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DYNAMICAL AND STATISTICAL ASPECTS OF GRAVITATIONAL CLUSTERING IN THE UNIVERSE

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ABSTRACT

We apply topological measures of clustering such as percolation and genus curves (PC & GC) and shape statistics to a set of scale free N-body simulations of large scale structure. Both genus amd percolation curves evolve with time reflecting growth of non-Gaussianity in the N-body density field. The amplitude of the genus curve decreases with epoch due to non-linear mode coupling, the decrease being more noticeable for spectra with small scale power. Plotted against the filling factor GC shows very little evolution – a surprising result, since the percolation curve shows significant evolution for the same data. Our results indicate that both PC and GC could be used to discriminate between rival models of structure formation and the analysis of CMB maps. Using shape sensitive statistics we find that there is a strong tendency for objects in our simulations to be filament-like, the degree of filamentarity increasing with epoch.

I. INTRODUCTION

Gravitational clustering in the Universe can be described by a number of statistical indicators including the two point correlation function, counts in cells statistics, genus curve, minimal spanning trees, percolation etc. In principle, a complete description of clustering is provided by a knowledge of the full hierarchy of N-point correlation functions, but this is impossible to achieve in practice. As a result one usually chooses a number of distinct ('orthogonal') statistical indicators to probe different aspects of clustering in galaxy catalogues and in simulations of large scale structure. As a practical tool the two point correlation function ξ has proved to be invaluable in discrminating between models of structure formation. However, although ξ is a very useful statistic, it ignores phase information and therefore provides a complete description of clustering only if galaxies sample a Gaussian random field. During late stages of gravitational clustering, non-linear correlations between phases grow and it becomes necessary to complement ξ with other statistical indicators which are more sensitive to geometrical and topological aspects of the distribution of matter such as percolation, the genus curve and shape statistics (Sahni *et al.* 1995).

II. PERCOLATION AND THE GENUS CURVE

Zeldovich was the first to notice that matter evolving under gravitational instability tends to percolate at much lower values of the filling factor (FF) than a Gaussian random field (Zel'dovich 1982; Zel'dovich *et al.* 1982; Shandarin 1983). The reason for this is easy to understand if one uses the Zeldovich approximation to follow particle trajectories (Sahndarin & Zel'dovich 1989):

$$\mathbf{x} = \mathbf{q} - D_+(t)\nabla\Phi_0(\mathbf{q}) \quad (1)$$

where $D_+(t)$ is the linear growing mode of the density contrast and $\Phi_0(\mathbf{q})$ is the initial (linear) velocity potential. The related density contrast is

$$\rho(\mathbf{x},t) = \frac{\rho_0}{a^3}\left[1 - D_+(t)\lambda_1(\mathbf{q})\right]^{-1}\left[1 - D_+(t)\lambda_2(\mathbf{q})\right]^{-1} \left[1 - D_+(t)\lambda_3(\mathbf{q})\right]^{-1}. \quad (2)$$

where λ_1, λ_2 and λ_3 are the eigenvalues of the deformation tensor $\frac{\partial^2\Phi_0}{\partial q_i \partial q_j}$. (We assume $\lambda_1 > \lambda_2 > \lambda_3$.) Caustics (regions of high – formally infinite – density) form in those regions of space which contract along at least one direction, so that $\lambda_1 > 0$. For a Gaussian random field such regions occupy 92% of the total volume in Lagrange (initial) space and hence percolate easily (a percolating structure in a Gaussian distribution must occupy at least 16% of the total volume). Since the Zeldovich approximation is topology preserving such regions will continue to percolate when mapped into Euler (real) space even though they now occupy a negligible fraction of the total volume. Thus structures which percolate easily such as pancakes and filaments form generically in the Zeldovich approximation (Sahndarin & Zel'dovich 1989).

Following Zeldovich's original insight we study percolation in Cosmological models using N-body simulations with scale invariant initial spectra $P(k) \equiv \left\langle|\delta_k|^2\right\rangle \propto k^n$, $n = -2, -1, 0, 1$, at different cosmological epochs characterised by the scale of nonlinearity k_{NL}^{-1} measured in units of the fundamental scale k_f^{-1}.

One of the main aims of peroclation analysis is to study the connectedness of structure as a function of the density threshold (equivalently – filling factor FF. FF is the total volume in all clusters/voids above/below the density constrast threshold divided by the simulation volume). Lowering of the density threshold leads to a 'percolation transition' characterised by the presence of an 'infinite' cluster spanning the entire simulation box. This occurs at a critical value of the threshold ρ_c. At high values $\rho > \rho_c$ most clusters are too insubstantial to link up and percolate, at very low values $\rho << \rho_c$ most of the matter in the simulation

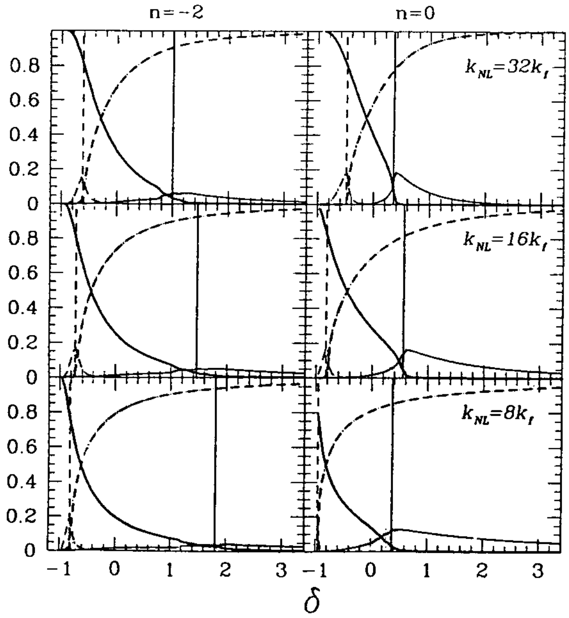

Fig. 1.— Percolation curves show the volume fraction in the largest cluster/void (thick solid/dashed line) as a function of the density contrast δ at increasing cosmological epochs (top to bottom) for N-body simulations with power spectra $P(k) = k^n, n = -2$ (left panel) and $n = 0$ (right panel). Also shown is the volume fraction in the remaining clusters/voids (light solid/dashed line) which peaks near the percolation transition. After percolation most clusters merge with the largest cluster causing their number to drop sharply.

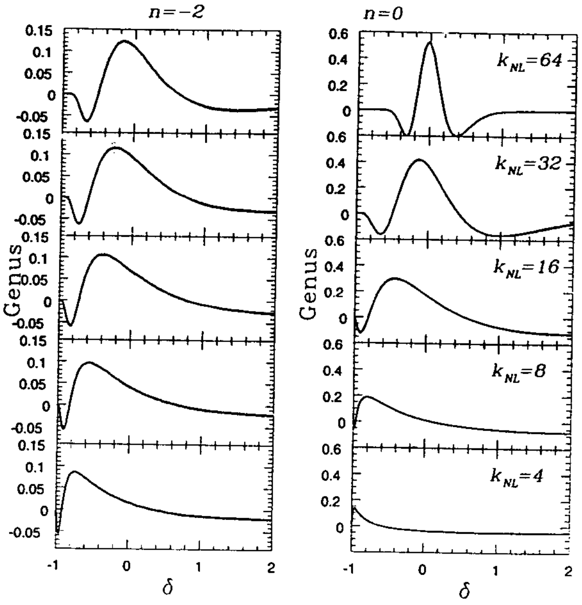

Fig. 2.— Genus curves are shown as functions of the density contrast δ at increasing cosmological epochs (top to bottom). Departure of the genus curve from its original Gaussian distribution is very noticeable. Reproduced, with permission, from (Sahni *et al.* 1996).

is in the percolating supercluster. As the simulation advances ρ_c increases (FF_c decreases) for spectra with large scale power (Klypin & Shandarin 1993; Yess & Shandarin 1996; Sathyaprakash *et al.* 1996; Shani *et al.* 1996).

The peroclation curve (PC) for clusters (solid line) and voids (dashed line) is shown plotted against the density contrast in fig. 1, also shown is the volume fraction in all clusters/voids with the exception of the largest cluster/void (light solid/dashed line). The vertical solid and dashed lines correspond to ρ_c for clusters and voids. For a Gaussian random field clusters and voids percolate at identical thresholds, hence the separation between $\rho_c(clusters)$ and $\rho_c(voids)$ is a measure of the extent of non-Gaussianity in the distribution. We find that as the simulation evolves $\rho_c(clusters)$ monotonically increases for $n = -2$ and increases then decreases for $n = 0$. The difference in the behaviour of $\rho_c(clusters)$ for the two spectra highlights differences in gravitational clustering in the two cases. For $n = -2$ there is sufficient power on large scales for percolation to become progressively easier with time. For $n = 0$ matter quickly collects into small clumps after which the distribution finds it difficult to percolate.

From fig. 3 (left panel) we find that, when plotted against FF, PC resembles a hysterisis curve. The area enclosed by the hysterisis curve (the separation between clusters and voids) grows monotonically with time demonstrating again the growth of non-Gaussianity as clustering advances. We also find that clusters find it easier to percolate than voids, as a result the large scale structure of the Universe resembles a network-like topology for clusters/superclusters and a bubble-like topology for voids. The smallness of the filling factor at percolation makes it likely for matter to be arranged in filament and sheet-like distributions since these occupy less space and so percolate more easily (for a fixed FF). We have checked this conjecture using statistical indicator's sensitive to shape and find that both pancakeness and filamentarity of structures grows with gravitational clustering (Sathyaprakash *et al.* 1996).

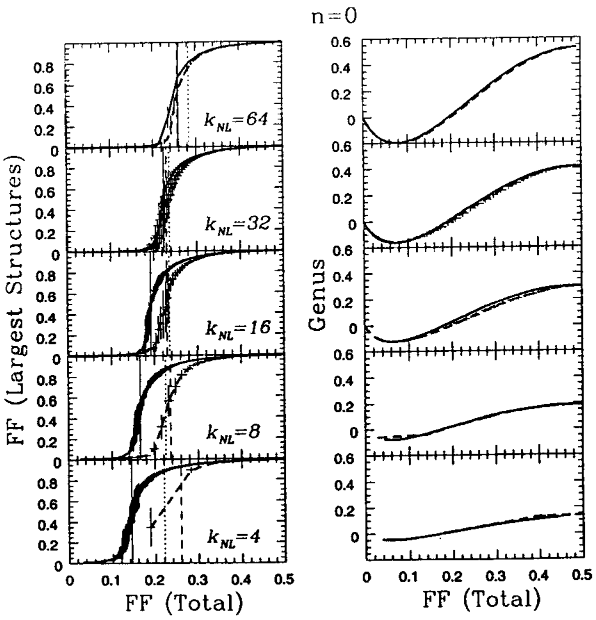

Fig. 3.— The solid/dashed percolation curves in left panels show the FF in the largest cluster/void plotted against the total FF. Solid/dashed lines in the right panels show the genus curve for clusters/voids as a function of the total FF. The vertical solid/dashed lines correspond to the filling factor at percolation for clusters/voids. Notice that the distinction between clusters and voids is more marked in PC, and that the amplitude of GC evolves with time.

Filamentarity appears to be the more dominant shape of the two at all epochs and for all spectra considered by us. It is particularly pronounced during late epochs possibly due to the alignement of neighboring clusters (Bond *et al.* 1996; Sathyaprakash *et al.* 1996).

A complementary measure of the connectedness of large scale structure is provided by the genus curve (GC) (Melott 1990). Simply connected surfaces have $G < 0$, multiply connected have $G \geq 0$. In fig. 2 and the right hand panel of figure 3 we show GC as a function of the density threshold and FF respectively. From fig. 2 we find that GC rapidly departs from its original 'bell-shaped' form, reflecting increasing departure from Gaussian initial conditions. In addition the amplitude of GC shows a significant decrease, caused by non-linear mode coupling and phase correlations. It is however important to point out that distributions related by a mapping $\delta \rightarrow f(\delta)$ will differ in the shape of their GC's despite the fact that topologically such distributions are identical. For this reason it is more appropriate to plot GC against FF as is done in fig. 3 (right panel). Comparing the left and right hand panels of fig. 3 we find a surprising result: the amplitude of GC shows much evolution, whereas there is greater distinction between underdense and overdense regions in PC. This result leads us to conclude that PC & GC may provide complementary probes of non-Gaussianity in distributions (Sahni *et al.* 1996).

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