

Simulation of Groundwater Flow in Fractured Porous Media using a Discrete Fracture Model

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ABSTRACT: Groundwater flow in fracture networks is simulated using a discrete fracture (DF) model which assume that groundwater flows only through the fracture network. This assumption is available if the permeability of rock matrix is very low. It is almost impossible to describe fracture networks perfectly, so a stochastic approach is used. The stochastic approach assumes that the characteristic parameters in fracture network have special distribution patterns. The stochastic model generates fracture networks with some characteristic parameters. The finite element method is used to compute fracture flows. One-dimensional line element is the element type of the finite elements. The simulation results are shown by dominant flow paths in the fracture network. The dominant flow path can be found from the simulated groundwater flow field. The model developed in this study provides the tool to estimate the influences of characteristic parameters on groundwater flow in fracture networks. The influences of some characteristic parameters on the fracture flow are estimated by the Monte Carlo simulation based on 30 realizations.

INTRODUCTION

In recent years, interest has increased considerably in the area of flow and transport in low permeable fractured rocks. One important reason for this is that many countries are seriously considering setting final repositories for nuclear waste at depth ranging from a few tens of meters for low and intermediate level wastes to 500m or even a kilometer or more for high level waste.

To simulate the flow through fractured rocks, many model approaches are suggested. They are categorized into three different models: EPM (Equivalent Porous Medium) model, DF (discrete fracture) model and DP (dual porosity) model (Bear *et al.*, 1993). The EPM model treats the fractured medium as a continuum and the hydraulic parameters can be obtained by lab or field tests as are applied to porous matrix. The hydraulic parameters, especially the hydraulic conductivity, are defined in the unit of the representative elementary volume (REV). The REV is very conceptual and is difficult in applying to the real system. The REV must contain many fractures such that each fracture in the REV can be treated as each pore in the porous matrix. Thus, the EPM approach may be valid when the ground water flow system of fractured media is represented in a regional scale.

The discrete fracture model assumes that fractures

are the conduits of water flow and matrix is the barrier of water flow. In general, permeability of fracture is $10^2 \sim 10^5$ times that of the porous matrix so that the flow through the porous matrix can be ignored (Cocas *et al.*, 1990a, b; Dverstorp and Anderson, 1989). The cubic law is generally applied to describe the flow through a fracture when the fracture is the opening between two parallel plates. Actual fracture surfaces are tough or rough to make the fracture flow more complicated (Witherspoon *et al.*, 1987). The application of the discrete fracture model has been in the domain of pure researches (Piggot and Elsworth, 1989; Tsang and Tsang, 1987, 1988; Long *et al.*, 1982; Smith and Schwartz, 1984) except a few researches applied to practical problems (Cocas *et al.*, 1990a, b; Dverstorp and Anderson, 1989).

The dual porosity model can be applied if the flow in the porous matrix can significantly change the flow in the fracture networks. The appropriate choice of the flow model between the dual porosity model and the discrete fracture model can be determined by aquifer tests. If most groundwater flow occurs in fracture networks, the discrete fracture model is more useful. Most groundwater flows in crystalline rocks can be analyzed using discrete fracture model.

The objective of this study is to develop a discrete fracture model and to investigate the sensitivities of the fracture flow to some characteristic parameters of fracture networks. The Monte Carlo simulations are used to investigate the statistical properties of fracture flow.

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FRACTURE NETWORK MODEL

Flow analyses of a fractured medium by the discrete fracture model require modeling of the fracture network. The complexity of reality does not allow a complete description of the actual field. It is therefore necessary to use stochastic models. The first task is the statistical analyses of fracture population: identification of sets and distribution of orientation, size, and aperture. The second task is the choice of a network model.

In this study, it is assumed that the individual fractures lie in a single plane. Characterization of a fracture system is considered complete when each fracture is described in terms of characteristic parameters: (1) hydraulic or effective aperture, (2) orientation, (3) location, and (4) length. In a two-dimensional case, it is assumed that all parameters are same along a specific direction. From this assumption, we can reduce a three-dimensional problem to a two-dimensional problem. So it is possible that two parameters for one fracture (strike and dip) are reduced to just one parameter (orientation). All fractures are assumed straight to have smooth surfaces.

The hydraulic behavior of fractures has been shown to be a function of their effective aperture. Unfortunately, it is very difficult to perform hydraulic tests on isolated fractures in the field. Because of the difficulty involved in hydraulically isolating a single fracture underground, what we know of fracture aperture distributions is limited to apparent apertures that have been observed directly in cores or well logs. The distribution of apertures measured by Bianchi and Snow (1968) was found to be very close to log-normal. It may be reasonable to expect hydraulic apertures are distributed log-normally.

The statistics of fracture orientations are perhaps the best understood of all the geometric properties of fractures. Orientations are easily measured in cores or in outcrops with simple tools. It is considered that the orientations are distributed normally.

The mathematical description of fracture location and fracture dimensions are interrelated. Fracture traces can be observed in excavation or in outcrops. The location of fractures can be determined in boreholes. We can figure out the location of fractures in space and their shape and dimensions from the trace length and intersection data. Baecher *et al.* (1977) have studied on fracture spacing and length distribution. Spacing and length have both been reported to vary exponentially and log-normally.

A general description of the fracture generation

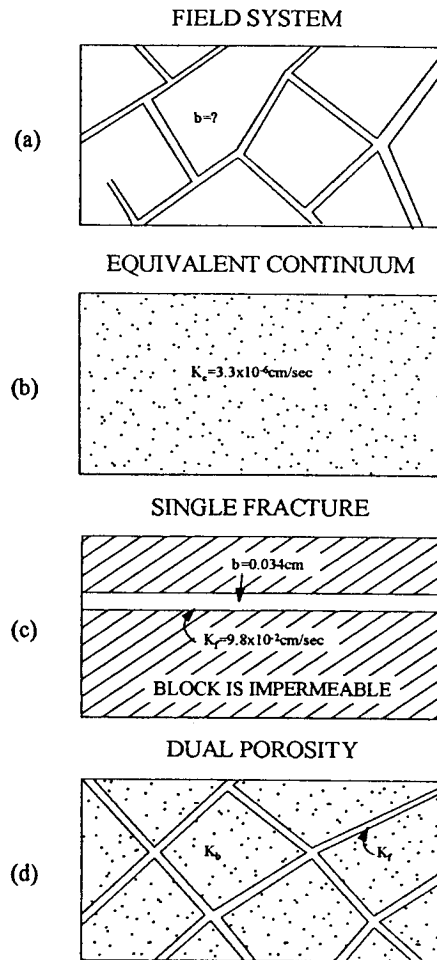


Fig. 1. Conceptual models of fractured rock system: (a) a simplified fracture network of aperture b with groundwater flow, (b) Equivalent porous medium model of (a), (c) Discrete fracture model of (a), and (d) Dual porosity medium model of (a) (from Anderson and Woessner, 1992).

process follows. Each set of fractures is generated independently followed by the individual sets being superimposed (Fig. 2). The location of each fracture in a set is found by assuming that the center of the fractures are randomly distributed (Poisson distribution) within the generation region (Fig. 2(a)). For each set a density value (number of fractures per unit area) must be supplied to determine the total number of fracture centers to be generated. The orientation of each fracture in a set is determined next (Fig. 2(b)). Orientation of fractures in a set has been assumed to be normally distributed so that the mean and variance of the orientation must be supplied for

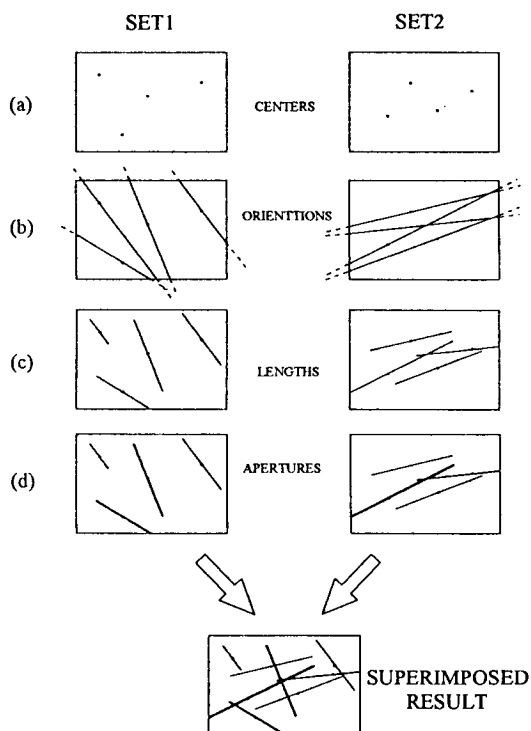


Fig. 2. Superimposing fractures (from Long *et al.*, 1982).

each set. The equation of the line on which the fracture lies is identified followed by determination of the fracture length (Fig. 2(c)). Fracture length within a set is assumed to be distributed log-normally (Long and Witherspoon, 1985). The mean and variance for length in the case of log-normal distribution must be supplied for each set. Fracture centers have been constrained to lie within the simulation region. However, with the assigned lengths, the part of the fractures may be outside the boundaries. These fractures are truncated at the boundaries of the simulation region. Finally, apertures are assigned to each fracture under the assumption that the apertures are log-normally distributed within a set (Fig. 2(d)). A mean and variance for aperture must be supplied for each set. When all the sets have been generated, a flow region is selected. The fractures which lie in the flow region are identified, and the coordinates of each intersection are calculated.

MATHEMATICAL MODEL

Flow through a single fracture may be idealized as the flow occurring between two parallel plates with a uniform separation equal to the fracture aperture

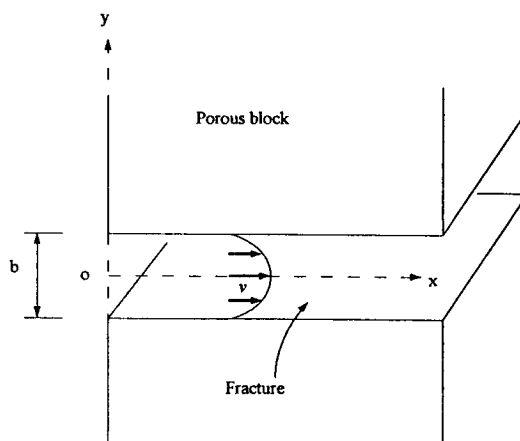


Fig. 3. A parallel plate model of fracture.

b. Fig. 3 depicts the parallel plate model. The flow rate in the direction s parallel to the fracture-matrix boundary is given by the Darcys equation

$$q_s = -k_f w b \frac{\rho g}{\mu} \frac{\partial h}{\partial s} \quad (1)$$

where k_f is the permeability, w is the width of the fracture, b is the effective aperture of the fracture, h is the hydraulic head, ρ is the fluid density, μ is the viscosity, and g is the gravitational acceleration. For an open fracture, the permeability of the fracture can be derived by applying the Navier-Stokes equation to straight duct problem as follows (Landau and Lifshitz, 1987)

$$k_f = b^2/12 \quad (2)$$

where b is the effective aperture of the fracture. By substituting (2) into (1), it is evident that q_s is proportional to the cube of the fracture aperture. The steady state equation of groundwater flow in a fractured medium is given by

$$\frac{\partial}{\partial s} \left(k_f w b \frac{\rho g}{\mu} \frac{\partial h}{\partial s} \right) = 0 \quad (3)$$

In this study, the width of fracture could be neglected because the fracture network is generated in a two-dimensional plane. The steady state equation of the groundwater flow in two-dimensional fractures is given by

$$\frac{\partial}{\partial s} \left(k_f b \frac{\rho g}{\mu} \frac{\partial h}{\partial s} \right) = 0 \quad (4)$$

NUMERICAL IMPLEMENTATION

The Galerkin finite element method is used to so-

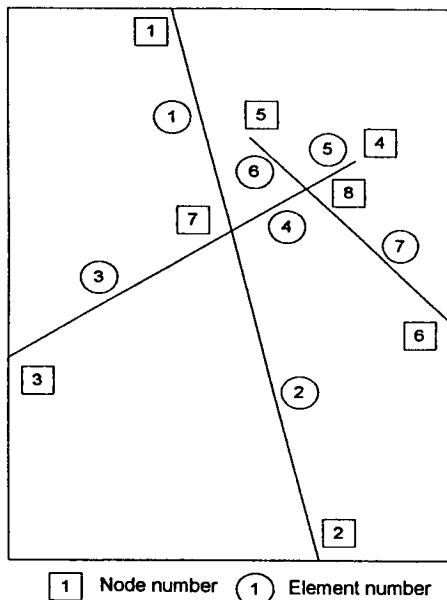


Fig. 4. A finite element mesh for a simple fracture network.

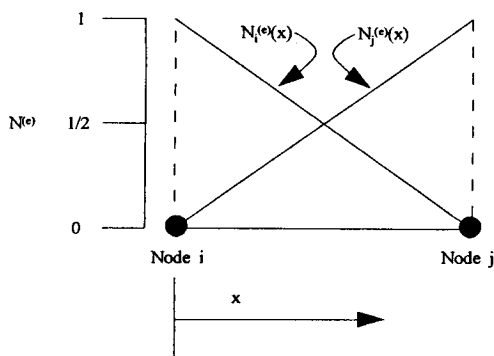


Fig. 5. A line element and its shape function.

Ive the equation of steady fluid flow. The fracture network generated by the stochastic procedure is converted into a two-dimensional finite element mesh with planar symmetry. The finite element mesh is composed of many line elements (Fig. 4). In the finite element mesh, every fracture is divided into small straight line elements. The line element has two-dimensional coordinates, but the shape function of the line element is one-dimensional (Istok, 1989). The shape function is defined proportional to the distance along the line element (Fig. 5).

The method to convert the fracture network into finite element mesh is followed.

(1) Each fracture must be in the inner part of the

domain. If the fracture is larger than the model domain, it must be truncated to fit the model domain.

(2) Each end point of fracture is registered as a nodal point of the finite element mesh.

(3) If one fracture intersects another fracture at one point, that point is registered as a new nodal point of the finite element mesh only if the point has not been registered before.

(4) For each fracture one of the end point is selected. Let the selected point be A. The nearest junction point from A in the direction of the other end point of the fracture is selected. Let the point be B. A new line element of the two selected points, A and B is generated. Next, let B be A, then the next new line element consists of the two points, A and B is generated. This element-making continues until the point B equals the other end point of the fracture.

(5) Step 4 is applied to all of the fracture lines.

In this procedure, it is very important not to register the same point twice. Each node must be registered only once. Because of the intersection points, several line elements share the same nodal point.

The element conductance matrix is constructed for every element and the global conductance matrix is assembled with these element conductance matrices. The global matrix is symmetric and banded. The entries outside the band are always zero and discarded. The vector storage reduces the memory space by storing only the entries within the band. The global matrix is solved using the Choleski method. The Choleski method is a direct method for solving a system of linear algebraic equations. It expresses any square matrix $[M]$ as the product of a lower triangular matrix $[L]$ and an upper triangular matrix $[U]$. From the above procedure, values of the total head at each point can be calculated.

PARAMETER ANALYSIS

Before the setup of a complex fracture network model, a simple fracture network model is set. A simple fracture network model is generated in the domain with the height of 90 m and the width of 60 m (Fig. 6). The number of fractures is limited to 50. The left boundary of the model has the constant head of 100 m. The right boundary of the model has the constant head of 0 m. The top and bottom boundaries of the model are assumed as the impermeable boundaries. It is not difficult to predict that the groundwater flows from left to right. Fig. 6 shows the result of simulation. The thick lines represent the fractures where the flow rate of groundwater is higher

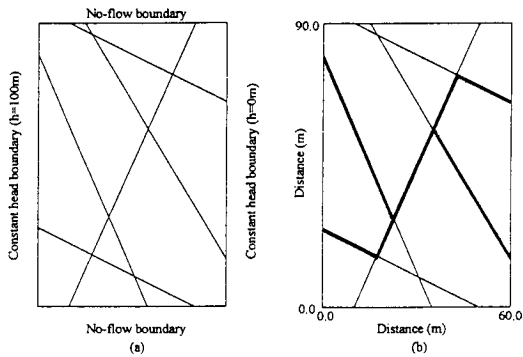


Fig. 6. A simple fracture network model and simulation result: (a) a fracture network and boundary conditions and (b) a map of groundwater flow paths. Thick lines represent the paths where groundwater flows faster than the average velocity.

than the mean flow rate. The thin line represents the fracture where the groundwater doesn't flow or the flow rate is lower than the mean flow rate. The groundwater flows through the connection of the fracture network. If a fracture is connected with another fracture, the groundwater flows as well. If a fracture is isolated, there is no flow of groundwater.

In this study, a complex fracture network model is developed under the assumption of Long *et al.*

(1982). The characteristic parameters are assumed as follows:

(1) Fracture centers are randomly distributed in the domain (Poisson distribution).

(2) Fracture orientations are normally distributed (mean of orientation = $N115^{\circ}E$, variance of orientation = 30°).

(3) Fracture lengths are log-normally distributed (mean of log-length = 2.0 m, variance of log-length = 0, 5 m).

(4) Fracture apertures are log-normally distributed (mean of log-aperture = -5.0 m, variance of log-aperture = 0.01 m).

Fig. 7 shows the result of a complex fracture network generated using the above parameters. Fracture centers are randomly distributed in the model domain. Major orientation of fractures is $N115^{\circ}E$. The fracture lengths vary greatly, but the fractures larger than the model domain are rare. Many fractures interconnected with each other, but most of them can not be used as the routes of the groundwater flow. In a global scale, the groundwater flows through the only little number of fractures. The thick lines represent the dominant flow paths of groundwater. The groundwater flows faster in the thick lines.

To draw general conclusion on the effects of characteristic parameters on the groundwater flow sys-

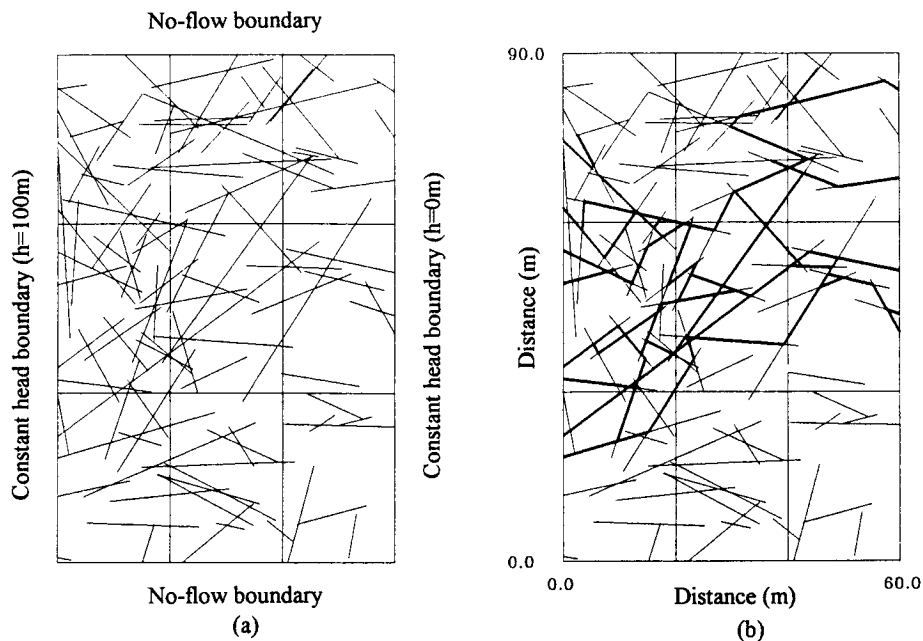


Fig. 7. A complex fracture network model and simulation result: (a) a fracture network and boundary conditions and (b) a map of groundwater flow paths. Thick lines represent the paths where groundwater flows faster than the average velocity.

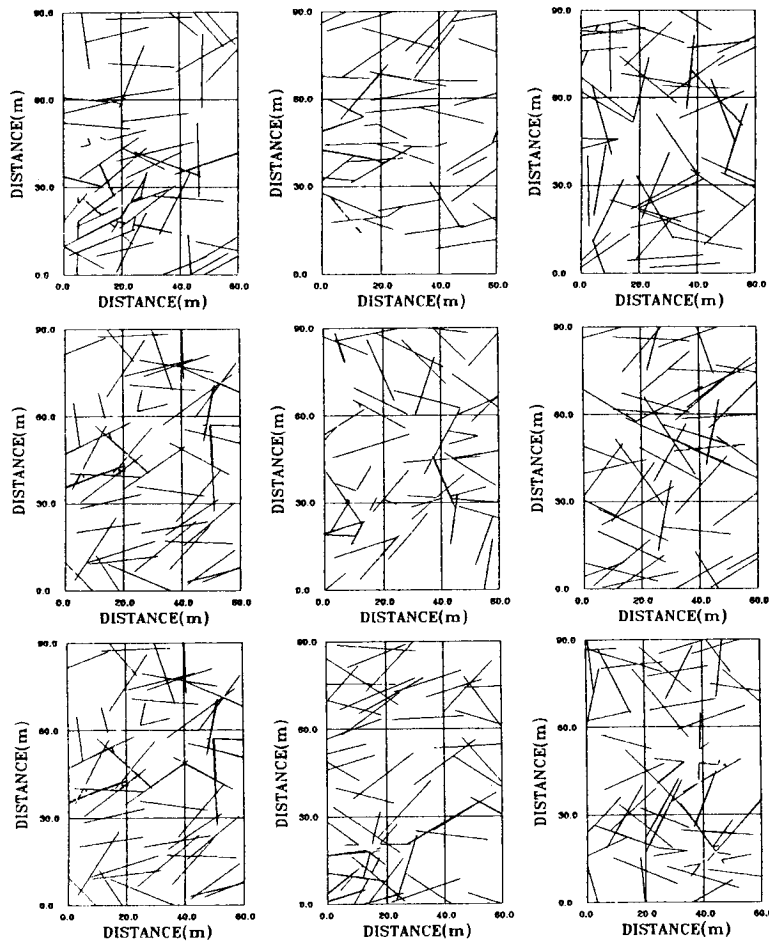


Fig. 8. 9 realizations of the Monte Carlo simulation. Each fracture network is generated with the same characteristic parameters set.

Table 1. The characteristic parameters of Run A.

Characteristic parameter	Distribution	Mean	Variance
number	random	50	—
orientation	normal (Gaussian)	80°	40°
length	log-normal	2.5 m	0.1 m
aperture	log-normal	5×10^{-5} m	1×10^{-5} m

tem, it is necessary to carry out Monte Carlo simulations. In this study, 30 realizations are generated and resulting flow systems are analyzed to estimate the effects of characteristic parameters. Fig. 8 shows the first 9 realizations of total 30 realizations that are generated using the parameters Table 1. The realizations are estimated in some statistical properties: nu-

mber of nodes, number of elements in the finite element mesh, total flux of groundwater, total length of fracture network, and total length of dominant groundwater flow paths. Table 3 shows the results of Monte Carlo simulations. Simulation parameters in Table 2 are designed to estimate the effect of characteristic parameters. The effect of the characteristic parameters on the groundwater flow system can be estimated by comparison of statistic results in Table 3.

To see how the density of fractures affects the groundwater flow, the following two models are established. Fig. 9 shows the two models, Run A and Run B. Run A has 50 fractures and Run B has 75 fractures in the model domain. The connection of fractures in Run B is better than that in Run A. The paths of groundwater flow increase proportional

Table 2. The characteristic parameters for the Monte Carlo simulations.

Run No.	No. of fractures	orientation		length		aperture	
		mean	variation	mean	variation	mean	variation
A	50	80°	40°	2.5 m	0.1 m	5×10^{-5} m	1×10^{-5} m
B	75	80°	40°	2.5 m	0.1 m	5×10^{-5} m	1×10^{-5} m
C	50	80°	40°	2.5 m	0.1 m	5×10^{-5} m	1×10^{-5} m
D	50	80°	40°	2.2 m	0.1 m	5×10^{-5} m	1×10^{-5} m
E	50	80°	40°	2.5 m	1.0 m	5×10^{-5} m	1×10^{-5} m

Table 3. The Monte Carlo simulation result.

Run no.	Nodes	Elements	Total flux (10^{11} m ² /s)	length 1 (km)	length 2 (km)	length 2/length 1
A	162.3	174.1	1.972	1.086	0.312	0.287
B	295.7	366.5	8.222	1.643	0.613	0.373
C	138.3	126.7	0.710	1.080	0.425	0.393
D	144.8	139.7	1.555	0.896	0.284	0.322
E	187.6	225.2	11.869	1.291	0.592	0.452

The result of each run is computed based on 30 realizations. Length 1 means the total length of fractures. Length 2 means the total length of major groundwater flow paths.

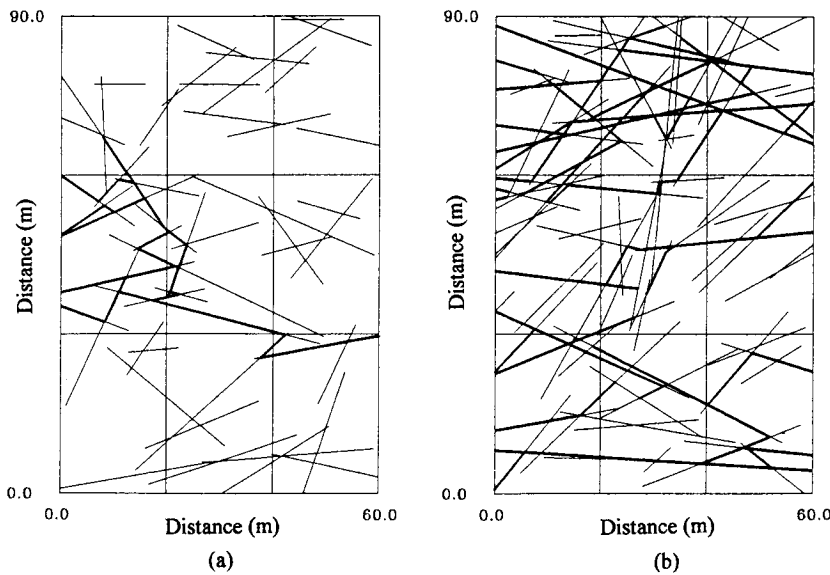


Fig. 9. Effect of fracture density on the groundwater flow system. (a) 50 fractures and (b) 75 fractures.

to the connection. Total flux of groundwater of Run B increases to 416.5% of Run A. As the density of fracture increases, the number of fractures which groundwater flows through increases and the flow rate of groundwater increases. The ratio of total path length to total fracture length increases from 0.287 to 0.374.

The effect of distributing orientation is illustrated in Fig. 10. The variance of the left model (Run C) is 20, and the variance of the right model (Run A)

is 40. In the left model, the only few fractures could be used as the paths of groundwater flow. In the right model, the total flux of groundwater flow increases as the variance of orientation increases. Total flux of groundwater of Run C is 35.9% of Run A. It could be said that the groundwater flow through fractures increases as the orientations of fractures are more arbitrarily distributed.

Fig. 11 shows the effect of the length of fractures on the groundwater flow. The left model (Run D)

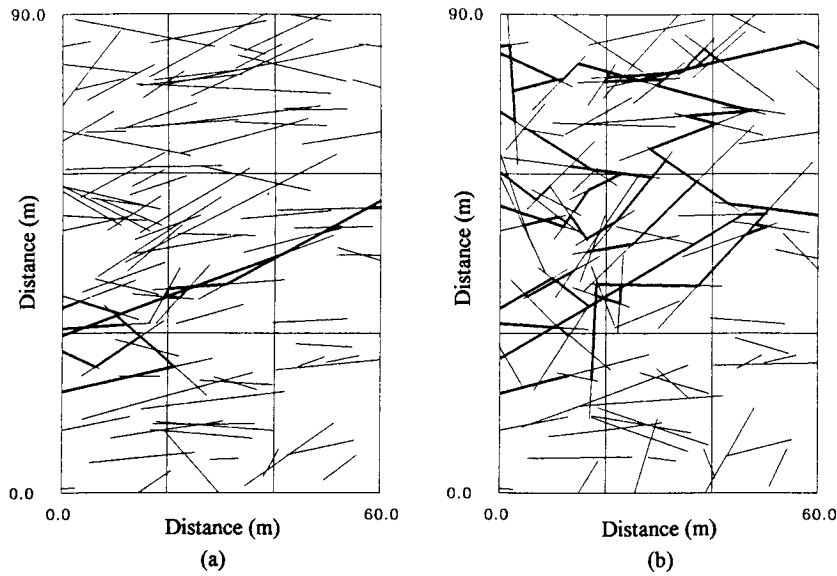


Fig. 10. Effect of fracture orientation distribution on the groundwater flow system: (a) variation=20° and (b) variation=40°.

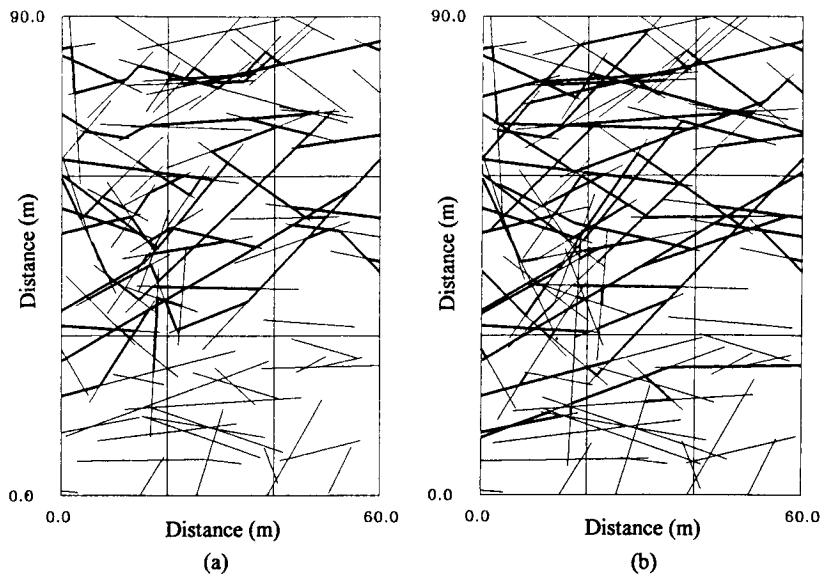


Fig. 11. Effect of fracture length on the groundwater flow system: (a) mean=2.2 m and (b) mean=2.5 m.

shows the fracture network where the mean of length of fractures is 2.2 m, and the right model (Run A) shows the fracture network where the mean of fractures is 2.5 m. As the lengths of fractures become longer, the fracture system becomes complicated. The number of paths of fluid increases and the great quantity of groundwater flows through these paths. Total flux of groundwater of Run D is 78.8% of Run A. The effect of the fracture length-distribution on

the groundwater flow is shown in Fig. 12. The variance of log-fracture length in the left model (Run A) is 0.1 m and that in the right model (Run E) is 1.0 m. As the variance of log-length increases with the same mean, the number of larger fractures increases. The paths of groundwater flow increase proportional to the complexity of fracture network. It could be said that the paths of groundwater flow increase as the mean and variance of fracture length increase.

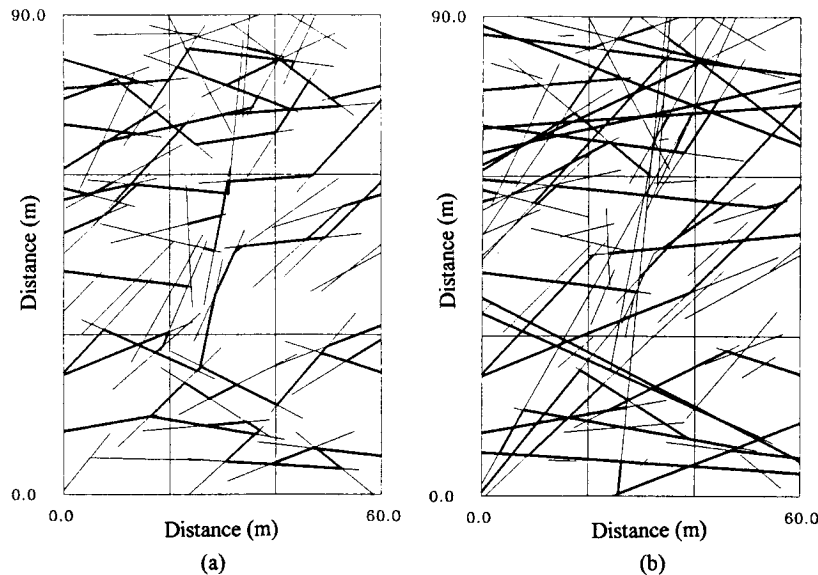


Fig. 12. Effect of fracture length distribution on the groundwater flow system: (a) variation=0.1 m and (b) variation=1.0 m.

Total flux of groundwater of Run E is 35.9% of Run A. The effect of aperture is also estimated. The distribution of aperture affects the formation of paths that the groundwater flows through. The number of paths varies very little, while the quantity of groundwater flow through the fracture increases significantly, as the aperture of the fracture becomes larger.

CONCLUSION

A discrete fracture (DF) model is developed to simulate the groundwater flow in fracture networks. The discrete fracture model assumes that groundwater moves only through the fracture network and there is no groundwater flow in the rock matrix. The discrete fracture model is especially useful if the permeability of rock matrix is very low as it is in crystalline rocks.

The stochastic model is convenient because it is impossible to describe fracture networks perfectly. The fracture networks are described with some characteristic parameters: center, orientation, length, and aperture. The characteristic parameters are assumed to have some distribution patterns.

The simulation for a simple fracture network is performed. The dominant flow paths in fracture network represent the groundwater flow system. The method developed in this study provides the tool to estimate the influence of characteristic parameters on groundwater flow in complex fracture networks. The

influences of some characteristic parameters are estimated by the Monte Carlo simulation. 30 realizations are generated for each characteristic parameters sets. The statistic results can be used to estimate the effect of the characteristic parameters on the groundwater flow system. The more groundwater flows as the density of fracture increase. The number of paths for groundwater flow increases as the length of fracture increases. The distribution of orientation has great influence on the formation of groundwater flow paths.

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불연속 파쇄모형을 이용한 파쇄 매질에서의 지하수 유동 시뮬레이션

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요 약 : 2차원 불연속 절리 모델(Discrete Fracture Model)을 사용하여 절리망 내에서 지하수 흐름을 시뮬레이션하였다. 불연속 절리 모델에서는 지하수가 오직 절리망을 통해서 흐른다고 가정한다. 이와 같은 분석은 결정질암 같이 지질 매체의 투수율이 매우 낮은 경우에 유용하다. 하지만 불연속 절리망을 완벽하게 구현하는 것이 불가능하므로, 이에 접근하는 방법으로 확률-통계적 모델이 제안되었다. 확률-통계적 모델은 특성인자(밀도, 방향, 길이, 틈새두께 등)가 특별한 분포 유형을 갖는다고 가정한다. 확률-통계적 모델은 가성된 분포를 따르도록 특성인자를 생성한다. 이 후 본 모델을 통해 분석된 몇몇 특성인자를 가지고 절리망을 생성한다. 절리망을 생성한 이 후 지하수의 유동을 계산하기 위해 유한요소법을 적용하였다. 이 때 일차원 선요소와 유한요소망의 주요 요소이다. 시뮬레이션 결과는 절리망 내의 주요 흐름 경로를 통해 보여진다. 절리망 내의 지하수 속도를 비교하여 주요 흐름 경로를 찾아낸다. 본 연구에서 개발된 모델은 절리망 내의 지하수 흐름에 특성인자들이 미치는 영향을 평가할 수 있는 방법을 제공한다. 이를 위하여 30번의 생성을 하는 몬테카를로 시뮬레이션을 통해서 여러 특성인자들이 지하수 흐름에 미치는 영향을 평가하였다.