■ 연구논문

Optimum Design of Accelerated Degradation Tests for Lognormal Distribution+

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Abstract

This paper considers the problem of optimally designing accelerated degradation tests in which the performance value of a specimen is measured only at one of three test conditions for a given exposure time. For the product having lognormally distributed performance, the optimum plan-low stress level and sample proportion allocated to each test condition - is obtained, which minimize the asymptotic variance of maximum likelihood estimator of a stated quantile at design stress. An illustrative example for the optimum plan is given.

1. Introduction

Accelerated life tests which are generally used to shorten the lives of test specimens or hasten the degradation of their performance, quickly provide information about the life distribution of products at use condition through a proper model. The performance of products will gradually degrade as the product ages. For example, the breakdown strength of electrical insulation degrades on age and temperature. Degradation processes of highly reliable products are usually slow but can be accelerated under high stress environment. Such tests are called accelerated degradation tests(ADTs).

In an ADT, test specimens are exposed to accelerated conditions and performance values are recorded instead of lifetime at each accelerated test condition, that is, specified exposure time and stress level. Data from ADTs are

+ This paper was supported in part by NON DIRECTED RESEARCH FUND, Korea Research Foundation, 1993.

then used to gain insight into the physical mechanisms that underlie the degradation process and to make inferences on the performance of the products at use condition and at operation times far beyond the length of the testing. These inferences imply extrapolation in two dimensions: stress and time.

ADTs have some advantages over accelerated life tests. The lifetime of a specimen, which is defined as the smallest time when performance reaches a reference value denoting the failure of products, can be gathered from extrapolating performance data. For highly reliable products, such as integrated circuits and lasers, the accelerated life tests provide little information since few failures may be observed within a given test period, even at very high level of stress. On the other hand, performance data from ADTs can be analyzed even though no specimen fails. ADTs can yield good insight into the degradation process and how to improve it. See for examples, Howes and Morgan(1981), Nelson(1990), and Nash(1992). Nelson(1981) provided an Arrhenius model and analysis for the breakdown strength data of electrical insulation which are measured only once at an accelerated test condition coupled with age and temperature. Ballado-Perez(1986) suggested a statistical model for the ADTs of adhesive-bounded wood composites. Carey and Koenig(1991) described an experimental and analytic strategy to extract reliability information from measuring the propagation delay of integrated logic devices submitted to accelerated condition. Lu and Meeker(1993) proposed statistical methods for not accelerated but degradation tests using degradation values to estimate a lifetime distribution for a broad class of degradation models. The optimum design of ADTs having three experimental points which differ slightly from ours was developed by Park(1993) using numerical searches method and also compared with optimum accelerated life tests. Boulanger and Escobar(1993) provided optimum design of ADTs under the assumption of sigmodal growth curve having random measurement error.

This paper considers the problem of optimally designing ADTs in which performance value of a specimen having lognormally distributed performance is measured only once at one of three test conditions including the measurements at the beginning of tests, within a specified exposure time. The lognormal distribution is widely used for the lifetimes of some products including electrical insulation, semiconductors, diodes, and adhesives. The ADT having three test conditions, which follows the suggestions by Nelson(1990) and will be called 3-point plan, is quite simple to apply it practically. Furthermore, our 3-point plan yields good estimates of the mean log performance at time zero and the amount of degradation till the given test period. The proportion of specimens allocated to each test condition and low stress level of 3-point plan are determined to minimize the asymptotic variance of maximum likelihood estimator(MLE) of the $100*q^{th}$ percentile of lifetime distribution at design stress which can be obtained from performance data.

The ADT model is introduced in Section 2 and the estimation procedure and optimization problem are described in Section 3. In Section 4, the optimum plan for ADT model is presented and an illustrative example is given.

2. The Model

The following assumptions are made:

1. The distribution of performance value $U(t, s)$ of a specimen at exposure time t and stress level s, is lognormal and U_i, $i=1, 2, \cdots, n$ are independently distributed. Thus the distribution of log performance $Y = \ln U$ is normal.
2. The standard deviation σ of log performance Y is constant, i.e., independent of exposure time and stress.
3. The relationship among the mean log performance μ, exposure time t and stress s, is

$$\mu(t, s) = \alpha - \beta * t * \exp\left(\frac{-\gamma}{s}\right), \quad t>0, \alpha>0, \beta>0, \gamma>0. \tag{1}$$

This is called Arrhenius model.

It is also assumed that specimens are tested at only two accelerated stresses and high stress is specified as the highest possible stress for which the assumed model is expected to hold and the longest possible exposure time t^* is pre-specified.

The following 3-point ADT plan for total test specimens n is considered:

1. Performance of $n\pi_0$ specimens randomly chosen from population are measured at the beginning of the test and design stress s_0.
2. Performance of $n\pi_1$ specimens randomly chosen from population are measured at exposure time $t(0 \le t \le t^*)$ and low stress s_1.
3. Performance of $n\pi_2$ specimens randomly chosen from population are measured at exposure time t^* and high stress s_2.

The object of an ADT for highly reliable products is to obtain performance data in a limited time. In particular, the above 3-point plan is useful for the experimenter who wants to carry out the ADTs as simple as possible. The performance data is extrapolated to estimate the lifetime distribution at design stress. The optimum 3-point plan specifies the optimum low stress, exposure time and proportions π_0, π_1 and π_2 $(=1-\pi_0-\pi_1)$ allocated to each test condition.

3. Estimation Procedure

Let $Y(t, s)$ be the random variable denoting the log performance at exposure time t and stress s, and the lifetime T at stress s be a random variable denoting the smallest time at which $Y(t, s)$ goes below a design value y_0 and failure of the specimen occurs. The population fraction $F(t, s)$ failed at exposure time t and stress s is the shaded fraction of distribution for $Y(t, s)$ as shown in ⟨Fig. 1⟩. Since $Y(t, s)$ is normally distributed with mean $\mu(t, s)$ and variance σ^2, for $t \geq 0$,

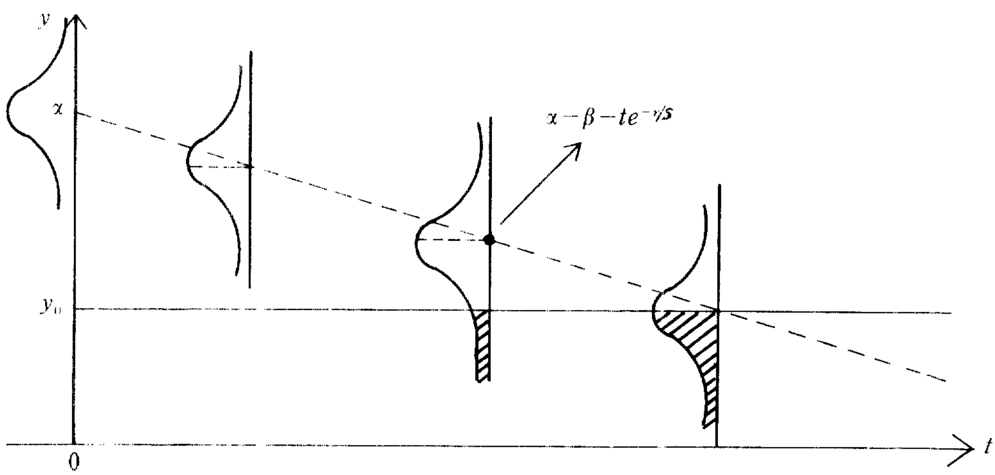
$$\begin{aligned} F(t, s) &= P[Y(t, s) \leq y_0] \\ &= \phi\,[\sigma^{-1}[y_0 - \alpha + \beta t \exp(-\gamma/s)]], \end{aligned} \tag{2}$$

where $\phi(\cdot)$ is the standard normal distribution function. Therefore,

$$F(t, s) = \begin{cases} 0, & \text{if } t<0 \\ \phi\,[\sigma_T^{-1}(t-\mu_T)] & \text{if } t \geq 0, \end{cases} \tag{3}$$

where $\mu_T = \beta^{-1}(\alpha - y_0)\exp(\gamma/s)$ and $\sigma_T = \beta^{-1}\sigma\exp(\gamma/s)$.

The lifetime T at stress s has a normal distribution with mean μ_T and variance σ_T^2.



⟨Fig. 1⟩ Distributions of log performance $Y(t, s)$.

Let p_f be the probability that a specimen fails at the beginning of tests and design stress s_0. The value of p_f may be very small because the event that the performance value of a specimen at $t=0$ is below y_0 will be rare in practice. Even though the value of p_f is very small, optimum design would be affected by this value. Furthermore, considering the existence of p_f is valid because the lognormal distribution of performance values in ADTs has left tail probability. The relationship between σ and p_f is

$$\alpha - y_0 = -\sigma z(p_f), \tag{4}$$

where $z(p_f)$ is the p_f^{th} quantile of standard normal distribution. The $100q^{th}$ percentile of the lifetime distribution at design stress s_0, say t_q, is

$$t_q = \begin{cases} 0, & \text{if} \quad q < p_f \\ \beta^{-1}[\alpha - y_0 + \sigma z(q)]\exp[\gamma/s_0] & \text{if} \quad q \ge p_f \end{cases} \tag{5}$$

The method of maximum likelihood can be used to estimate parameters α, β, γ and σ from performance data. The MLE of t_q, say $\hat{t}_q$, is

$$\hat{t}_q = \hat{\beta}^{-1}[\hat{\alpha} - y_0 + \hat{\sigma} z(q)]\exp[\hat{\gamma}/s_0], \tag{6}$$

where $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$ and $\hat{\sigma}$ are MLEs of α, β, γ and σ, respectively.

The optimization criterion used in this paper is to minimize the asymptotic variance of $\hat{t}_q$ in (6), which is a function of MLEs of $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$, $\hat{\sigma}$ and 3 test conditions $(0, s_0)$, (t, s_1), (t^*, s_2) and the proportions π_0, π_1, π_2.

It is convenient to define transformed stress $x_i = 1/s_i$, $i=0$, 1, 2 and the standardized transformed stress $\eta = (x - x_2)/(x_0 - x_2)$. Then, $\eta = 0$ for high stress s_2 and $\eta = 1$ for design stress s_0.

We also define standardized exposure time $\tau = t/t^*(0 \le \tau \le 1)$. Then $\mu(t, s)$ in (1) may be written in terms of η and τ as

$$\mu(\tau, \eta) = \alpha_0 - \beta_0 \tau \exp[-\eta \gamma_0], \tag{7}$$

where parameters α_0, β_0 and γ_0 mean that

$$\begin{aligned} \alpha_0 &= \alpha, \\ \beta_0 &= \beta t^* \exp[-x_2 \gamma], \\ \gamma_0 &= \gamma(x_0 - x_2). \end{aligned} \tag{8}$$

and scale parameter σ_0 is equal to σ. The MLE of t_q may be written as

$$\hat{t}_q = t^* \hat{\beta}_0^{-1} [\, \hat{\alpha}_0 - y_0 + \hat{\sigma}_0 z(q)] \exp(\hat{\gamma}_0) \tag{9}$$

where $\hat{\alpha}_0$, $\hat{\beta}_0$, $\hat{\gamma}_0$ and $\hat{\sigma}_0$ are MLEs of α_0, β_0, γ_0 and σ_0, respectively.

It is assumed that the independent random variable $Y_i(\tau, \eta)$, $i=1, 2, \cdots, n$, are identically distributed at the same test condition. The log likelihood L of an observation $y(\tau, \eta)$ at a transformed test condition (τ, η) is

$$L = -\ln \sigma_0 - \frac{1}{2}\left(\frac{A}{\sigma_0}\right)^2 + constant, \tag{10}$$

where $A = y - \alpha_0 + \beta_0 \tau \exp(-\eta\gamma_0)$. The second partial derivatives of the log likelihood with respect to the model parameters are needed in order to obtain variance-covariance matrix for the MLEs $\hat{\alpha}_0$, $\hat{\beta}_0$, $\hat{\gamma}_0$ and $\hat{\sigma}_0$. For a single observation, the first derivatives are

$$\begin{aligned} &\sigma_0^2(\partial L/\partial\sigma) = A, && \sigma_0^2(\partial L/\partial\beta_0) = -\tau A \exp(-\eta\gamma_0), \\ &\sigma_0^2(\partial L/\partial\gamma_0) = \eta\tau\beta_0 A \exp(-\eta\gamma_0), && \sigma_0^2(\partial L/\partial\sigma_0) = -\sigma_0 + \sigma_0^{-1}A^2. \end{aligned} \tag{11}$$

The following $F(\tau, \eta)$ for an observation will be Fisher information matrix whose elements are negative expectations for the second partial derivatives.

$$F(\tau, \eta) = (\sigma_0^{-2}) \begin{vmatrix} 1 & & & (symmetric) \\ -B & B^2 & & \\ \eta\beta_0 B & -\eta\beta_0 B^2 & (\eta\beta_0 B)^2 & \\ 0 & 0 & 0 & 2 \end{vmatrix} \tag{12}$$

where $B = \tau \exp(-\eta\gamma_0)$. Since $n\pi_0$ specimens are tested at the transformed test condition (0, 1) and $n\pi_1$ specimens at (τ, η) and $n\pi_2$ specimens at (1, 0), Fisher information matrix F for our 3-point plan with a sample of n independent observations is as follows;

$$\begin{aligned} F &= n\pi_0 F(0, 1) + n\pi_1 F(\tau, \eta) + n\pi_2 F(1, 0) \\ &= (n\sigma_0^{-2}) \begin{vmatrix} f_{11}, & f_{12}, & f_{13}, & 0 \\ f_{12}, & f_{22}, & f_{23}, & 0 \\ f_{13}, & f_{23}, & f_{33}, & 0 \\ 0, & 0, & 0, & 2 \end{vmatrix} \end{aligned} \tag{13}$$

where $f_{11}=1$, $f_{12}=-(\pi_1 B+\pi_2)$, $f_{13}=\pi_1\eta\beta_0 B$,
$f_{22}=\pi_1 B^2+\pi_2$, $f_{23}=-\pi_1\eta\beta_0 B^2$, and $f_{33}=\pi_1(\eta\beta_0 B)^2$.

The asymptotic variance-covariance matrix of the MLEs $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$ and $\hat{\sigma}$ is the inverse of the Fisher information matrix. Let H be the row vector whose elements denote the partial derivatives of t_q w.r.t. parameters α_0, β_0, γ_0, and σ_0. Then H can be obtained from (4) and (8) as follows:

$$H=t^*\beta_0^{-1}\exp(\gamma_0)*[\,1,\ -\sigma\beta_0^{-1}(z(q)-z(p_f)),\ \sigma(z(q)-z(p_f)),\ z(q)\,] \tag{14}$$

The corresponding asymptotic variance of $\hat{t}_q$ is of the form

$$Asvar(\hat{t}_q)=HF^{-1}H' \tag{15}$$

where the prime denotes a vector transpose.

4. Optimum Plan

4.1 3-point Optimum Plan

The asymptotic variance of $\hat{t}_q$ depends on the model parameters α_0, β_0, γ_0 and σ_0. To obtain optimum design one must know the value of α_0, β_0, γ_0 and σ_0, which is impossible. Many authors(1, 5, 11, 13) use pre-estimates of unknown parameters to overcome such difficulties and obtain optimum plans. These pre-estimates can be approximated from past experience, similar data, or a preliminary test. Chernoff(1953) calls such plans "locally optimum" since they are optimum only for the assumed or estimated parameter values.

Let p_d and p_h be the probabilities that a specimen will fail at maximum exposure time t^* at design stress x_0 and high stress x_2, respectively. From (3) and (4), we have

$$\begin{aligned}\beta t^*\exp[-x_0\gamma]&=\sigma(z(p_d)-z(p_f))\\ \beta t^*\exp[-x_2\gamma]&=\sigma(z(p_h)-z(p_f))\end{aligned} \tag{16}$$

resulting $p_h>p_f$ and $p_d>p_f$. We have from the relationships (8),

$$\alpha_0=y_0-\sigma_0 z(p_f)$$

$$\beta_0 = \sigma_0 [z(p_h) - z(p_f))]$$
$$\gamma_0 = \ln[(z(p_h) - z(p_f)) / (z(p_d) - z(p_f))] \quad (17)$$

The asymptotic variance of $\hat{t}_q$ is a function of η, τ, π_0, π_1, and model parameters. It can be shown that the optimum value of τ minimizing the asymptotic variance is 1 provided $p_d < p_h \leq q$. This fact is coincide with our intuition that the larger is the value of τ, the more information about performance degradation can be obtained. See Appendix for the proof.

Therefore, the following design problem is induced;

Given the values of q, p_f, p_d and p_h, find the values of π_0, π_1 and η minimizing the asymptotic variance of $\hat{t}_q$.

The Powell method(1964) of conjugate directions for finding the minimum of a function without using derivatives is used to solve the design problem. The computer program of Jensen(1985) was modified and coded in FORTRAN and run on an IBM PC compatible.

We have chosen the values $p_f = 1*10^{-5}$, $2*10^{-5}$, $4*10^{-5}$ and $p_d = 5*10^{-5}$, $1*10^{-4}$, $1.5*10^{-4}$ and $p_h = 0.01$, 0.05, 0.1 because those are very small in practice. And we have also chosen the values $q = 0.1$, 0.3, and 0.5.

We have obtained unique solutions of η, π_0 and π_1 for the selected values p_f, p_d, p_h, and q. The optimum values η^*, π_0^*, π_1^* and $Asvar^*(\hat{t}_q)$ is in ⟨Table 1⟩. It can be known by the numerical searches that η is not varied for any value of q, but the analytic proof on this fact cannot be given because of complexity of (A1). It can be also known that 1) the larger is the value of p_f, the smaller are the values of π_0^*, π_1^*, η^* and also the larger is the variance of $\hat{t}_q$, and 2) the larger is the value of p_d, the larger are the values of π_0^*, π_1^*, η^* and also the smaller is the variance of $\hat{t}_q$, and 3) the larger is the value of p_h, the smaller are the values of π_0^*, η^* and variance of $\hat{t}_q$, and also the larger is the value of π_1^*, and 4) the larger is the value of q, the smaller are the values of π_1^* and also the larger are the value of π_0^* and variance of $\hat{t}_q$.

4.2 An illustrative example

Nelson(1981) gives measurement data on the dielectric breakdown strength of insulation specimens. The performance of four specimens was measured at each combination of four accelerated conditions (180°C, 225°C, 250°C, and 275°C) and eight exposure times (1, 2, 4, 8, 16, 32, 48, and 64 weeks). Nelson obtained the values of MLEs $\hat{\beta}$ and $\hat{\gamma}$ as $2.96*10^{11}$ and 16652.6, respectively. For finding optimum design, these values and $\alpha=2.473$, $\sigma=0.451$ will be used as the pre-estimates of parameters. If the maximum exposure time t^* is 64 weeks and the value of $\exp(y_0)$ is $2.0KV$ in the ADTs, then the values of $p_f=4*10^{-5}$, $p_d \doteqdot 5*10^{-5}$ and $p_h=0.10$ are obtained from (16) and (17) using the pre-estimates. If a reliability analyst want to minimize the asymptotic variance of tenth percentile of lifetime

〈 Table 1 〉 Optimum plans for the given values of p_f, p_d, p_h and q.

q	p_h	p_d	$p_f=1*10^{-5}$				$p_f=2*10^{-5}$				$p_f=4*10^{-5}$			
			π	π_1	η	$V(\hat{t}_q)$	π_0	π_1	η	$V(\hat{t}_q)$	π_0	π_1	η	$V(\hat{t}_q)$
	.01	.00005	.383	.539	.608	163.6	.369	.529	.475	295.4	.349	.517	.294	868.2
		.0001	.405	.552	.789	93.0	.390	.541	.657	148.1	.374	.530	.507	277.6
		.00015	.421	.561	.915	67.2	.405	.550	.778	102.7	.389	.538	.628	176.3
	.05	.00005	.357	.545	.514	122.8	.349	.535	.412	201.2	.337	.522	.266	513.4
.10		.0001	.368	.557	.637	76.7	.360	.548	.541	111.5	.351	.537	.431	186.4
		.00015	.376	.566	.717	59.1	.367	.555	.621	81.6	.358	.544	.514	127.1
	.10	.00005	.347	.549	.482	106.6	.340	.539	.390	168.7	.332	.525	.256	409.3
		.0001	.354	.561	.589	68.6	.348	.551	.504	96.6	.341	.540	.405	155.2
		.00015	.359	.569	.656	53.9	.353	.559	.572	73.1	.347	.548	.478	108.3
	.01	.00005	.393	.531	.608	264.3	.376	.523	.475	485.4	.354	.513	.294	1452.5
		.0001	.417	.541	.789	151.3	.401	.532	.657	245.2	.382	.523	.507	468.9
		.00015	.435	.548	.915	109.7	.418	.538	.778	170.9	.399	.529	.628	299.5
	.05	.00005	.369	.535	.514	199.0	.358	.527	.412	332.1	.343	.517	.266	862.3
.30		.0001	.383	.545	.638	125.0	.373	.537	.541	185.3	.361	.528	.431	316.3
		.00015	.392	.551	.717	96.7	.382	.543	.621	137.9	.371	.534	.514	216.8
	.10	.00005	.359	.538	.482	173.0	.351	.530	.390	278.9	.339	.519	.256	688.9
		.0001	.370	.547	.589	112.0	.362	.539	.504	160.9	.353	.531	.405	263.9
		.00015	.377	.553	.656	88.3	.369	.545	.572	122.2	.360	.537	.478	185.1
	.01	.00005	.397	.527	.608	348.6	.380	.520	.475	645.1	.356	.511	.294	1945.4
		.0001	.423	.535	.789	200.4	.406	.528	.657	327.3	.386	.520	.507	631.2
		.00015	.442	.541	.915	145.9	.423	.533	.778	228.8	.403	.525	.628	404.3
	.05	.00005	.374	.531	.514	263.1	.363	.524	.412	442.3	.346	.515	.266	1157.2
.50		.0001	.389	.539	.638	165.9	.379	.532	.541	247.9	.366	.524	.431	426.8
		.00015	.399	.544	.717	128.6	.388	.537	.621	185.0	.376	.529	.514	293.4
	.10	.00005	.365	.533	.482	229.0	.356	.526	.390	371.9	.342	.516	.256	925.4
		.0001	.377	.541	.589	148.8	.368	.534	.504	215.5	.358	.526	.405	356.7
		.00015	.385	.546	.656	117.7	.376	.539	.572	164.1	.366	.531	.478	250.8

distribution at design stress $150^0 C$, the proportions for optimum allocation are $\pi_0^*=0.332$, $\pi_1^*=0.525$, $\pi_2^*=0.143$ and optimal low stress level η^* is 0.256 from ⟨Table 1⟩. When high stress is specified as $275^0 C(548^0 K)$, optimum low stress is $243^0 C$ $(516^0 K)$.

If 1000 test specimens are available for ADTs, then optimum test procedure is that the performance of 332 specimens are measured at the beginning of the test and design stress $150^0 C$, and the performance of 525 specimens at $243^0 C$ and 143 specimens at $275^0 C$ are measured after 64 weeks. The asymptotic variance of MLE for tenth percentile in this optimum design is 409.3.

5. Concluding Remarks

We have presented optimum ADTs in which the performance value of a test specimen is measured only once at the one of three test conditions. We have proved the fact that optimal exposure time at low stress is the maximum exposure time under the condition $p_n \le q$. The low stress level and proportions to be allocated at each test condition are determined numerically to minimize the asymptotic variance of the MLE for the percentile of lifetime distribution at design stress. The optimum ADTs can be used in a kind of destructive tests in which the performance of a test specimen is measured only once at a particular inspection time. The design problems for the cases where the performance of specimens are measured at more than three test conditions and the performance can be continuously monitored, should be solved in the future.

Appendix

Proof for $\tau=1$

From (15), we can obtain the following equation:

$$\pi_0 Asvar(\hat{t}_q) = (\frac{Q}{\eta\beta_0})^2 (\frac{1-\pi_2}{\pi_1})(\frac{1}{B})^2 + (\frac{2Q}{\eta\beta_0})(\frac{Q}{\beta_0} - \frac{Q}{\eta\beta_0} - 1)(\frac{1}{B})$$
$$+ 1 - \frac{2Q}{\beta_0} + \frac{2Q}{\eta\beta_0} + (\frac{1-\pi_2}{\pi_1})(\frac{Q}{\beta_0})^2 (1-\frac{1}{\eta})^2 + \frac{\pi_0 (z(q))^2}{2} \tag{A1}$$

where $B=\tau * \exp(-\eta\gamma_0)$ and $Q=\sigma * [z(q)-z(p_f)]$. The variance in (A1) has an unique minimum at

$$B=(\frac{Q}{\eta\beta_0})(\frac{1-\pi_2}{\pi_1})(1-\frac{Q}{\beta_0}+\frac{Q}{\eta\beta_0})^{-1} \quad \text{(A2)}$$

because $\pi_0 Asvar(\hat{t}_q)$ is the quadratic form for B^{-1} and the coefficient of B^{-2} is positive. If the B in (A2) is solved about τ, then we have the following solution for τ;

$$\tau=(1+\frac{\pi_0}{\pi_1})(1-\eta+\frac{\eta\beta_0}{Q})^{-1}\exp(\eta\gamma_0)$$

$$=(1+\frac{\pi_0}{\pi_1})\exp(\eta\gamma_0)[1+\frac{z(p_h)-z(q)}{z(q)-z(p_f)}\eta]^{-1} \quad \text{(A3)}$$

Since $[z(p_h)-z(q)]/[z(q)-z(p_f)]\leq 0$, provided $p_h \leq q$ and the $(1+\pi_0\pi_1^{-1})>1$ and $\exp(\eta\gamma_0)\geq 1$ in (A3), we have the fact that $\tau\geq 1$. But τ is bounded on [0, 1], and so we conclude that $\tau=1$, provided $p_h \leq q$.

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