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Application of the Numerical Integration Method in a Repair Facility Using SIMAN and FORTRAN

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Abstract

This paper presents a decision model that will estimate the expected number of failed units in a repair facility in accordance with the varying demand, and determine the required number of personnel for repairing components. The demand is related to the failure process which follows a reliability growth phenomenon in service. The information in this paper is useful for selecting appropriate scheduling rules and spares stocking policies. SIMAN and FORTRAN were used for computing the time dependent performance measures in the repair facility. The numerical integration method that is presented in this paper will provide accurate performance measures with any dynamic pattern of demand, service rates, and any number of servers.

1. INTRODUCTION

When the complexity and increasing cost of many modern industrial or military systems are considered, the importance of reliability as an effectiveness parameter has become apparent. Organizations such as airlines, the military and public utilities are aware of the costs of unreliability. Manufacturers often suffer high costs of failure under warranty. In the weapons field, if an anti-aircraft missile has a less than 100 percent probability of functioning correctly throughout its engagement sequence, operational planners must consider deploying the appropriate extra quantity to provide the required level of defense.

It is common for new products to be less reliable during early development than later in

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the program, when improvements have been incorporated as a result of failures observed and corrected. Similarly, products in service often display reliability growth. The concept of reliability growth in service recognizes that increased usage will identify product deficiencies through failures. All failures are analyzed fully and corrective actions are taken in design or production to ensure that such failures do not occur on products in the next service period. Therefore, the failure rate of a product is seen to be a decreasing function of total operational time.

A type of reliability contract which has recently attracted a lot of attention is the reliability improvement warranty (RIW). The RIW contract requires that the supplier provides all spares needed, and carries out all repairs which include the growth (improvement) of a system's reliability, for an extended period for a once-off fee. The fundamental purpose of the RIW contract is to encourage reliability growth in fielded equipment. Under the RIW, the failure rate is a decreasing function of time based on the growth profile inherent in the reliability guarantee.

The problems treated here are : i) estimation of the expected number of failed units in the repair depot under reliability growth, and ii) determination of an adequate number of repair personnel for repairing components. The level of expected number of failed units in i) is essential to characterize outstanding orders. Variation in the level depends on demand rate and number of servers. Since demand is driven by a field failure and the failure process follows a reliability growth phenomenon, the personnel requirements for a depot to provide necessary repair for a specific component must be planned in accordance with the varying demand. In fact, any actual repair depot has important options available for temporary expansion of capacity via over-time, additional shifts, or subcontracting. Without making any additional assumptions about the repair discipline, such as deterministic repair time or ample repair capacity, it is possible to obtain the required number of personnel with the varying demand at the depot. The following sections present a decision model which incorporates the mechanism that determines when to increase shop capacity. This information is extremely useful for selecting appropriate scheduling rules and spares stocking policies.

2. EQUIPMENT FAILURE PROCESS

To model the system improvement, it is assumed that the expected number of failures in any initial interval is no less than the expected number of failures in any interval of the same length occurring later. One popular stochastic process to represent this situation is the nonhomogeneous Poisson process (NHPP). The properties of NHPP satisfy all the conditions for a Poisson process except that the mean rate varies with time. The NHPP has been used widely as a model for a system subject to improvement. Within the class of NHPP models, the power law model is most commonly discussed in the literature. See Ascher and Feingold [1] for more details. In this model, the instantaneous failure rate at cumulative utilization time T, $r(T)$, has a functional form,

$$r(T) = K\beta T^{\beta-1} \quad (1)$$

where K is a constant value and β is a growth rate.

Equation (1) shows that the future failure rate is a function of the utilization time length and the growth parameters. Estimation of the parameters of a failure rate is presented in Crow [4], and dependent on how well the product development program is progressing and how much resources and time are required to meet the end specified reliability target.

If we let $N(T_1, T_2)$ be the expected number of failures over the time interval $[T_1, T_2]$, then we would expect $N(T_1, T_2)$ to be

$$N(T_1, T_2) = \int_{T_1}^{T_2} r(T)dT = KT_2^{\beta} - KT_1^{\beta} \quad (2)$$

Under the NHPP assumption, the probability that exactly m units will fail in any interval $[T_1, T_2]$ has a Poisson distribution with mean $N(T_1, T_2)$. That is, for all $0 \leq T_1 \leq T_2$

$$P_r\{X = m\} = \frac{[N(T_1, T_2)]^m e^{-N(T_1, T_2)}}{m!} \quad (3)$$

where X is the number of failures in (T_1, T_2).

Now it could be show how this distribution might be used in a typical situation. Suppose that a hardware demonstrates reliability growth with the following parameter values : $K=0.0079$, $\beta=0.82$. Then, the mean value function is found by substitution into Equation (2) and becomes

$$N(T_1, T_2) = 0.0079(T_2^{0.82} - T_1^{0.82}) \quad (4)$$

Consider a case where there are three military sites, with the forecasted utilization hours over the first three years as given in Table 1. The demand for repair is directly related to the utilization hours which vary from site to site. Since the superposition of the three independent Poisson is also a Poisson, assume that demand from the different sites can be aggregated. For example, if a single item operating at site j with known demand rate, $N_j(T_1, T_2)$ is considered, then the aggregated field failures in (T_1, T_2) follows a Poisson distribution with $N(T_1, T_2) = \sum_{j=1}^{3} N_j(T_1, T_2)$. Hence, the number of aggregated failures is a Poisson random variable with a mean rate $N(T_1, T_2)$.

In this example, for the first year, the aggregated utilization hours per month would be

(42,000 hr + 30,800 hr + 56,800 hr) / 12

= 10,800 hr

Then, the expected number of failures during the first calendar month of the program would be

$N(0,\ 10800\ \text{hr}) = 16.03$ failures

Also, as expected, the number of failures in the second calendar month is given by

$N(10800\ \text{hr},\ 21600\ \text{hr}) = 12.27$ failures,

and so on.

The quantity $N(T_1, T_2)$ will be used in the EXAMPLE section as demand rate for repair through transit process.

Table 1. Utilization Statistics

Year	Expected Utilization Hours.			Aggregated Util. Hrs./Month
	Site 1	Site 2	Site 3	
1	42,000	30,800	56,800	10,800
2	44,000	30,800	57,200	11,000
3	50,400	33,600	58,800	11,900

3. MODELING FRAME WORK

Since the demand follows an NHPP process, the personnel requirements for a repair depot must be planned in accordance with the varying demand. With general shipment times, the transit process for the site-to-depot can be modeled as an $M/G/\infty$ queueing system. Mirasol [10] showed that the output of an $M/G/\infty$ system is a Poisson process, regardless of the distribution of service times. Therefore, using a common scenario the repair process at the depot can be modeled as a nonstationary M/M/s system. In this modeling approach, the probability distribution for the number of failed units either in queue or in service is calculated by the occupancy level of the nonstationary M/M/s system. The nonstationary M/M/s system experiences time dependent Poisson arrivals at rate, $\lambda(t)$, and has a single first-in-first-out queue feeding $s(s \geq 1)$ parallel servers. Each server provides identical, exponential service with mean service time $1/\mu$. All interarrival and service times are assumed to be independent of each other.

4. IN-TRANSIT PROCESS TO THE REPAIR FACILITY

Mirasol [10] and Newell [12] show that the number of customers in service for an $M/G/\infty$ system follows a Poisson distribution, and that the departure process is also Poisson. Gross and Harris [6] presented the system-size and the departure process in the stationary distribution for the $M/G/\infty$ system. We will extend the results for the case where the arrival process is an NHPP. In this section it is our intention to derive two results for an $M/G/\infty$ system, viz. the transient distribution for the number of failed items in the system at time t, and the transient distribution for the number of failed items which have completed service by time t. The latter distribution is the departure counting process of the $M/G/\infty$ system that will become the arrival process at the depot.

We define

$S(h)$ probability that a failed item leaves the transit system less than or equal to the time length h

$H_j(t)$ probability that a failed item from site j which was shipped at t_1 is still in transit at time t

$N_j(0,t)$ cumulative expected number of failures at site j by time t

$I_j(t)$ expected number of in-transit items from site j at time t

$D_j(0,t)$ cumulative expected number of arrivals at the depot from site j by time t

Suppose the in-transit time has a distribution function

$S(h) = \Pr\{\text{transit time} \leq h\}$

Then the probability of a failed item which has been shipped from site j at time t_1 still being in service at time t is given by $1-S(t-t_1)$. From the Poisson property, the probability of an arbitrary failure entering the transit service process at time t_1 is uniformly distributed on $(0,t_1)$. Therefore, the conditional probability of a failed item from site j which was shipped at t_1 for a transit service still in transit becomes

$$H_j(t) = \frac{\int_0^t [1-S(t-t_1)]dt_1}{t} \tag{5}$$

From (5) we see that the number of in-transit items from site j at time t has a Poisson distribution with mean rate

$$I_j(t) = H_j(t)N_j(0,t) \tag{6}$$

We also find that the cumulative distribution of the arrival process at the depot for site j is

$$D_j(0,t) = \{1-H_j(t)\}N_j(0,t) \tag{7}$$

If we consider time intervals (t_1,t_2), $(t_2,t_3),\cdots$, (t_{n-1},t_n), then the arrival rate for time interval (t_{n-1},t_n) to the depot is Poisson with the mean rate $D_j(0,t_n)-D_j(0,t_{n-1})$.

5. REPAIR PROCESS UNDER NONSTATIONARY DEMAND

Various papers written on queueing theory have discussed the transient behavior of M/M/s queueing systems. For example, Saaty [17] obtained the Laplace transform of the transient probabilities of the ordered queueing problem for the M/M/s system. However, he was only able to invert the transform for the two server case, and that case still showed some computational difficulties. Kolesar et al. [8] used numerical integration method for solving the Chapman-Kolmogorov differential equations of nonstationary M/M/s queueing problems. However, their model was found to be cumbersome for the large number of equations integrated to represent congested queues. Rothkopf and Oren [16] studied a computational method for finding the time dependent mean and variance of the number of customers in a multi-server system. They followed an approach for the M/M/1 queue used earlier by Clarke [3] and generalized it to the M/M/s queue with time varying arrival and service rates. To solve a pair of differential equations for the mean and variance of the number in the system, they used the standard numerical integration method applied piecewise over continuous segments of the time dependent arrival and service rates.

Clark [2] presented an approximation method to the solution of a nonstationary M/M/s queue. He considered the Polya-Eggenberger distribution as a surrogate for the true distribution of the number in the queueing system at time t. Kelton and Law [7] studied the transient behavior of the M/M/s queue with an arbitrary number of customers present at time zero. By following an approach used earlier by Morisaku [11], they carried out their analysis in discrete time i.e., indexing by customer number, and obtained probabilities that can be used to evaluate several measures of system performance, including the expected delay in queue of each arriving customer. Other works in this area can be found in Grassman [5], Marks [9], Odoni and Roth [14], and van Doorn [18].

Although many transient solutions exist, the available analytical results are quite restricted and impractical to use in our site-depot support model. For example, the results obtained by the existing models are approximations, and these models have no analytic bounds on the approximations. Moreover, the mathematics involved in solving the transient problem are complex and intractable to use as a part of a larger site-depot support system.

In modeling the nonstationary M/M/s system, we implemented SIMAN for computing the time dependent number of units in the depot (units in queue plus units in repair) with an arbitrary number of units present at time zero. SIMAN [15] uses the Runge-Kutta-Fehlberg (RKF) procedure to integrate differential equations numerically. This procedure provides results for a direct numerical integration of the Chapman-Kolmogorov Equations. These integrated values are the exact probability distributions of the system state at any time t that are founded by tracking the time dependent arrival and/or service rates. The following is a brief discussion on the basis of numerical integration methods that perform in increment of time.

A simple numerical technique for solving a first-order ordinary differential equation is called Euler's method. This method is based on the first two terms of the Taylor series expansion of function about time t. Although Euler's method is conceptually simple and relatively easy to analyze, this method is rarely used because of its low order of accuracy. One method for increasing the order is to carry more terms in the Taylor expansion from which Euler's method is derived.

The Runge-Kutta methods use the high-order local truncation error of the Taylor methods while elimination the computation and evaluation of the derivatives of functions. The advantage of the high-order Runge-Kutta methods is that they can achive higher level of accuracy for the same number of function evaluations.

Fehlberg presented the RKF method in 1970. This algorithm uses Runge-Kutta methods of order four and five together, and reduces the number of evaluations per step. SIMAN uses the RKF procedure with an automatic reduction of step size until the estimated truncation error on each step is within allowable limits.

The Chapman-Kolmogorov equations for the birth and death process are simultaneous differential-difference equations involving the time-state probabilities, $P_i(t)$ which denotes probability that exactly i failed units are in depot at time t.

These differential equations are given as

$$\begin{aligned}
P'_o(t) &= -\lambda p_o(t) + \mu p_1(t),\\
P'_i(t) &= -(\lambda(t) + i\mu)P_i(t) + (i+1)\\
&\quad \mu P_{i+1}(t) + \lambda(t)P_{i-1}(t),\\
&\quad \text{for} \quad i = 1,2,\cdots,s-1\\
P'_i(t) &= -(\lambda(t) + s\mu)P_i(t) + s\mu P_{i+1}(t)\\
&\quad +\lambda(t)P_{i-1}(t), \quad \text{for} \quad i = s,s+1,\cdots \qquad (8)
\end{aligned}$$

where $P'_i(t) = \frac{d}{dt}P_i(t)$ for all i.

The above equations can be translated into the continuous framework of SIMAN. Our objective is to find the probability distribution of the system state over time under the given parameter values $(\lambda(t) \& \mu)$ of the system. The calculation of $\lambda(t)$ and the differential equations must be coded in FORTRAN and are coded in subroutine STATE in order to be recognized by SIMAN. $\lambda(t)$ and the derivative values of each variable are passed between SIMAN and the subroutine STATE using the arrays stated in the COMMON block. This subroutine computes either $\lambda(t)$ or the derivative value of each continuous variable in the model. With those variables defined by derivative values, SIMAN automatically integrates the derivatives over time to yield values for the state probabilities at time t.

The methodology presented here will give an accurate estimation of the dynamic distribution of the level of failed units in the depot repair system, and eventually provide measures to plan the spare requirements at the depot and sites under reliability growth.

6. ILLUSTRATIVE EXAMPLE

The following example demonstrates the effect of time-varying demand on the expected number of failed units, and the required number of servers at the repair depot. For this example, we will use the mean value function given in Equation (4) and the utilization data given in Table 1. Further, the transit service time distribution from the sites to the repair depot is assumed to be exponential with mean transit times 10 days, 15 days, and 10 days for sites 1, 2, and 3 respectively.

For determining the required number of servers, a planned level of the utilization factor, u, (i.e., the expected fraction of time the servers are busy) is used. In Table 2, the required number of servers in the repair depot is presented for the case where 0.6<u<0.8, and the mean service time is identically 15 days for each server. Figure 1 presents dynamic results for the expected number of failed units in the repair depot through the end of year 3. The run started in the empty and idle condition in year 1. Year 2 and year 3 are continuations of the situation in year 1.

From Table 2, we see an initial increase of 7 to 8 in the required number of servers at the depot from month 1 to month 2. This initial increase occurred even though the field failures have decreased from 16.03 to 12.27.

Table 2 Required Number of Servers by Month

	1	2	3	4	5	6	7	8	9	10	11	12
Year 1	7	8	8	8	8	8	7	7	7	7	7	7
Year 2	7	7	7	7	7	7	6	6	6	6	6	6
Year 3	7	7	7	6	6	6	6	6	6	6	6	6

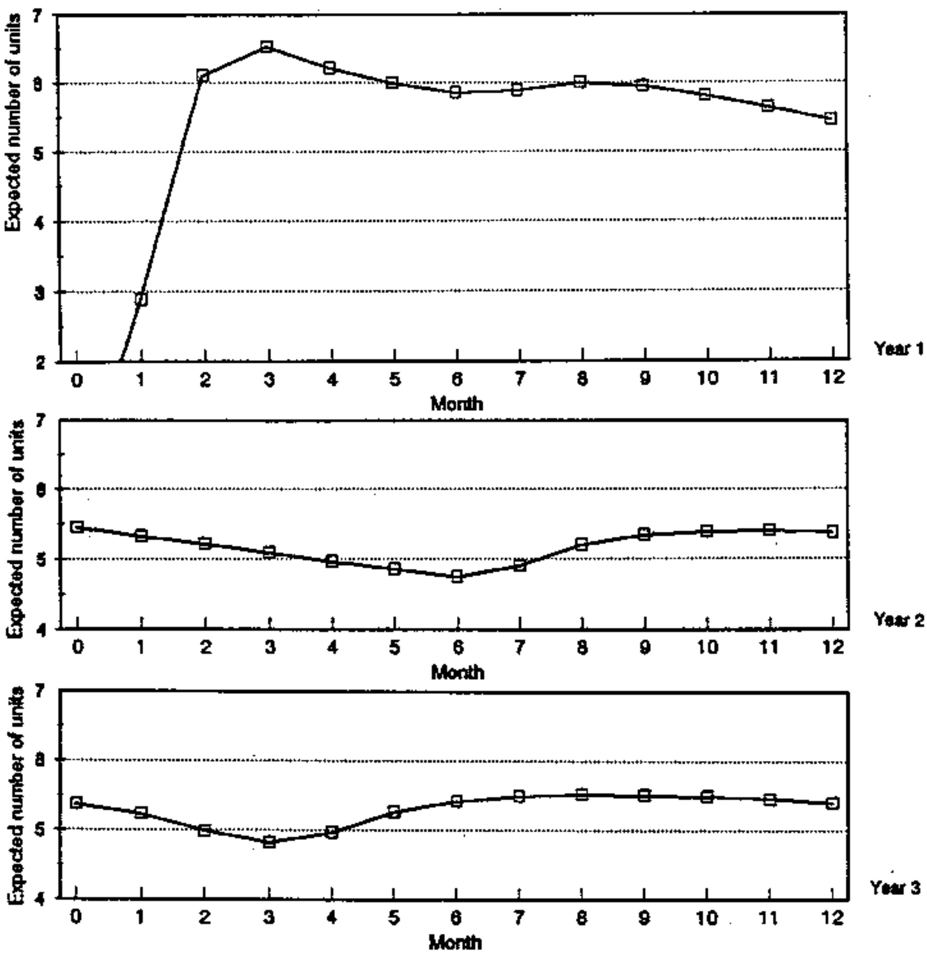

Figure 1. Month-by-Month Expected Number of Failed Units at the Repair Depot

Much of this phenomenon is due to delay involved in shipping the units from the site to the depot. It is evident from Table 2 and Figure 1 that the changing rate of demand affects the choice of the number of required servers, the utilization factor, and the expected number of failed units in the repair system. For example, decreasing the demand rate through year 1 allows for the removal of one service personnel at month 7 for the choice of u>0.6. This increases the utilization factor and therefore increases the expected number of failed units in the system as shown in Figure 1. The same phenomenon is observed at month 7 in year 2 and at month 4 in year 3. The increased number of servers in the beginning of year 3 is due to the increment of operating hours at the sites and resulted in higher number of servers for the time being.

For the numerical integration of the nonstationary M/M/s system, we formulated the problem using a double precision version of SIMAN implemented on a 486 PC microcomputer. When integrating the Chapman-Kolmogorov equations, we used $AERR = 10^{-4}$, where AERR is the Absolute single step truncation Error for the RKF algorithm. The expected number of failed units in the system at time t was computed by sampling the process at each integrated value of time, and the values were plotted through the end of the planning horizon. The running data for the SIMAN program and frame listing are as follows;

Model Frame Listing of Program in SIMAN

```
  BEGIN;
  ;
 SYNONYMS; DUMMY=X(1);
  ;
           CREAT;
;
           ASSIGN: 'DUMMY'=0: DISPOSE;
END;
```

Experimental Frame Listing of Program in SIMAN

```
  BEGIN; ; PROJECT, DEPOT, W.JUNG, 8/24/94;
  CONTINUOUS, 50, 1, 0.025, 0.25, 1.0, 0.0001, 0.0001;

    c * Number of differential equations 50
```

```
c * Number of state equation 1
c * Minimum allowable step size 0.025
c * Maximum allowable step size 0.25
c * Time between save points 1.0
c * Absolute single step trunc. error 0.0001
c * Relative single step trunc. error 0.0001

INITIALIZE, S(1)=1.0, S(2)=.0, S(3)=.0, S(4)=.0, S(5)=.0,
            . . . . . . . . . . . . . . . . . . . . . . . . . . ...
            S(45)=.0,S(46)=.0,S(47)=.0,S(48)=.0,S(49)=.0;

c * System starts with the empty & idle condition
c * i.e., S(1) =P0(0) = 1.0

REPLICATE, 1, 1, 1080;
c * Number of simulation runs 1
c * Beginning time of the first run 1
c * Max length of each run 1080
END;
```

FORTRAN subroutines for the numerical integration are as follows;

```
      SUBROUTINE USER
c
      COMMON/SIM/D(50),DL(50),S(50),SL(50),X(50),DTNOW,TNOW,TFIN,J,NRUN
      COMMON/JUNG/M,MS,TOT,CNT,NNS,ZLAM,QLAMDA
c * Calculate expected number of items in the system
      SUM=0.0
      DO 200 N=0,46
         SUM=SUM+N*S(N+2)
  200 CONTINUE
      IF (TSAVE.EQ.TNOW) THEN
         IF (INT(TNOW/30.).EQ.(TNOW/30.)) THEN
            QLAMDA=QLAMDA+ZLAM
```

```
            WRITE(*,*)'TNOW,M,QLAMDA',TNOW,M,QLAMDA
            QLAMDA=0
        ELSE
            QLAMDA=QLAMDA+ZLAM
        END IF
    END IF

c * Calculate average number of items for each month
    IF(M.EQ.MSAVE) THEN
        TOT=TOT+SUM
        CNT=CNT+1
    ELSE
        AVG=TOT/CNT
        TOT=SUM
        CNT=1
    END IF
    RETURN
    END

    SUBROUTINE STATE
c
   COMMON/SIM/D(50),DL(50),S(50),SL(50),X(50),DTNOW,TNOW,TFIN,J,NRUN
   COMMON/JUNG/M,MS,TOT,CNT,NNS,ZLAM,QLAMDA
   REAL TEMP(50),RATEJ(10,10), DELTA(10)
   REAL DHOUR(10),CUM(10),SUBCUM(10),T(10)
   REAL LAMDA, LAMDAJ,MU,Q,DAY,CUMFL,CUMFL1,T1,T2
   INTEGER NS(50)
c *  DJ(J)=expected transit time from site J to the depot in days
c *  RATEj(I,J)=proportion of failures at site J in year I
c *  NS(M)=number of servers at period M
   DATA DELTA(j)
   DATA RATEJ(i,j)
   DATA NS(j)
300    CONTINUE
```

```
c * Calculate arrival rate at the depot
    IF (TNOW.LE.360.) THEN
          I=1
          M=1+INT(TNOW/30.)
          MSAVE=1+INT((TNOW-1)/30.)
          MM=M
          DAY=TNOW-(M-1)*30.
          DSAVE=(TNOW-1)-(MSAVE-1)*30.
          DHOUR(I)=FTIME(J)/360.
          T(I)=(M-1)*(FTIME(J)/12.)+DAY*DHOUR(I)
          T(I+5)=(MSAVE-1)*(FTIME(J)/12.)+DSAVE*DHOUR(I)
    ELSE
          I=2
          M=1+INT((TNOW-360.)/30.)
          MSAVE=1+INT(((TNOW-1)-360.)/30.)
          MM=M+12
          DAY=TNOW-(M-1)*30.-360.
          DSAVE=(TNOW-1)-(MSAVE-1)*30.-360.
          DHOUR(I)=FTIME(J+1)/360.
          T(I)=FTIME(J)+(M-1)*(FTIME(J+1)/12.)+DAY*DHOUR(I)
          T(I+5)=FTIME(J)+(MSAVE-1)*(FTIME(J+1)/12.)+DSAVE*DHOUR(I)
    END IF
    DO 150 K=1,3
        Q=(DELTA(K)/TNOW)*(1-EXP(-TNOW/DELTA(K)))
        QQ=(DELTA(K)/(TNOW-0.9999))*(1-EXP(-(TNOW-1)/DELTA(K)))
        CUM(K)=(1-Q)*CUMFL
        SUBCUM(K)=(1-QQ)*CUMFL1
        LAMDAJ=(CUM(K)-SUBCUM(K))*RATEJ(I,K)
        LAMDA=LAMDA+LAMDAJ
150     CONTINUE

c *     Set state value
          S(50)=LAMDA
c *     Calculate state probabilities
```

```
c *    If N=0
         D(1)=  -S(50)*S(1)+MU*S(2)
c *    If N< NS(MM)
       DO 100 N=1,46
       IF(N.LT.NS(MM)) THEN
       D(N+1)=-(S(50)+N*MU)*S(N+1)+(N+1)*MU*S(N+2)+S(50)*S(N)
c *    If N>=NS(MM)
       ELSE
            D(N+1)=-(S(50)+NS(MM)*MU)*S(N+1)+NS(MM)*MU*S(N+2)
                        +S(50)*S(N)
       ENDIF
100      CONTINUE
       RETURN
       END
```

7. CONCLUSION

In this paper we have discussed an application of the numerical integration method to the spare parts inventory model under the time varying demand situation. Applying such a method using SIMAN and FORTRAN is remarkably practical since in most cases it is extremely difficult to consider the analytical solutions directly in the frame work of nonstationary systems. In our model we also incorporated reliability growth with repairable items. A search was made to find all possible implementations of repairable inventory models. A common assumption to the literature is that failures are generated by a stationary compound Poisson process. By considering growth phenomena in hardware reliability, this research would lead to a basis for a new approach to repairable inventory planning.

Assuming unlimited space at the repair depot, the transit and repair processes in the system are viewed as a network series of queues, i.e. $M/G/\infty$ and M/M/s. For transit process, we extended stationary results for $M/G/\infty$ system in Gross and Harris [6] to the case where the arrival process is nonhomogeneous.

The method we suggested here performs well both for steady and for nonstationary demand to provide appropriate scheduling rules at the repair facility. The information can be applied for inventory sizing needed at each location, by month, for various levels of associated stock-out risk. A further contribution of this research is that the developed methodology for this situation has been fully defined and integrated into a single compact package. We believe that the model could be used by many companies that design and develop for the industrial and military hardware systems.

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95년 1월 최초 접수, 95년 9월 최종 수정