stars N is the only cluster parameter for self-similar models, the gravothermal instability occurs for $N < N_{\rm crit} \approx 7000$. In real clusters with a mass spectrum, however, the energy generation rate in the core and absorption rate in the outer part are not fuctions of N only, and thus instability should depend on parameters other than N too.

Considering the fact that instability is caused by the unbalance between energy generation in the core and absoprtion in the outer part, Goodman (1993) suggested that the quantity

$$\epsilon \equiv \frac{E_{\rm tot}/t_{rh}}{E_c/t_{rc}} \qquad (1)$$

should describe the degree of stability universally (regardless of the presence of mass spectrum), where

$$E_c \equiv \frac{2\pi}{3}\rho_c r_c^3 v_c^2 \qquad (2)$$

is the energy in the core, v_c^2 is the three-dimensional velocity dispersion in the core, and t_{rh} and t_{rc} are half mass and core relaxation time, respectively.

The feasibility of ϵ as a measure of stability of clusters with a mass spectrum can be tested most easily with two-component models owing to its simplicity. We have performed total of 12 runs of time integration of two-component Fokker-Planck equation with three-body binary heating and no tidal cut-off. The initial conditions were Plummer's models with 3 different mass ratios (m_2/m_1 = 1.5, 2, 3) and 4 different total numbers ($N = 3 \times 10^4, 10^5, 3 \times 10^5, 10^6$). In all our runs, $N_1/N_2 = 100$. Not only the central density but also ϵ oscillates during the instability period. However an equilibrium ϵ can be obtained by appropriately enlarging integration timesteps. Such equilibrium ϵ is almost constant during the whole instability period and

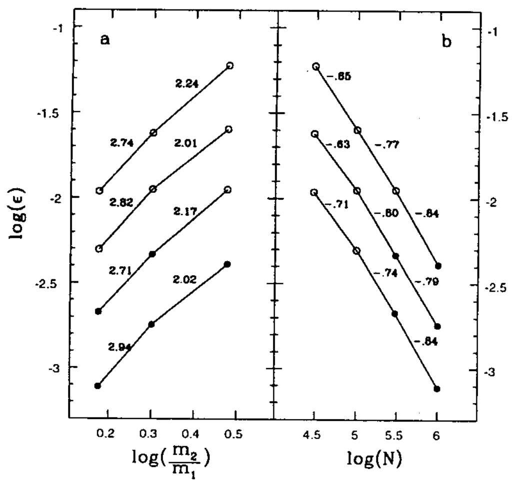

Fig. 1.— Oscillation-suppressed ϵ values for our runs over mass ratio (a) and total number (b). Filled circles are for runs that showed gravothermal oscillation and open circles are for those that did not. There is a boundary near $\epsilon \simeq 0.05$ only below which oscillations take place.

can be used as a representative value for the postcollapse phase. These representative ϵ's of our runs are plotted in Figure 1a over m_2/m_1, and in Figure 1b over N. As marked with filled circles in the Figure, 5 out of 12 runs showed gravothermal oscillations in the postcollapse expansion phase. There seems a clear boundary between clusters which show gravothermal oscillation and clusters which do not, and this proves that ϵ indeed is a measure of stability during postcollapse expansion. The boundary, $\epsilon_{\rm crit}$, resides near 0.005 and this value is about a half of that for single mass clusters ($\epsilon_{\rm crit} = 0.013$, Goodman 1993).

III. CLUSTER DEPENDENCY OF ϵ

In their study on the stability of clusters with three-body binaries and a broad mass spectrum, Murphy, Cohn & Hut (1990) found that stability persists to much further N than in single-mass clusters if the mass function is steep. They suggested that stability depends mainly on the total number of the heaviest stars. However, Goodman (1993) presented a different interpretation that instability depends on the mass function itself.

With such an idea, he derived an interesting theoretical prediction on the relationship between ϵ and cluster parameters such as m_2/m_1 and N for two-component clusters with three-body binary heating. By balancing the energy generation rate by three-body binaries in the core and the power required by the expansion of the cluster, he obtained

$$\left(\frac{r_c}{r_h}\right) \propto \left(\frac{m_2}{m_1}\right)^{7/6} N^{-2/3}, \tag{3}$$

and accordingly

$$\epsilon \propto \left(\frac{m_2}{m_1}\right)^{5/3} N^{-2/3}. \tag{4}$$

In the above derivation, equipartition in the core and $M_2/M > r_c/r_h$ have been assumed (cf. Goodman 1987, 1993 for detail).

We test the above theoretical prediction with the results of the same 12 runs as in the previous section. The dependencies of ϵ on m_2/m_1 and N for our runs are shown in Figure 1, where the numbers represent the slopes of the corresponding lines, i.e. exponents in the right-hand-sides of equations (3) and (4). Although the slopes over N of our runs agree fairly well with theoretical prediction, the slopes over m_2/m_1 of our runs are quite higher than the predicted values. This discrepancy is due to the approximations used in each step of derivation of equations (3) and (4), especially that used for $\dot{r}_h$.

Although numerical simulations do not precisely agree with Goodman's (1993) prediction on the cluster parameter dependency of ϵ (equation 2), it is certain that the post-collapse instability depends on the cluster's mass function, rather than just the number of heaviest stars in the cluster, because the instability parameter ϵ, whose validity has been already shown in the previous section, does differ for clusters with the same N_2 but with different m_2/m_1, and is a monotonic function of m_2/m_1 such that clusters with a steeper mass function is less exposed to instability.

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