

## RECENT PROGRESS IN STRING INFLATIONARY COSMOLOGY

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### ABSTRACT

Super-inflation driven by dilaton/moduli kinetic energy is naturally realized in compactified string theory. Discussed are selected topics of recent development in string inflationary cosmology: kinematics of super-inflation, graceful exit triggered by quantum back-reaction, and classical and quantum power spectra of density and metric perturbations.

*Key Words* : string theory, super-inflation, density perturbations, gravitational waves

### I. MOTIVATIONS FOR STRING INFLATION

Despite impressive success, the Standard Model of big-bang cosmology is known to suffer from two 'naturalness' problems: observed homogeneity and spatial flatness of the present universe that cannot be explained in a natural way. Rather they point to a period of inflationary expansion in the past. The evolution of Friedmann-Robertson-Walker (FRW) scale factor is governed by the Einstein equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{k}{a^2} \quad (1)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G(\rho + 3p) \quad (2)$$

where  $\rho$  and  $p$  are the energy density and the pressure of the matter and  $k$  measures the spatial curvature. The spatial flatness problem is solved naturally if the matter density has grown much larger than the spatial curvature. Thus inflation is characterized by a period during which the ratio  $\frac{8\pi}{3}G\rho/(1/a^2) = \dot{a}^2 + k$  has increased with time, viz.,  $\dot{a} > 0$  and  $\ddot{a} > 0$ . Eq.(??) shows that such an accelerated expansion is possible only for exotic matter satisfying  $\rho + 3p < 0$ . Inflation solves the horizon problem automatically. The physical distance for a fixed comoving separation scales as  $a$ . The cosmic horizon is inversely proportional to the Hubble parameter. Thus, during inflation, their ratio  $a/(a/\dot{a}) = \dot{a}$  grows with time since  $\ddot{a} > 0$ . This implies that the physical distance scale is stretched outside the horizon so that the correlation encompasses an enormous spatial volume, hence, solves the horizon problem.

There are three possible types of inflation. The first, de Sitter inflation  $a(t) = \exp(Ht)$ , arises from nonzero vacuum energy during weakly first-order phase transition. The second, power-law inflation  $a(t) = t^p$  for  $t > 0, p > 1$  arises in many models of supergravity with exponential potential. The third, super-inflation  $a(t) = (-t)^p, t > 0, p < 0$  is the least familiar one but arises for Brans-Dicke-type gravity theories including string theory. The novelty of the super-inflation is that it is driven by kinetic energy rather than vacuum

potential energy as is required for the former two inflations. This is gratifying since it has been known that the vacuum potential energy has to be fine-tuned in order to achieve an observationally successful inflation. As such, for inflations driven by potential energy, the naturalness problems of observational cosmology have been traded for the naturalness problems of underlying microscopic physics.

### II. KINEMATICS OF STRING SUPER-INFLATION

Classical string dynamics at sub-Planck scale is described by a Brans-Dicke type effective Lagrangian expressed as a power-series expansion of spacetime derivatives:

$$L = e^{-2\phi}[-R - 4(\nabla\phi)^2 + (\nabla T)^2 + \dots] + L_{\text{matter}}. \quad (3)$$

The ellipses denote (classical) higher-derivative terms,  $L_{\text{eff}}$  is the Lagrangian describing matter coupling, and  $\phi$  and  $T$  are dilaton and moduli fields (associated with the size and the shape of compactified space) respectively. Eq.(3) indicates that, in string theory, the Newton's constant is not a fixed quantity but determined dynamically by the dilaton  $\phi$ :  $G = e^{2\phi}/16\pi$ .

As first pointed out by Veneziano (Veneziano, 1991), the string theory gives rise to the super-inflation naturally. Veneziano has shown that Einstein equation and  $\phi, T$  equations of motion derived from Eq.(3) have always two cosmological branches. For example, for vacuum without matter, the first branch exhibits decelerating expansion and growing Newton's constant

$$a(t) = t^{+1/\sqrt{3}}, \quad e^\phi = t^{-1+\sqrt{3}}, \quad t > 0, \quad (4)$$

while the second branch represents accelerating expansion and growing Newton's constant

$$a(t) = (-t)^{-1/\sqrt{3}}, \quad e^\phi = (-t)^{-1-\sqrt{3}}, \quad t < 0. \quad (5)$$

Similarly, for  $p = \pm\rho/3$  matter, the first branch ( $p = +\rho/3$ ) represents a radiation-dominated FRW universe

and frozen Newton's constant

$$a(t) = t^{1/2}, \quad \phi = \text{constant}, \quad t > 0. \quad (6)$$

The second branch ( $p = -\rho/3$ ) represents the universe with accelerated expansion and growing Newton's constant

$$a(t) = (-t)^{-1/2}, \quad \phi = -3 \log(-t), \quad t < 0. \quad (7)$$

Veneziano has shown that the two branches are related each other by simultaneous time-reversal  $t \rightarrow -t$  and 'scale-factor duality':  $a \rightarrow 1/a$ ,  $\phi \rightarrow \phi - 6 \log a$  and  $(p/\rho) \rightarrow -(p/\rho)$ . The duality is a consequence of the underlying string theory symmetries, hence, is the most distinguishing feature of string cosmology from the others. More interestingly the scale-factor duality may offer a stringy mechanism for exiting from inflation: by duality the branch can flip from the super-inflation to the FRW-type one. For physically sensible branch change, metric, Newton's constant and all other physical quantities should interpolate smoothly across the moment of fixed point of the scale-factor duality  $t^*$ :  $a(t^*) = 1/a(-t^*)$ ,  $\phi(-t^*) = \phi(t^*) - 6 \log a(t^*)$ .

Kinematical distinction of the super-inflation compared to the other two inflations is most clearly seen from the behavior of the cosmic horizon (Fabbri et.al., 1985). The particle horizon given by the inverse Hubble parameter  $R_H = 1/H = (-t)/p$  shrinks to zero size asymptotically as  $t \rightarrow 0^-$ . In de Sitter inflation  $R_H = 1/H$  remains frozen, while in power-law inflation  $R_H = t/p$  grows large. This distinction bears an important consequence to the generation of the primordial density and metric perturbations. The physics behind generating these perturbations is the quantum fluctuation produced inside the subluminal horizon. Once produced, the quantum fluctuation is stretched outside the horizon and behaves as classical density and metric perturbations (Rey, 1987). Since the horizon for super-inflation shrinks during inflation, quantum fluctuations are parametrically squeezed toward shorter wavelengths. More precisely, the first horizon-crossing condition implies that the higher frequency modes cross the horizon at later time. With increasing Hubble parameter in time, the power spectra of the higher frequency modes are amplified with respect to lower frequency mode ones. This has an important bearing on the formation of the large-scale structures as discussed later.

### III. GRACEFUL EXIT VIA QUANTUM BACK REACTION

As mentioned the string theory offers an extremely attractive possibility of exit from inflation utilizing the scale-factor duality. Unfortunately, extensive study has shown it impossible (Brustein and Veneziano, 1994) because the curvature and the Newton's constant have turned out discontinuous and divergent at the fixed point moment  $t^* = 0$ . This problem seems very generic

and is now known as the 'graceful exit problem' of string inflationary cosmology.

On a closer look, however, the problem arises entirely within the classical approximation. Near the fixed point moment, both the curvature and the Newton's constant grow indefinitely so that higher-order curvature and quantum corrections become important. Therefore, whether a smooth transition between the two branches is possible or not should be determined only after a full-fledged quantum stringy effect is taken into account.

Antoniadis, Rizos and Tamvakis (Antoniadis et.al., 1994) have initiated the study of the quantum effect to the graceful exit problem. They have found that under certain initial conditions the quantum back reaction of the fluctuating matter leads to a smooth transition from the super-inflation branch to the FRW-type branch. Their analysis has been further extended (Easler and Maeda, 1996) for nonzero spatial curvature and have reached essentially the same conclusion.

In fact, it is possible to obtain an 'exact' quantum analysis for a two-dimensional truncation (Rey, 1996), which still captures all the underlying essential physics of four dimensions. After the truncation, the classical Lagrangian is given by

$$L_{\text{classical}} = e^{-2\phi}(-R - 4(\nabla\phi)^2) + \frac{1}{2}(\nabla\bar{f})^2 \quad (8)$$

where  $\phi$  and  $\bar{f}$  refer to the dilaton and the  $N$ -component (Ramond-Ramond) matter field. By solving the equations of motion, two branches are found exactly. The super-inflation branch:

$$(ds)^2 = [d\tau^2 - (\frac{\tilde{M}}{-\tau})^2 dx^2]; \quad -\infty < \tau \leq 0 \quad (9)$$

with a growing dilaton:  $\phi = -\log(-2\tau)$ , and the FRW-type branch:

$$(ds)^2 = [d\tau^2 - (M\tau)^2 dx^2]; \quad 0 \leq \tau < \infty \quad (10)$$

with a frozen dilaton.

The two branches also show discontinuous and divergent curvature and Newton's constant at the fixed point moment  $\tau^* = 0$ , hence, a two-dimensional version of the graceful exit problem. In two dimensions, the quantum corrections are entirely specified by the conformal anomaly arising at one loop only, hence, exactly solvable for a given spin and multiplicity content of the massless fields. It is found that

$$L_{\text{quantum}} = L_{\text{classical}} + \frac{\kappa}{2} [R \frac{1}{\partial^2} R + 2\phi R] \quad (11)$$

where  $\kappa = (N - 24)/24$  and is assumed negative definite (this last condition has been relaxed for a more general class of truncations (Gasperini and Veneziano, 1996)). For detailed analysis of the quantum effects, we refer to the original work (Rey, 1996). Here, we

sketch the main result. Due to the quantum corrections, both the scale factor and the dilaton evolves beyond the classically allowed region and interpolates between the two branches. A straightforward calculation shows the scalar curvature evolves as

$$R = 16e^{2\phi}/(1 + |\kappa|e^{2\phi}/2)^3, \quad (12)$$

which vanishes at past and future infinity at which  $\phi \rightarrow \pm\infty$ . Hence, the classical singularity at  $\tau = 0$  is now completely erased out and the inflation has ended gracefully! The  $\kappa$  dependence of the curvature clearly indicates that the graceful exit is a quantum mechanical effect.

Similar conclusion is reached for the quantum corrected string vacua in four dimensions. The scalar curvature again vanishes at asymptotic past/future infinity but approaches a finite positive maximum at the fixed point moment  $\tau = 0$ . We thus conclude that quantum corrected string theory resolves the classical singularity and exit super-inflation gracefully.

#### IV. POWER SPECTRA OF DENSITY AND GRAVITATIONAL WAVES

Further indication that the quantum back reaction is an essential element for a successful string inflation comes from the constraints of the large-scale observational cosmology.

As emphasized above, if the quantum back reaction effect is ignored, the cosmic horizon shrinks with time during the super-inflation epoch. The shrinking horizon amplifies quantum fluctuations parametrically as they are stretched outside the horizon. Because of this effect, it is expected that the power spectra of primordial scalar and tensor perturbations are enhanced characteristically to higher frequency than the spectra for de Sitter or power-law inflations. Explicit calculations (Brustein et al., 1995, Hwang, 1996) have confirmed this expectation. The power spectrum at the moment of re-entrance inside the horizon  $aH|_{\text{HC}} = k$  during matter-dominated epoch is given by

$$P(k, t_{\text{HC}}) = \frac{(aH)^4 |\delta(k)|^2}{2\pi^2 k}. \quad (13)$$

Here  $|\delta(k, t)|^2 \equiv A(t)k^n$  denotes the conventionally normalized power spectrum of the density contrast  $\delta \equiv \delta\rho/\rho_0$ . Up to logarithmic corrections, the spectral index is found to  $n = 4$ , hence, strongly tilted to higher frequency modes. This should be contrasted to the observationally supported near-Harrison-Zeldovich spectrum  $n \approx 1$ . Density perturbation with such a high spectral index is problematic to seed the large-scale structure formation. For instance, consider the temperature fluctuation of the cosmic microwave background radiation  $\delta T(\Omega_2)/T = \sum_{l,m} a_{lm} Y_{lm}(\Omega_2)$  induced by the scattering off the gravitational potential perturbations. Power spectrum of the  $l$ -th spherical mode is

given by

$$|a_l|^2 = \pi \int_0^\infty \frac{dk}{k} \left( j_l\left(\frac{2k}{H_0}\right) \right)^2 P(k). \quad (14)$$

For the spectral index  $n = 4$  the short wavelength contribution is so large that Eq.(14) diverges for all  $l$ . Recent COBE observations clearly contradicts this, hence, seems to rule out the string super-inflation as a seed for the large-scale structure formation.

However, the above calculations have not taken into account of the important quantum mechanical effects to the dynamics of the horizon during inflation. Especially, since the relevant density perturbations at the present large-scale observations have left the horizon near the very end of the super-inflation epoch, the quantum back reactions should have become significant by then. Eq.(12) shows that the back reaction tends to retard the rate the horizon shrinks. It is now easy to understand that any change of horizon dynamics affects directly the shape of the power spectra.

The super-inflation has started essentially classically initially. Therefore low-frequency fluctuations generated during earlier stage should show the characteristic  $n = 4$  spectral index. On the other hand, toward the end of inflation, the back reaction has slowed down significantly the rate the horizon shrinks to the point  $\dot{H} \rightarrow 0$ , hence, evolves de Sitter-like essentially. Therefore the spectral index of higher frequency quantum fluctuations that have left the horizon at this later stage should be close to that of the de Sitter inflation, viz.,  $n \approx 1$ . The crossover from the classical, low-frequency regime to the quantum mechanical, high frequency regime takes place at some intermediate scale  $k = k^*$ , and is model-dependent. As a result, the fully quantum corrected power spectra of density perturbation should exhibit frequency-dependent spectral index  $n(k \ll k^*) \approx 4$ ,  $n(k \gg k^*) \approx 1$  which interpolates monotonically between the two limits. Consequently the quantum effects keep CMBR partial wave power spectra Eq.(14) from diverging at high frequency.

The gravity wave power spectrum can be calculated in a similar manner and exhibit classically a strongly tilted spectra with  $n = 4$ . Again quantum back reaction will curb down the spectral index at higher frequency regime, hence, the actual gravitational wave signal would not be as strong as what the classical power spectra shows.

The above discussions clearly point to the importance of calculating fully quantum corrected power spectra for density and metric perturbations. In addition, stochastic dynamics (Rey, 1987) of inflaton during the super-inflation exhibits distinct non-Gaussian signals from the de Sitter or power-law inflations. A full exposition of these calculations will be reported elsewhere.

In this talk I have summarized basic features of the string inflationary cosmology. I have emphasized that an essential element to string cosmology is the

full-fledged quantum back reaction effect. The features should confront present and future observations and experiments. The prospect is quite exciting for both string theorists and observational cosmologists. For string theorists, observational cosmology offers the first direct observation of relic cosmological string effects. For observational cosmologists, string theory offers the first natural model of inflationary cosmology and unique signature of relic density and gravitational waves.

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