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CAUSTICS AND GRAVITATIONAL FOCUSING

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ABSTRACT

When we follow the lines of the trajectory of photons which intersect the circle, the circle may suffer some deformation as approaching to the observer. We consider an infinitesimal light bundle suffering gravitational bending. We examine the deformation of the deflected light bundle due to the gravitational lens. The size of the deformation is expressed in terms of the focal length of the gravitational lens.

I. INTRODUCTION

The doubly imaged Q0957+561 A and B were discovered and identified as a gravitational lensed image by Walsh, Carswell and Weynmann in 1979 (Walsh et al. 1979). Since then many efforts have been made to detect gravitational lens systems in various wavelengths ranges.

During the last decade, several tens of new gravitational lensing system have been found and confirmed(Surdej 1993, other references therein). Theoretical modelling and analysis on the recently discovered lensing objects have been intensively carried out. In a consequence, it has been confirmed that gravitational lens effects may provide us various important astrophysical and cosmological information.

The observable effects of gravitational lensing are very closely related to optical properties of the lens. We, therefore, would like to discuss the effect of propagation of a small circle perpendicular to the light beam traveling from the source to the observer. We restrict our discussion to gravitational focusing with a compact source acting as a point source at infinite distance.

II. GRAVITATIONAL FOCUSING

(a) The lens equation

Gravitational bending of photon trajectory leads to gravitational lensing effects. When a deflector lies at the distance D_{od} acting as a gravitational lens. Light from a distant source will deflect toward the centre of the gravitational lens as passing nearby it. The observable effects due to such a gravitational bending are referred to be the gravitational lens effects. To analyze the gravitational lensing we need the set of the transformation equations which describe the mapping from the deflector plane to the observer plane.

When there is no deflection due to gravitational potential field, the normalized equations for the impact parameter at (ξ, η) and the observer at (x,y) are

$$x=\xi,$$
$$y=\eta, \tag{1}$$

within the geometrical optics approximation. With the gravitational deflection, we may write the lens equations as follows:

$$x=\xi(1+\frac{\partial \alpha_\xi}{\partial \xi}D_{od}),$$

$$y = \eta(1 + \frac{\partial \alpha_\eta}{\partial \eta} D_{od}). \tag{2}$$

The right hand side of eq.(2) possesses the first order partial derivatives of deflection angle $\alpha(\xi, \eta)$ which represent the deformation of the circular light bundle.

(b) Focal length

To show the relation between the deformation and the focal length, we for the moment consider a single macro-gravitational lens. We suppose an infinitesimal light bundle from the source, the cross section of which is a unit circle before suffering bending effects. Gravitational lens effect causes deformations of this circular light bundle along the light path. The deformation varies as approaching to the caustic in which the circular bundle forms a line, which we call focal line.

The deformation of the unit circular bundle may be expressed by focal length f (see Fig.1). Then eq.(2) can be rewritten as

$$x = \xi(1 + \frac{D_{od}}{f}),$$
$$y = \eta(1 - \frac{D_{od}}{f}), \tag{3}$$

where D_{od} is the distance between the deflector and the observer, and f, the focal length to the caustic.

By definition, the focal length of the gravitational lens is then the distance between the deflector to the observer (i.e. the point in the observer plane where the cross section of the deflected light bundle approaches (or equal) to null). We may define the limiting value of the focal length as D_{od} for the source at the distance D_{os} from the observer. Also for the incident parallel light bundle from the source, the focal length would be D_{od}.

Focal length of a gravitation lens, in general, could be expressed by deflection angle, such that

$$f = \frac{r(\xi, \eta)}{\alpha(r)},$$
$$f' = \left| \frac{d\alpha(r)}{dr} \right|^{-1}, \tag{4}$$

where $\alpha(r)$ is the deflection angle with an impact parameter $r(\xi, \eta)$; and f', the focal length to the anticaustic.

Let us now introduce the so called γ-parameter, representing the size of the deformation in the ξ-axis and the η-axis (Chang and Refsdal 1979, Chang 1981). The focal lengths may also be given by

$$f = D \mid \gamma_\eta \mid^{-1},$$
$$f' = D \mid \gamma_\xi \mid^{-1},$$
$$D = \frac{D_{od} D_{os}}{D_{ds}}, \tag{5}$$

where D_{os} is the distance between the observer and the source ; D_{ds} , the distance from the deflector to the source .

Let us consider several individual cases. First, we discuss the case of the deflecting galaxy acting as a point mass deflector. The mass as a point gravitational lens is concentrated at one point, such that

$$M(r) = M \tag{6}$$

where r is the impact parameter; $M(r)$, the mass within the impact parameter $r(\xi, \eta)$.

If there is no matter inside the light bundle, the light bundle will be deformed; that is, it diverges in the direction parallel to the planes containing the observer, the deflector, and the source; and converges in the orthogonal direction. At a certain distance - the so-called focal length - this bundle forms a focal line. As is well known, the point mass deflector acts as a perfect astigmatic lens which has two mutually perpendicular focal lines l and l' at the focal distances f and f', respectively (see Fig.1).

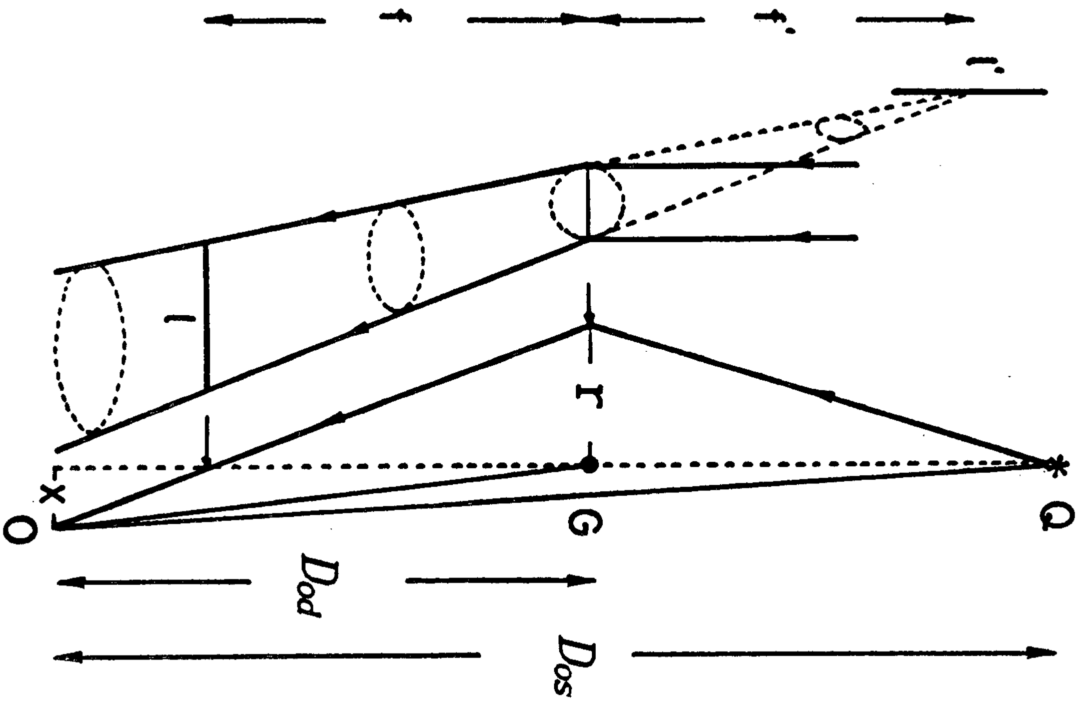

Fig. 1. Forcal length and focal line: When G actiong as a gravitational lens(deflactor), the light bundle from the source Q, the cross section of which is circle in the deflector plane, suffers deformation along the light path. The deformation varies as approaching to the pbserver O (or Q) and forms focal line at the focal length f(or f').

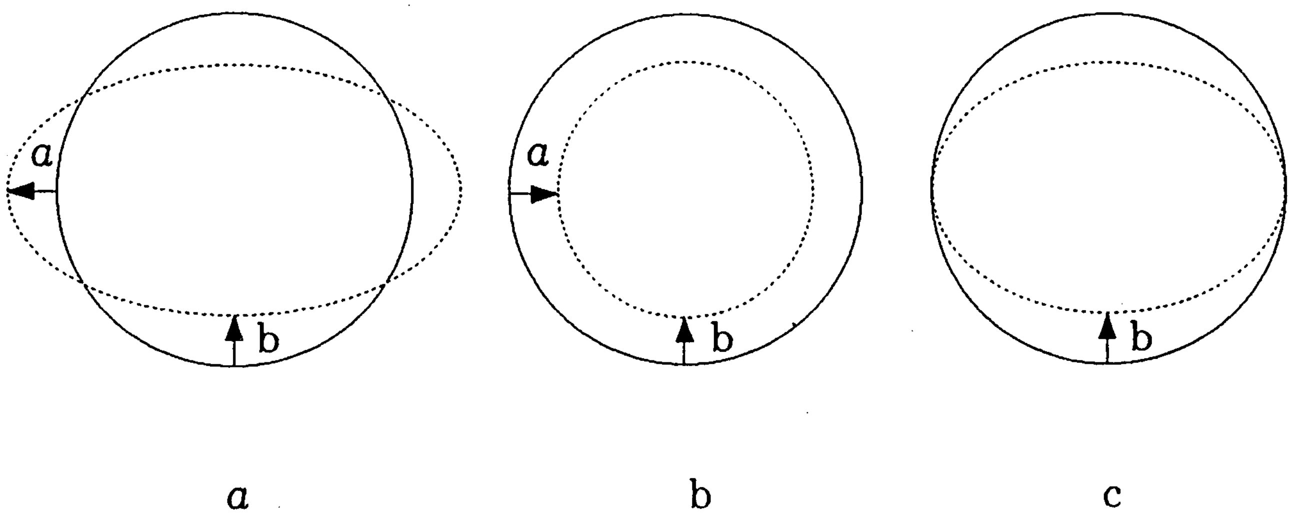

Fig. 2. Size of deformation: The size of deformation a and b are equal to D_{od}/f. (a) point mass gravitational lens, (b) uniform disk gravitational lens, (c) King galaxy model gravitational lens.

The optical properties of such a lens are very similar to those of the foot of glasses of wine. A point mass gravitational lens has no single focal length, but focal length depends on the impact parameter under consideration; i.e.

$$f = \frac{c^2 r^2}{4GM} \tag{7}$$

where M is the mass of the gravitational lens, and other symbols have their usual meaning. A disk form with a constant mass density will act as an optical converging lens. Such a gravitational lens of a uniform disk will make a circular light bundle circular with equally converged cross sectional area in all direction. The focal length of uniform disk is constant, which is given by

$$f = \frac{c^2}{4\pi G\sigma_o} \tag{8}$$

where σ_o is uniform surface mass density. The disk gravitational lens with a uniform mass density uniquely possesses a focal point, such as a optical conversing lens. We now consider a mass of gravitational lens linearly depending on impact parameter, i.e.

$$M(r) \sim r \tag{9}$$

In such a model (the so-called-singular-isothermal-sphere, hereafter SIS) there is no single focal length, but each impact parameter defines its own focal length. Optical properties of this model are those between point mass lens and uniform disk lens. The circular beam may be deformed in tangential direction as well as in radial direction. In some cases, for example, King galaxy model, a circular light bundle suffers a convergence in tangential direction but no change (no deformation) in radial direction.

In more realistic approach, we take a model of gravitational lens as a spiral galaxy. If we suppose that the most galactic matter is concentrated at the central core, then for convenience we may take the core as a focal point. From the focal point the caustic lines are then spreading towards the observer's plane. For such a galaxy, the mass would be defined by

$$M(r) = 2\pi \int_0^r \rho(r')r'dr' \tag{10}$$

This model also has a focal length depending on impact parameters under consideration.

Let us now consider the case of microlens. For simplicity we consider Chang-Refsdal model which describes the effects of a massive galaxy combined with a star lying close to the deflected light ray. The evolution of the caustic is then very chaotic. The simplest caustic is generated when the point mass macrogravitational lens combined with a point mass ($0.6M_\odot < M < 10^6 M_\odot$) acts simultaneously as a gravitational lens. In such a case, the focal point of the macrogravitational lens evolves as the areas surrounded by four branchs of hyper cyclic curves (see for details Chang 1981; Schramm 1985).

III. CAUSTICS AND IMAGES

Caustic lines are defined by the points where the Jacobean determinant of the lens equations vanishes. The image properties strongly depend on the configuration between the position of the source (or the observer) and the caustic lines in the source plane (or the observer's plane).

With the symmetry geometry of gravitational lens system, the observer in general sees the so-called Einstein ring along the critical circular line in the deflector plane. Note that the observer moves away from the symmetry axis Einstein ring will break into two images for the SIS and the point mass gravitational lens. Inside the caustic cone two images always would be seen for the SIS and the point mass lens.

However, with the uniform disk model of gravitational lens the observer will see always one image whether he is inside or outside the caustic cone. The most realistic case is spiral galaxy model, in which observer will see the images inside the caustic cone. Outside the caustic cone the observer in above cases will see only one image.

To estimate the number of all lensed images is not the simple problem. There are only couple of gravitational lens models provide us analytical expressions to determine the number and the position of all the lensed images, for example Chang-Refsdal lens(Chang and Refsdal 1979, 1984 ; Chang 1981), a single lens as a galaxy (Bourassa and Kantowski 1975), two-body gravitational lens (Bartels 1981).

Therefore, in general, numerical procedure is a preferable method in order to find all the solutions of the lens equations. For instance, spheroidal model of gravitational lens may produce a maximum number of images upto seven images (Bourassa, Kantowski, and Norton 1976).

Above all, the number and the position of lensed images must be obtained by solving the inverse mapping. The other difficulty, however, is to determine which solution is responsible to observable image. It is worthwhile once again to mention that the optical properties of the lensed images are of strong dependence on the configuration of caustics in the observer's plane (or the source plane). Some examples are presented in Fig 3.

IV. DEFORMATION AND ORIENTATION

The deformation of the circular light bundle due to the gravitational being also determined by means of the eigen value of the transformation matrix of the gravitational lens equations. This also may be expressed by the Dehnungsmatrix

$$A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \tag{11}$$

The element of A depends on the focal length of the given lens model, as well as on the deflection angle. In most cases, the absolute value of the element a is equal to that of b. That is ,

$$| a |=| b |= D\frac{1}{f} \tag{12}$$

where D is given eq.(5). In eq.(11) a and b are the size of the deformation of a unit circular light bundle in the radial- and in the tangential-direction, respectively. Let us return to Chang-Refsdal lens problem, i.e. the two deflector with a large mass ratio. With some mathematical derivation, we obtain the elements of the Dehnungsmatrix as follows :

$$\begin{pmatrix} a \\ b \end{pmatrix} = \pm\sqrt{\gamma^2 + 2\gamma\frac{\xi^2 - \eta^2}{(\xi^2 + \eta^2)^2} + \frac{1}{(\xi^2 + \eta^2)^2}} \tag{13}$$

For the type of Chang-Refsdal lens, the focal lengths are then

$$f = \frac{D}{| b |}, \qquad f' = -\frac{D}{| a |}. \tag{14}$$

The orientation of the deflected circular light bundle is also determined by the eigen vector of the Dehnungsmatrix. With using the lens equations and the orientation matrix, the orientation angle would be given as

$$\varphi = \frac{1}{2}\tan^{-1}\left(\frac{\frac{\partial x}{\partial \eta} + \frac{\partial y}{\partial \xi}}{\frac{\partial x}{\partial \xi} - \frac{\partial y}{\partial \eta}}\right) \tag{15}$$

The orientation angle φ is measured counter clockwise from the radial axis of the deflected light bundle.

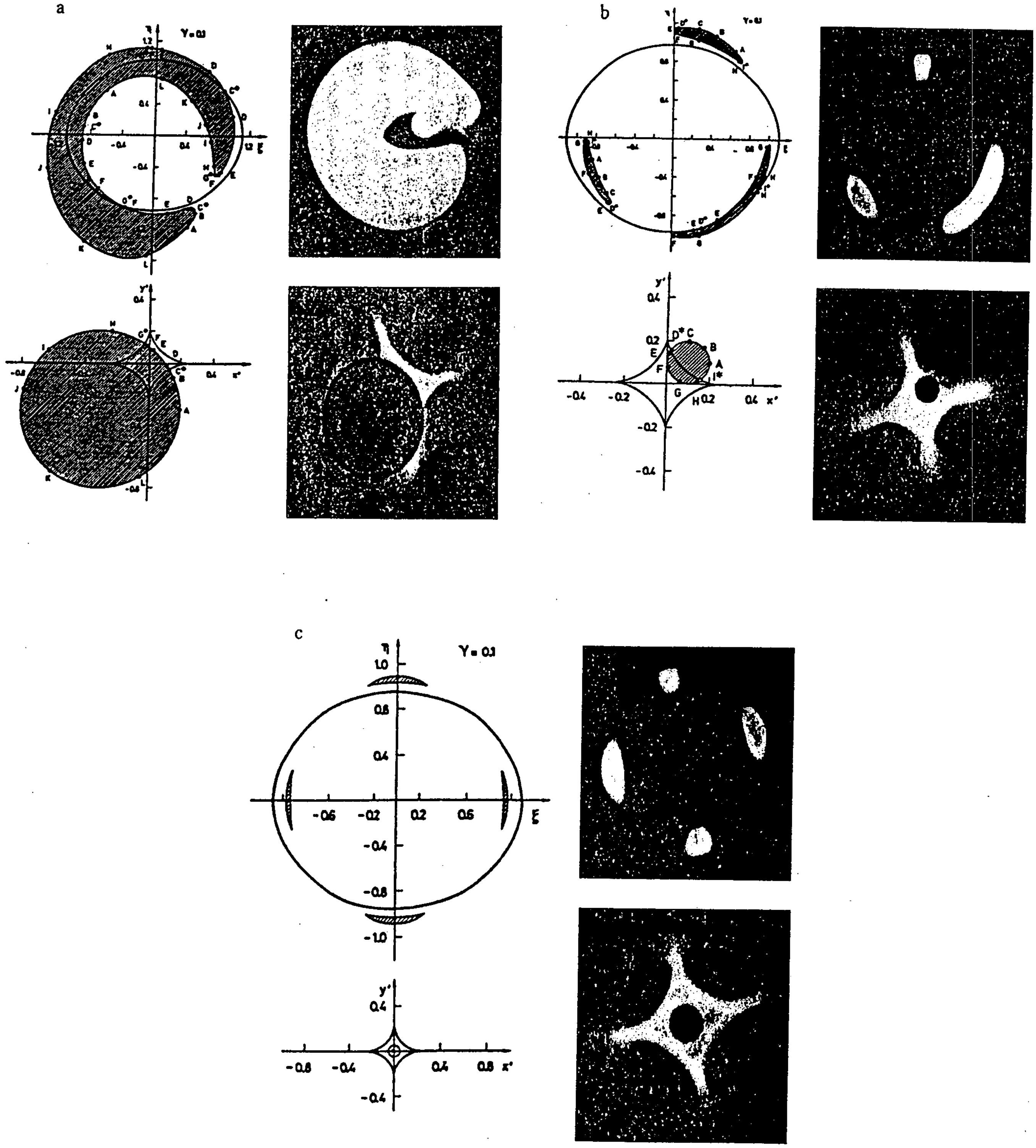

Fig. 3. Image configuration as source crossing the caustic: Left hand side in a -c shows the image obtained analytically (courtesy of Chang(1981)). Right hand side in a -c shows the image obtained by optical gravitational lens experiment. The position of the extended source is expressed by shaded (or dark) circular disk. The optical gravitational lens has been manufactured by Hamburger Sternwarte Technical Labouratory, which is designed by Dr. Schroeder based on Chang-Refsdal lens. One can easily see the diamond shape of caustic for the case of $\gamma-$ parameter less than 1.

Table 1. Gravitational Lens Model and Focal Length

Gravitational Lens Model	Focal length	Deflection angle
Point mass	$\frac{c^2 r^2}{4GM}$	$\frac{4GM}{c^2 r^2}; M = M(r)$
Transparent uniform disk	$\frac{c^2}{4\pi G\Sigma_o}$	$\frac{4\pi G\Sigma_o r}{c^2}$
SIS	$\frac{4\pi\sigma^2}{c^2 r^2}$	$\frac{4\pi\sigma^2}{c^2} = const.$
Spiral galaxy	$\frac{c^2 r^2}{4GM(r)}$	$\frac{4GM(r)}{c^2 r^2}; M(r) = 2\pi \int_0^{r'} \rho(r) r dr$

σ : one component observable velocity dispersion of galaxy
Σ_o : Surface mass density of galaxy
r : impact parameter
$M(r)$: mass of the deflector within the radius of the impact parameter r
M : Total mass of the deflector

V. CONCLUDING REMARKS

Since 1979, it has been known more than 20 proposed cases of multiply imaged distant source. There are also several tens of giant arcs and arclets. The two major configurations are two and four images. In the two image configuration the lensing galaxy is used to be at the centre of two images.

For the case of four image configurations the lensed images are located, as if the images surround the lensing galaxy. The galaxy is used to be at the centre of the images. The image configurations, such as, four-image, two-image, arclet, or arc are closely related to morphology of the caustic lines (critical lines) as shown in Fig.3 (for details see Chang 1981).

We conclude, therefore, that it is important to examine thoroughly the caustic structure in order to obtain the information on the lensing galaxy and the source structure, as well. We note that for any given pair of light source and compact deflecting objects in the universe, there exists a two dimensional caustic of light and that an observer located very near to it will actually see the distant source subject to the gravitational lensing effects. Such a case observer will actually see the distant source amplified and usually see also multiple imaged.

We need above all a continuous monitoring in deep multi-wavelength (optical, radio, x-ray, gamma, etc) for long period to determine any significant astrophysical information. Speckle imaging and adoptive optics would at best dedicate to determine the luminous arc and dark matter distribution in the lens as well as the physical structure of the source, for example, the size of the continuum, the emission line region, the radio jet, etc. In next generation, the gravitational lens would definitely offer us better understanding of the universe.

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