

# A Stochastic Cost-Volume-Profit Approach to Investment Risk in Advanced Manufacturing Systems

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## Abstract

Conventional discounted cash flow techniques fail to capture the risk associated with investments. This paper proposes an annual cash flow model that considers risk, cost structure and inventory liquidation in the evaluation of investment alternatives. The risk differential of investments is included using the capital asset pricing model while the stochastic version of the cost-volume-profit approach is used to consider inventory liquidation and cost structure. Tradeoffs between fixed and variable costs have been investigated, and portrayed using iso-cash flow curves. The proposed cash flow model has been developed, in particular, to enable an accurate evaluation of advanced manufacturing systems.

## 1. Introduction

Conventional discounted cash flow techniques (DCF) are widely used to evaluate investment opportunities. DCF employs the concept of *time value of money*, and "actual" cash flows. Despite being straightforward and theoretically reasonable, some deterrents exist in the deterministic DCF approach in relation to the economic assessment of advanced

manufacturing technologies (AMTs)

The cost structure of AMTs is quite different from that of traditional manufacturing systems (TMSs). AMTs typically require a large initial outflow, and hence have a large break-even point and long payback period. Investment in AMTs is, therefore, generally regarded as a risky venture. AMTs, however, require less expenses in subsequent years of use, whereas TMSs rely heavily on direct labor and involve

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a large proportion of operating costs. Operating costs usually consist of fixed costs (repair, maintenance, factory overheads, etc) and variable costs (direct material and labor). Annually, AMTs tend to require less fixed costs and less unit variable cost than TMSs.

AMTs often involve process automation which helps to streamline human control by enhancing operational flexibilities. This in turn may lead to a high utilization rate, and an appreciable decrease in annual operating costs which is primarily due to a decline in the unit variable cost. The cost structure of AMTs can provide added flexibility in terms of responsiveness to demand variations. When demand is uncertain, the lesser variable cost requirement could significantly reduce the losses incurred due to excess production. AMTs may thereby reduce the risk due to demand variations, and could generate more income (profit) than TMSs. AMTs thus have the potential to absorb demand fluctuations more effectively owing to their flexibility and low operating costs, and would be less risky on the long run which is a key aspect to their justification.

In DCF, annual cash flows are the main determinants of an investment. Many suggestions have been proposed to resolve the limitations of DCF with regard to justifying AMTs [1, 8, 12, 15]; these, however, do not address the risk differential of investment options. The risk associated with an individual investment is usually incorporated, in DCF, into a discount rate. However, the same

discount rate is used for all alternatives and, thus, DCF implicitly assumes the same degree of risk for all competing investments. Due to their unique cost structure, AMTs may have different sensitivity to risk (future uncertainty). The risk associated with an AMT alternative may have profound effects on investment decisions. The objective of this paper is to expand the DCF approach to include the different cost structure and risk levels associated with investment choices.

This paper is organized as follows: the next section discusses some of the issues in considering the risk and cost structure associated with investments in DCF, and reviews some relevant literature. Section 3 presents the features of our proposed cash flow models by including risk, cost structure and inventory liquidation. Mathematical expressions to determine the optimal production volume and annual cash flows are given. Section 4 discusses the nature of the tradeoff between fixed and variable costs, and illustrates the use of the proposed model via a numerical example and a case study. Section 5 offers some general conclusions and a brief summary of our proposed approach.

## 2. Investment Risk and Cost Structure

For a prudent and accurate analysis, the different degrees of risk and cost structure associated with the competing alternatives must be considered in the evaluation. The risk

involved in investment options can be included, in DCF, in two ways. One way is to incorporate the risk in terms of a discount rate. However, it is not easy to estimate the discount rate involving risk. Also, different discount rates may have to be employed to portray the different degrees of risk associated with investment alternatives - it is quite difficult to estimate these discount rates. The other approach is to include the differences in the degrees of risk into the cash flow. Risk can be incorporated into the cash flows in three different ways, namely probabilistic cash flow measures [7, 14], utility theoretic approaches [3] and capital asset pricing model (CAPM) [5, 11].

Probabilistic measures may require certain unrealistic assumptions regarding the distribution of cash flow elements and thus may not provide a meaningful result, while utility based approaches, though conceptually superior, are difficult to apply in practice. On the other hand, CAPM assumes a common, pure (risk-free) rate of return (ROR), and homogeneity of investor expectations [10, 13]. As discussed earlier, AMTs may exhibit a different sensitivity to risk, than that of TMSs, due to their different cost structure. Since we need to consider the different risk levels associated with investments, the CAPM approach appears to be useful; in doing so, we propose a risk-free ROR for discounting, and the use of *certainty equivalents* of annual cash flows. The publicly reported risk price can be used to derive the

certainty equivalent of risky cash flows (via the risk-return tradeoff relationship of CAPM). Since our intent is primarily to investigate the effects of including risk, we employ *before-tax* cash flow analysis - for simplicity and ease of computation, income taxes are not considered.

We shall use the *stochastic* version of the *cost-volume-profit approach* (CVP) [7]; sometimes called *break-even analysis*, this is a useful tool for short-term profit analysis when both cost structure and production volume determine operating profits. Several researchers have studied the stochastic CVP; many, however, have supposed that inventory has little or no influence on cash flows under stochastic demand. Most studies have assumed either that (a) inventory does not exist or even if it does, it can be sold *without* loss [4, 6, 7, 9], or that (b) inventory has *no* salvage value (newspaper boy problem) [2, 11, 14]. These assumptions may lead to over- or under-estimation of the realized profit. This paper employs a stochastic CVP relationship that considers *end of period* inventory liquidation. This assumption is valid for perishable goods and products subject to obsolescence. Also, this type of liquidation occurs frequently in the "real" world; for instance, in the automobile market, this year's model(s) should be sold by the end of the year.

### 3. Proposed Annual Cash Flow Model

Below, we present a mathematical model for

cash flow analysis, using the CVP relationship, to investigate the effects of risk and cost structure on cash flows. We use the annual cash flow model for multi-period capital budgeting that considers stochastic demand, cost structure of investments, and end of period inventory liquidation. The following notations will be used in the proposed DCF model.

- $Z$  is the net annual cash flow,
- $Z_{ce}$  is the certainty equivalent of net annual cash flow,
- $D$  is the demand,
- $D_{ce}$  is the certainty equivalent demand,
- $x$  is the production volume
- $p$  is the unit price,
- $f$  is a fraction less than 1, possibly negative and decreasing over  $x$ ,
- $v$  is the unit variable cost,
- $h$  is the holding cost rate per unit,
- $F$  is the fixed cost,
- $R_f$  is the risk-free ROR,
- $E(R_m)$  is the expected ROR on the market portfolio,
- $E(R)$  is the expected ROR on an investment,
- $\delta$  is the market price per unit of risk, and is given by  $[E(R_m) - R_f] / \delta_m^2$ ,
- $\beta$  measures the sensitivity of individual asset's ROR to market ROR, and is given by  $COV(R, R_m) / \delta_m^2$  (*systematic risk*)

### 3.1 Stochastic Demand and Inventory Liquidation

Under stochastic demand, the net annual cash

flow can be expressed using the following CVP relationship.

$$Z = \begin{cases} p(1-f)D + (fp-v)x - F, & \text{if } D \leq x, \\ ((p-v)x - F), & \text{if } D > x. \end{cases} \quad (1)$$

The price per unit,  $p$ , variable cost,  $v$ , and fixed cost of production,  $F$ , are assumed to be known. Demand is the only uncertain random variable. Any inventory resulting from surplus production is liquidated at a reduced price,  $fp$ ; the fractional price,  $f$ , is a decreasing function of production volume, and includes holding costs. We assume no loss or penalty due to production shortage (unmet demand).

### 3.2 Risk Resolution Using Market Certainty Equivalent

In a typical DCF analysis, we consider the riskiness of an investment held in isolation. However, an investment held as part of a corporate investment portfolio is less risky than the same investment held in isolation. This fact has been incorporated into a generalized framework for analyzing the relationship between risk and rates of return; this framework is called the capital asset pricing model, or CAPM [13].

The random risk associated with an individual investment can be overcome by balancing two types of assets (or portfolios), one a risk-free asset ( $R_f$ ) and the other a risky asset ( $R_m$ ). Note that the asset could be a single investment or a portfolio. By combining the two, the investor can achieve a higher expected

return. This relationship of expected return for the combination can be expressed, using CAPM [10,13], as

$$E(R) = R_m + \beta[E(R_m) - R_f] \quad (2)$$

The following expression, derived from CAPM, can be used to obtain the market certainty equivalent of net annual cash flow [5, 9, 11]

$$Z_{ce} = E(Z) - \delta COV(Z, R_m) \quad (3)$$

The certainty equivalent of net annual cash flow can be derived using expressions (1) and (3) (see Appendix A for derivation). No specific statistical distribution is assumed. We may write  $Z_{ce}$  as

$$Z_{ce} = A - B \quad (4)$$

where

$$A = p(1-f)[E(D) - \delta COV(D, R_m)] + (pf - v)x - F, \quad (5)$$

$$B = p(1-f)C,$$

$$C = \int_x^\infty (D-x)g_D(D)dD - \delta \left[ \int_{R_m} R_m \int_x^\infty (D-x)g(D, R_m) dD dR_m - E(R_m) \int_x^\infty (D-x)g_D(D)dD \right], \quad (6)$$

$g(D, R_m)$  = joint p.d.f. of  $D$  and  $R_m$   
 and  $g_d(D)$  = marginal p.d.f. of  $D$

From Eq. (3), the following relationship holds.

$$D_{ce} = E(D) - \delta COV(D, R_m) \quad (7)$$

Now,  $A$  of Eq. (4) can be written as

$$A = pD_{ce} + pf(x - D_{ce}) - vx - F. \quad (8)$$

Thus,  $A$  denotes the net annual cash flow under demand certainty. Without loss of generality, the fractional price,  $f$ , can be taken as unity when production shortage occurs ( $x < D_{ce}$ ).

As shown in Appendix B, Eq. (6) can be rewritten as

$$C = E(L) - \delta COV(L, R_m) = L_{ce}$$

$$\text{where } L = \int_x^\infty (D-x)g(D|R_m)dD$$

and  $L_{ce}$  = certainty equivalent of  $L$ .

Since

$$E(L) = \int_x^\infty (D-x)g_D(D)dD$$

and  $p(1-f)E(L)$  is the expected loss due to unmet demand,  $B$  is the certainty equivalent of the loss. The certainty equivalent of annual cash flow can be obtained as

$$\begin{aligned} Z_{ce} &= \text{Net Cash Flow under Demand Certainty-Certainty Equiv. Loss} \\ &= A - B \\ &= pD_{ce} + pf(x - D_{ce}) - vx - F - p(1-f) \end{aligned} \quad (9)$$

$$\int_x^\infty (D-x)g(D|R_f)dD$$

### 3.3 Optimal Production Volume

From the annual cash flow model given in Eq. (9), we derive the optimal production volume and optimal cash flow. Let the fractional price,  $f$ , decrease linearly as

$$f = \begin{cases} 1, & \text{if } x \leq D_{ce} \\ -(1+h) \frac{(x-D_{ce})}{(x_{max}-D_{ce})} + 1, & \text{if } D_{ce} < x \leq x_{max} \\ -h, & \text{if } x > x_{max} \end{cases} \quad (10)$$

When production shortage occurs, the price is unaffected. We assume a maximum limit,  $x_{max}$ , on production volume under which inventory is sold at a fractional price. Above this limit, inventory has no salvage value, and leads only to holding costs,  $h\phi$ . This limit can be determined from historical sales data, or from management's judgement on maximum sales.

The certainty equivalent of net annual cash flow, under linearly decreasing inventory salvage value, can be obtained by combining expressions (9) and (10).

$$Z_{ce} = \begin{cases} (\phi-v)x - F, & \text{if } x \leq D_{ce} \\ (\phi-v)x - F - \phi \frac{(1+h)}{(x_{max}-D_{ce})} [(x-D_{ce})^2] \\ + (x-D_{ce}) \int_x^\infty (D-x)g(D|R_f)dD, & \text{if } D_{ce} < x \leq x_{max} \\ \phi D_{ce} - \phi h(x-D_{ce}) - vx - F \\ - \phi(1+h) \int_x^\infty (D-x)g(D|R_f)dD, & \text{if } x > x_{max} \end{cases} \quad (11)$$

Note that, as production volume reaches demand certainty, the cash flow increases

linearly after which the function is concave until  $x_{max}$ . Furthermore, above production volume  $x_v$ , where fractional price equals variable cost  $v$ , the function values are lesser than those when production volume equals demand certainty ( $D_{ce}$ ). We need only investigate the region  $[D_{ce}, x_v]$  to find the optimal production volume. The volume at which the fractional price equals  $v$  can be written as

$$x_v = (\phi - v) \frac{(x_{max} - D_{ce})}{\phi(1+h)} + D_{ce} \quad (12)$$

Since  $x_v < x_{max}$ , the function remains concave in the region  $[D_{ce}, x_v]$ . The condition for optimality can be established, from the derivative, as

$$2(x - D_{ce}) + \int_x^\infty (D-x)g(D|R_f)dD - (x - D_{ce}) \int_x^\infty g(D|R_f)dD = (\phi - v) \frac{(x_{max} - D_{ce})}{\phi(1+h)} \quad (13)$$

Since the derivative is monotonically decreasing in the region  $[D_{ce}, x_v]$ , the optimal production level that satisfies Eq. (13) can be obtained by numerical solution. If no such  $x$  exists,  $Z_{ce}$  at  $x = D_{ce}$  is the optimal.

### 4. Tradeoff Between Fixed and Variable Costs

A numerical example is presented to illustrate the use of the proposed annual cash flow model. Technology investments can be perceived as a tradeoff between fixed and variable

costs which can be used as a comparative measure when evaluating investment opportunities. Through sensitivity analysis, we investigate the effect of fixed and variable cost tradeoff on annual cash flows.

#### 4.1 Numerical Example

Product data, in the form of price, costs and demand, and market data are given in Table 1. The risk premium of investments is found using market data. For analytical simplicity, the joint distribution of demand,  $D$ , and market return,  $R_m$ , is assumed to be bi-variate normal (it can be any distribution, though). Figure 1 depicts the shape of the cash flow function for this numerical example, and represents the typical shape assumed by Eq. (11). To find the optimal production volume, the range  $[D_{ce}, x_v]$  within which it exists is first established. Using expressions (7) and (12), this range was found to be  $[111,905 \ 151,948]$ . The optimal production volume can be found from Eq. (13). However, since the equation is not a closed form expression of production volume, a numerical analysis was required, and the optimal production level was obtained as  $x_0 = 136,680$  units. A commercial software, MATLAB, was used for numerical analysis. From Eq. (11), the optimal certainty equivalent of net annual cash flow is \$228,520 ( $Z_{ce}$ )

In Figure 1, note that, as production volume increases, so does the annual cash flow upto a production level of  $x_0 = 136,680$  units. For

Table 1. Data for Numerical Example

Production Data		Market Data	
$p$	\$10	$E(R_m) - R_f$	8.5%
$v$	\$4	$\delta m$	21.0%
$F$	\$500,000	$R_f$	6.0%
$h$	0.1	$Corr(D, R_m)$	0.5
$x_{max}$	200,000	$E(D)$	120,000
		$\delta D$	40,000

any additional production volume, the annual cash flow decreases

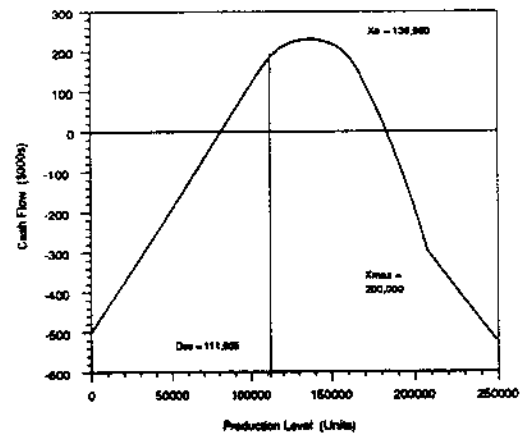


Figure 1. Cash Flow Plot for Numerical Example

#### 4.2 Iso - Cash Flow Curves

To investigate the relationship between fixed and variable costs, we develop tradeoff curves named as *iso-cash flow curves* which define the set of fixed and variable cost combinations that give the optimal cash flow. Thus, these denote the assortment of fixed and variable costs that yield the same net annual cash flow. Since the optimal production volume is virtu-

ally independent of the fixed cost, iso-cash flow curves can be obtained by finding the optimal cash flows for various levels of variable cost. The fixed cost, then, is found by adjusting for differences in the optimal cash flows. Figure 2 shows the iso-cash flow curves for the numerical example.

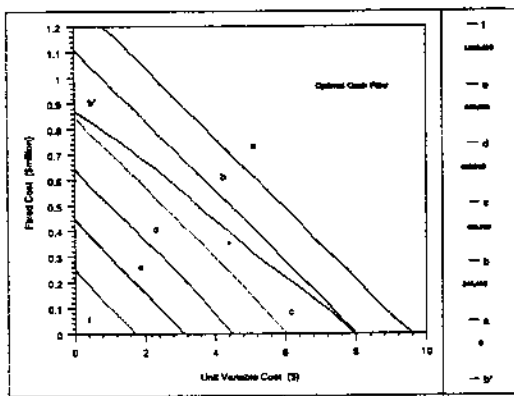


Figure 2. Iso-Cash Flow Curves for Numerical Example

Suppose, for instance, the unit variable cost is known to be \$6 for a production volume of 400,000 units. Then the optimal investment in fixed cost would be about \$0.85 million as indicated by the iso-cash flow curve “c” in Figure 2. A fixed and variable cost combination that lies below an iso-cash flow curve yields higher cash flows than the ones that lie above it. Investment alternatives can be contrasted by comparing their relative position in the iso-cash flow curves. The iso-cash flow curves plummet with increase in variable costs; however, their decreasing pattern is atypical and varies in relation to a variable cost,  $v_{max}$ ,

where

$$v_{max} = p - \frac{p(1+h)}{(x_{max} - D_{ce})} \int_{D_{ce}}^{\infty} (d - D_{ce})g(D|R_f)dD$$

Here,  $v_{max}$  is the variable cost above which the certainty equivalent demand always equals the optimal production level. When the variable cost exceeds  $v_{max}$ , the tradeoff exactly follows that of conventional CVP relationship, and the curves dip linearly. But, when  $v < v_{max}$ , the curves subside with a decreasing rate. In the context of technology investments, this concavity can be interpreted as the leverage to incur more as fixed costs in order to reduce the variable costs. The leverage gained is more than that alluded to by the linear tradeoff relationship of conventional CVP.

To illustrate, consider the iso-cash flow curves  $b$  and  $b'$  both of which yield the same annual cash flow of \$200,000. Curve  $b$  is through our proposed method while  $b'$  is due to conventional CVP tradeoff. When variable costs are \$2 per unit, we can annually spend \$115,018 more as fixed costs than the amount allowed with conventional tradeoff (see Table 2). For an economic life of 10 years and discount rate of 6%, we can spend  $\$846,543 = \$115,018(P/A, 6\%, 10)$  more for initial investment than expected.

Since technology investments usually require a large initial investment and since they generally lead to a significant reduction in the variable cost (over that of TMSs), allocating more money to initial investments appears to



**Table 2. Fixed and Variable Cost Tradeoff (\$)**

<i>Variable Cost</i>	2	4	6	8
<i>Fixed Cost (b)</i>	810,258	528,515	265,297	24,103
<i>Fixed Cost (b')</i>	695,240	471,430	247,620	28,810
<i>Difference</i>	115,018	57,085	17,677	293

be justifiable in AMTs.

### 4.3 Case Study

Here, we extend the proposed model for multi-period capital budgeting to evaluate an AMT case example. Along with the certainty equivalent of cash flows found from the proposed model, we use a risk-free ROR as the discount rate for capital budgeting (since the risk factor can be resolved through our proposed model). Once again, we assume end of period inventory liquidation: if inventory carryover is plausible, an inventory balance equation should be included. Also, if the investment will affect multiple products, demand needs to be aggregated thus reducing the evaluation to a single-product instance.

A case study from literature [9] is analyzed to illustrate the use of our cash flow model for investment evaluation. Two mutually exclusive alternatives, namely an AMT and a TMS option, are evaluated. Table 3 summarizes the data pertinent to these options. The same market information, as given in Table 1, is employed. We assume both investments have

**Table 3. Cost Structure of Investments at a U. S. firm**

<i>Item of Comparison</i>	TMS	AMT
Number of direct workers	35	21
Average in-process time/part	9 weeks	3 days
Average in-process inventory(\$)	260,000	35,000
Finished Goods inventory(\$)	818,000	204,000
Number of part types	3,000	3,000
Average No. of pieces produced	544,000	544,000
Variable labor cost/part(\$)	2.15	1.30
Variable material cost/part(\$)	1.53	1.10
Total variable cost/part(\$)	3.68	2.40
Annual overhead costs(\$)	3,150,000	1,950,000
Annual tooling costs(\$)	470,000	300,000
Annual inventory costs(\$)	141,000	31,500
Annual fixed operating costs(\$)	3,760,000	2,280,000
Incremental investment(\$)	0	7,500,000

Source : Lederer and Singhal(1988)

a 7 year service life and zero salvage value. Demand increases linearly over the first 5 years from 200,000 to 600,000 units, and remains constant thereof (product maturity). For the AMT option, there is a decrease in variable costs due to a reduction in direct material and labor costs; the lead time is shorter as well. Note that the AMT is also attractive in terms of annual overheads, fixed costs, tooling costs and inventory charges. The only visible demerit is that it requires a huge initial investment.

**Table 4. Results of Stochastic CVP Analysis**

Year	TMS Option		AMT Option	
	CF(\$)	$x_o$ (units)	CF(\$)	$x_o$ (units)
0	0	-	(7,500,000)	-
1	(2,065,014)	220,536	(299,690)	224,940
2	(1,233,014)	320,536	660,310	324,940
3	(401,014)	420,536	1,620,309	424,940
4	430,986	520,536	2,580,309	524,940
5	1,262,985	620,536	3,540,309	624,940
6	1,262,985	620,536	3,540,309	624,940
7	1,262,985	620,536	3,540,309	624,940
NPV	(366,733)		3,705,049	

$x_o$  is the Optimal Production Volume; CF denotes Cash Flow; Numbers enclosed in brackets are negative

**Table 5. Incremental Analysis of AMT**

over TMS(\$)

Year	$\Delta CF$	Cause for $\Delta$ (B)		$\Delta CF$ (C)	AMT CF inc.
	(A)	Fixed	Variable	over Conv.	incl. risk(D)
1	1,765,323	1,480,000	285,323	39,685	138,024
2	1,893,323	1,480,000	413,323	39,685	138,024
3	2,021,323	1,480,000	541,323	39,685	138,024
4	2,149,323	1,480,000	669,323	39,685	138,024
5	2,227,323	1,480,000	797,323	39,685	138,024
6	2,227,323	1,480,000	797,323	39,685	138,024
7	2,227,323	1,480,000	797,323	39,685	138,024
NPV				\$221,537	\$770,501

We obtain the annual cash flows and optimal production volumes using expressions (11) and (13) respectively (see Table 4). Note that the annual cash flows of the AMT investment are larger, and increase more rapidly with time.

Table 5 illustrates the results of Table 4 in more detail. Column A denotes the incremental cash flows of the AMT option over the

traditional investment, while column B classifies the cause for this difference. The cash flow increments rise with demand. It is clear that the cause for this increase is the smaller variable cost requirement of the AMT; as production volume increases, this distinction becomes more pronounced. The increase in the optimal production level coincides with the presumption that as variable costs decrease, production volume can be increased if sufficient demand exists.

Column C shows the net increase in incremental cash flows using our annual cash flow model instead of the conventional CVP relationship which overlooks risk. Column D shows the increase in the cash flows of the AMT investment using the annual cash flow model. By considering the risk, the present worth of the AMT option increases by \$770,501, a 26.3% increase. The incremental benefit of the AMT investment over the traditional one increases by \$221,537, a 7.5% increase in NPV.

## 5. Summary and Conclusions

AMTs have a cost structure that is unlike TMSs, and typically require a large initial investment. This large outflow, however, aids in decreasing the variable costs (both material and labor) leading to a net reduction in operating expenses. Due to their different cost structure, AMTs exhibit a different sensitivity to risk. Analyzing AMTs in terms of their

long-term profitability requires the consideration of their cost structure and sensitivity to risk (future uncertainty). A stochastic annual cash flow model that considers risk, cost structure and inventory liquidation was developed. Inventory is inevitable in stochastic ("real" world) situations, and consideration of inventory in an inapt manner can lead to over- or under-estimation of annual cash flows. A certainty equivalent cash flow expression was derived using the risk-return tradeoff relationship of CAPM. An optimality condition was also derived.

To visualize the tradeoff between fixed and variable costs over the competing investment alternatives, iso-cash flow curves were developed. While the tradeoff is linear in conventional CVP relationship, it is concave for stochastic situations signifying that more money can be incurred as fixed costs for a net reduction in variable costs. The concavity tradeoff correlation is a potentially useful tool in evaluating/justifying AMT investments. Finally, a case study was examined to illustrate the workings of the proposed DCF model.

## Appendices

### A. Derivation

Here, we illustrate the derivation of equations (4) through (6) from equation (3). The expected value of the net annual cash flow,  $E(Z)$ , in equation (3) can be written as

$$\begin{aligned} E(Z) &= \int_0^x [\phi(1-f)D + (f\phi-v)x - F]g_D(D)dD \\ &+ \int_x^\infty [x(\phi-v) - F]g_D(D)dD \\ &= (\phi-v)x - F + \phi(1-f)[E(D) - x] - \phi(1-f) \\ &\int_x^\infty (D-x)g_D(D)dD. \end{aligned}$$

Also,

$$\begin{aligned} COV(Z, R_m) &= E(ZR_m) - E(Z)E(R_m) \\ &= \phi(1-f)COV(D, R_m) - \phi(1-f) \left[ \int_{R_m} R_m \int_x^\infty (D-x) \right. \\ &\left. g(D, R_m)dD dR_m - E(R_m) \int_x^\infty (D-x)g_D(D)dD \right]. \end{aligned}$$

Thus,

$$\begin{aligned} Z_{ce} &= E(Z) - \delta COV(Z, R_m) \\ &= \phi(1-f)[E(D) - \delta COV(D, R_m)] + (\phi-v)x - F \\ &- \phi(1-f) \left\{ \int_x^\infty (D-x)g_D(D)dD - \delta \left[ \int_{R_m} R_m \int_x^\infty \right. \right. \\ &\left. \left. (D-x)g(D, R_m)dD dR_m - E(R_m) \int_x^\infty (D-x)g_D(D)dD \right] \right\} \\ &= A - B \end{aligned}$$

### B. Illustration

Let

$$L = \int_x^\infty (D-x)g(D|R_m)dD.$$

Then,

$$E(L) = \int_x^\infty (D-x)g_D(D)dD$$

and

$$\begin{aligned} \text{COV}(L, R_m) &= E(L, R_m) - E(L)E(R_m) \\ &= \int_{R_m} R_m \int_x^\infty (D-x)g(D, R_m)dD dR_m \\ &\quad - E(R_m) \int_x^\infty (D-x)g_D(D)dD. \end{aligned}$$

Thus, equation (6) can be written as

$$C = E(L) - \delta \text{COV}(L, R_m).$$

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