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Determination of Shallow Velocity-Interface Model by Pseudo Full Waveform Inversion

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ABSTRACT: This paper presents a new approaching method to determine the velocity and geometry of shallow subsurface from seismic refraction events. After picking the first breaks from seismic refraction data, we assume that field refraction seismogram can be replaced by the unit delta function having time shift of first break. Time curves are generated by shooting ray tracing. The partial derivatives seismogram for a damped least squares method is computed analytically at each step of the forward ray tracing. The technique is successfully tested on synthetic and real data. It has the advantage of real full waveform inversion, which is robust at low frequency band even if the initial guess is far from the true model.

INTRODUCTION

Seismic refraction techniques have been widely used to delineate the shallow subsurface. These techniques depend on first break picks for near surface studies of depth and velocity.

According to the method used to solve the Green's function of the wave equation, seismic inversion may be classified into two groups. One approach is waveform inversion based on the wave equation. Another approach is traveltime inversion by a ray tracing algorithm.

The seismic traveltime inversion can be classified into two methods depending upon the choice of objective function. One is to minimize the error (or difference) between the field data and the synthetic seismograms. The other is to maximize the cross-correlation between the synthetics and the observations (Sen and Stoffa, 1991). And the full waveform inversion can be broadly divided into the direct inversion methods (Clarke, 1984, Yagle and Levy, 1985) and the iterative inversion schemes or nonlinear least-squares inversion methods (Tarantora, 1987; Pan *et al.*, 1988).

In this paper, we assume that every geologic models can be subdivided into polygon regions having constant velocity, and designed the shooting raytracing algorithm which is applicable to the complex topographical problem and the analytic calculation of the partial derivative seismogram.

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THEORY

Inversion in the Frequency Domain

This inversion technique is the nonlinear inversion method in the frequency domain. Since the first breaks of head waves are function of the velocity and the coordinates of the interface coordinates, unknown parameters such as velocity and depth are taken simultaneously to invert for the head waves. In this case, solutions become unstable because the partial derivative seismogram with respect to velocity and that of depth have different scale values. To avoid this scale problem, we employed the logarithmic variation developed by Madden (1972). Taking the logarithmic variation of the Taylor series expansion of the model response around a priori parameter p gives

$$\log(D_i)=\log(U_i)+\sum_{j=1}^{M}\frac{\partial\log(U_i)}{\partial\log(p_j)}\Delta\log p_j \qquad (1)$$

where, D_i is the observation data, U_i is the calculation data and $i=1, 2, 3, \cdots, N, j=1, 2, 3, \cdots, M$

After algebraic manipulation Eq. (1) becomes

$$\log\left(\frac{D_i}{U_i}\right)\simeq\sum_{j=1}^{M}\frac{p_j}{U_i}\frac{\partial U_i}{\partial p_j}\Delta\log p_j \qquad (2)\text{-}1$$

or in matrix notation

$$\begin{bmatrix}\log\left(\frac{D_1}{U_1}\right)\\ \log\left(\frac{D_2}{U_2}\right)\\ \vdots \\ \log\left(\frac{D_N}{U_N}\right)\end{bmatrix}=$$

$$\begin{bmatrix} \frac{p_1}{U_1}\frac{\partial U_1}{\partial p_1} & \frac{p_2}{U_1}\frac{\partial U_1}{\partial p_2} & \cdots & \frac{p_M}{U_1}\frac{\partial U_1}{\partial p_M} \\ \frac{p_1}{U_2}\frac{\partial U_2}{\partial p_1} & \frac{p_2}{U_2}\frac{\partial U_2}{\partial p_2} & \cdots & \frac{p_M}{U_2}\frac{\partial U_2}{\partial p_M} \\ \vdots & \vdots & & \vdots \\ \frac{p_1}{U_N}\frac{\partial U_N}{\partial p_1} & \frac{p_2}{U_N}\frac{\partial U_N}{\partial p_2} & \cdots & \frac{p_M}{U_N}\frac{\partial U_N}{\partial p_M} \end{bmatrix} \begin{bmatrix} \Delta \log p_1 \\ \Delta \log p_2 \\ \vdots \\ \Delta \log p_M \end{bmatrix} \tag{(2)-2}$$

where the N is the number of data points, and M is the number of parameters. Unlike the usual residual vector and the ∂ derivatives, we have scale free residual (E) and partial derivatives (S) defined as below

$$E = \log D - \log U = \log \frac{D}{U}, \tag{(3)-1}$$

$$S = \frac{p_j}{U_i}\frac{\partial U_i}{\partial p_j} \tag{(3)-2}$$

Applying standard least squares method to Eq. (2)-2 and adding damping factor beta give the normal equation given below

$$(S^T S + \beta I)\ \Delta \log p = S^T E \tag{4}$$

Solving Eq. (4) updating the parameter space by a general iterative rule, we can find the optimum parameters which minimize the residual between the field seismograms and the model response. Fig. 1 shows the flowchart of inversion algorithm.

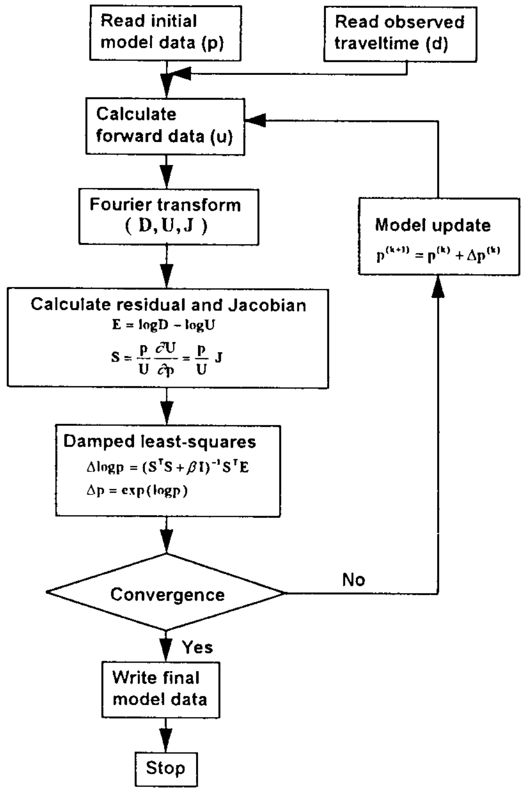

Fig. 1. The flowchart of inversion algorithm.

Traveltime and Wavefield

To calculate the traveltime of head wave, we assume that subsurface can be divided into the blocky regions consisting of straight line segments and head waves travel along the interface, as a model shown in Fig. 2. The model is characterized by the model parameter vector

$$P = [v_n, x_k, z_k]$$

Unlike the conventional shooting ray tracing of fan of rays from the source, a series of rays along the interface at infinitesimal distance interval are shooted according to the Snell's law. After collecting the rays which is nearest to the source and the receivers, calculating the traveltime corresponding to the those ray paths and adding the traveltime along the interface gives the traveltime of head wave from a source to a receiver as below

$$T = \sum_{k=1}^{l} \left(\frac{r_k}{v(r_k)} \right)$$

where, l is the number of ray path segments, $v(r_k)$ is the velocity of medium which ray passes through, r_k is the distance defined as

$$r_k = \sqrt{(x_{k+1} - x_k)^2 + +(z_{k+1} - z_k)^2} \quad k = 1,\ 2,\ 3,\ \cdots,\ l$$

where, k is the number of ray paths segments, and x_k and z_k are the coordinates of the points which ray passes through.

Since we are dealing with the first break in seismic refraction prospecting, we discard the wave events following the first break and assume that seismogram can be a unit delta function without any reflection, diffraction and other wave events. Picked seismogram can be given as

$$u(t) = \delta(t - T) \tag{6}$$

where T is the traveltime of head wave. Because the traveltime of head wave is a function of the velocity and the coordinates of the interface, we can take the ∂ derivatives of Eq. (5) with respect to velocity and coordinates.

The key point to note is that we will take Fourier transform of Eq. (6) in the frequency domain. Fou-

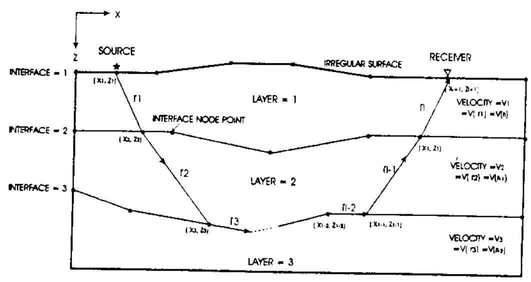

Fig. 2. Ray path in a layer with irregular interface. Interface is assumed to consist of straight line segments and velocity in each layer is constant.

rier transforming Eq. (6) gives

$$U(\omega)=e^{i\omega T}=e^{i\omega\sum_{k=1}^{l}\left(\frac{r_k}{v(r_k)}\right)}. \quad (7)$$

Calculation of Analytical Partial Derivatives

For a three layered model shown in Fig. 2, wavefield can be represented as below

$$U(\omega)=e^{i\omega T}=e^{i\omega\left(\frac{r_1}{V_1}+\frac{r_2}{V_2}+\frac{r_3}{V_3}+\cdots+\frac{r_{l-2}}{V_3}+\frac{r_{l-1}}{V_2}+\frac{r_l}{V_1}\right)} \quad (8)$$

$$=e^{i\omega\left(\frac{r_1}{V(r_1)}+\frac{r_2}{V(r_2)}+\frac{r_3}{V(r_3)}+\cdots+\frac{r_{l-2}}{V(r_{l-2})}+\frac{r_{l-1}}{V(r_{l-1})}+\frac{r_l}{V(r_l)}\right)}$$

$$=e^{i\omega\left(\frac{\sqrt{(x_2-x_1)^2+(z_2-z_1)^2}}{V(r_1)}+\frac{\sqrt{(x_3-x_2)^2+(z_3-z_1)^2}}{V(r_2)}+\frac{\sqrt{(x_4-x_3)^2+(z_4-z_3)^2}}{V(r_3)}\cdots\frac{\sqrt{(x_{l+1}-x_l)^2+(z_{l+1}-z_l)^2}}{V(r_1)}\right)}$$

In frequency domain, the partial derivatives of Eq. (8) with respect to the interval velocity

$$\frac{\partial U(\omega)}{\partial v_1}=-\frac{i\omega}{v_1^2}(r_1+r_l)\,U(\omega)$$

$$\frac{\partial U(\omega)}{\partial v_2}=-\frac{i\omega}{v_2^2}(r_2+r_{l-1})\,U(\omega)$$

$$\frac{\partial U(\omega)}{\partial v_3}=-\frac{i\omega}{v_3^2}\sum_{k=3}^{l-2} r_k\,U(\omega)$$

are calculated by simple algebra

$$\frac{\partial U(\omega)}{\partial v_n}=-\frac{i\omega}{v_n^2}(r_n+\cdots+r_{l-n+1})\,U(\omega). \quad (9)$$

The partial derivatives of Eq. (8) with respect to interface coordinates

$$\frac{\partial U(\omega)}{\partial x_1}=\frac{\partial U(\omega)}{\partial T}\frac{\partial T}{\partial x_1}=i\omega\left(0-\frac{x_2-x_1}{v(r_1)\,r_1}\right)U(\omega)$$

$$\frac{\partial U(\omega)}{\partial x_2}=\frac{\partial U(\omega)}{\partial T}\frac{\partial T}{\partial x_2}=i\omega\left(\frac{x_2-x_1}{v(r_1)\,r_1}-\frac{x_3-x_2}{v(r_2)\,r_2}\right)U(\omega)$$

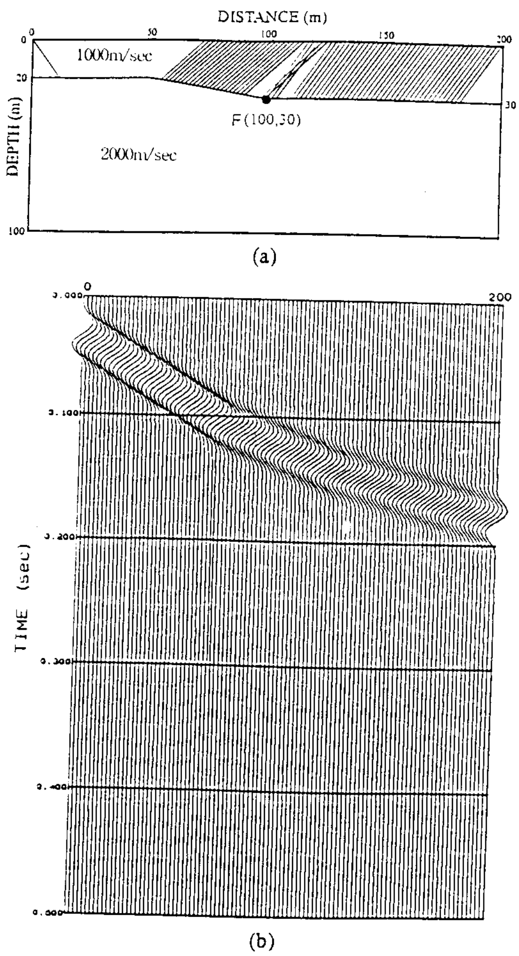

Fig. 3. (a) Raypaths in a fault model. (b) Synthetic seismogram computed by ray tracing. Source is Gaussian first derivative wavelet having 50 Hz major frequency.

$$\frac{\partial U(\omega)}{\partial x_3}=\frac{\partial U(\omega)}{\partial T}\frac{\partial T}{\partial x_3}=i\omega\left(\frac{x_3-x_2}{v(r_2)\,r_2}-\frac{x_4-x_3}{v(r_3)\,r_3}\right)U(\omega)$$

$$\frac{\partial U(\omega)}{\partial x_{l+1}}=\frac{\partial U(\omega)}{\partial T}\frac{\partial T}{\partial x_{l+1}}=i\omega\left(\frac{x_{l+1}-x_l}{v(r_l)\,r_l}-0\right)U(\omega)$$

$$\frac{\partial U(\omega)}{\partial x_k}=i\omega\left(\frac{x_k-x_{k-1}}{v(r_{k-1})\,r_{k-1}}-\frac{x_{k+1}-x_k}{v(r_k)\,r_k}\right)U(\omega) \quad (10)$$

$$\frac{\partial U(\omega)}{\partial z_k}=i\omega\left(\frac{z_k-z_{k-1}}{v(r_{k-1})\,r_{k-1}}-\frac{z_{k+1}-z_k}{v(r_k)\,r_k}\right)U(\omega) \quad (11)$$

where, $k=1, 2, 3, \cdots, l$, l is the number of segments.

From Eq. (9), (10) and (11), we can see that the phase and the amplitude of partial derivative seismogram in frequency domain change according to the frequency, velocity and coordinates of raypath.

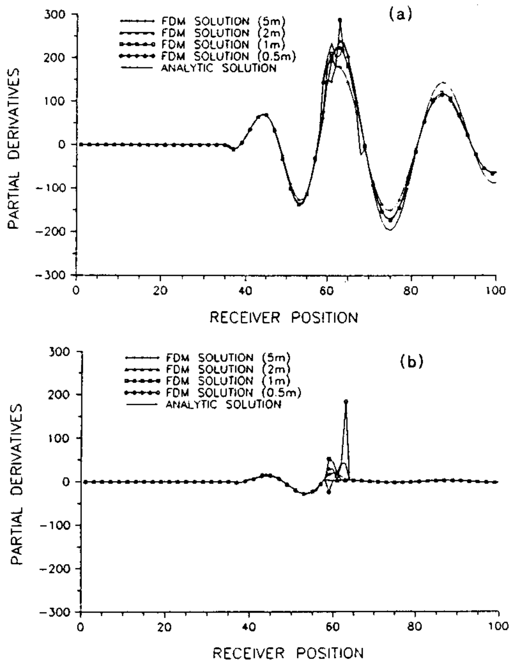

Fig. 4. Comparison between analytic and numerical partial derivative wavefield (real components) with respect to (a) z-coordinate and (b) x-coordinates of node point F shown in Fig. 3 where frequency is at 40 Hz.

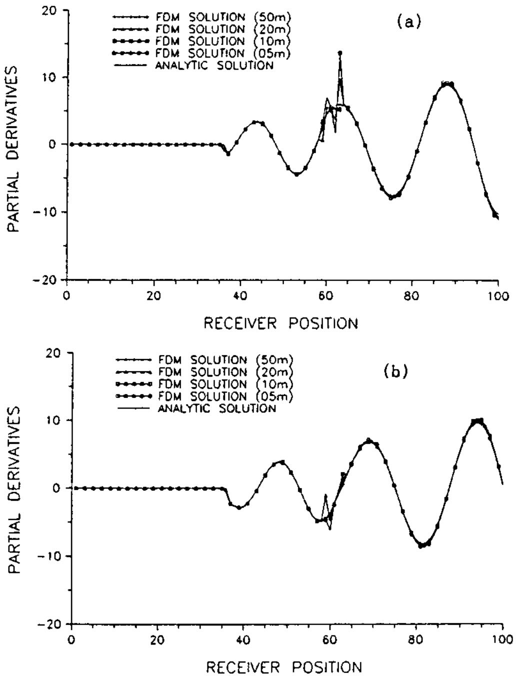

Fig. 5. Comparison between analytic and numerical partial derivatives of wavefield with respect to second layer velocity in Fig. 3 where frequency is at 40 Hz. (a) Real component and (b) Imaginary component.

EXAMPLES

Fig. 3(b) is a synthetic head wave seismogram, convolved with Gaussian first derivative wavelet having 50 Hz major frequency whereas Fig. 3(a) shows the ray paths computed by shooting ray tracing. We compared the analytical partial derivative wavefield to the partial derivative wavefield by numerical differencing. By adjusting the difference interval of the vertical coordinate of point F in Fig. 3, we calculated the partial derivative wavefield by central difference method, and compared it to the analytic partial derivative wavefield. As shown in Fig. 4 and 5, the analytical partial derivative wavefield is matching with the partial derivative wavefield using difference method.

Fig. 6 shows the inversion result for a curved layer model with irregular surface. Input data of 6 shot records are used for seismic inversion. As a starting initial model, three horizontal layer is taken as shown in Fig. 6. After 31th iteration, the velocity and depth model is converged to the true model.

After successful experiments on synthetics, we applied this inversion scheme to field data provided by IPRG (The Institute for Petroleum Research and

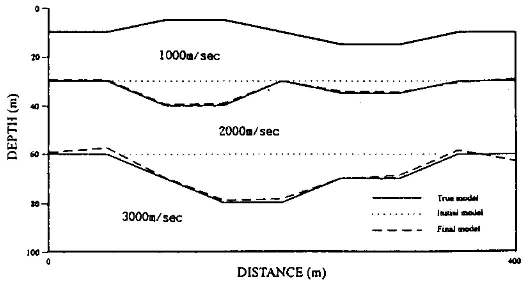

Fig. 6. Inversion result for a three layer model with irregular interfaces.

Geophysics, Israel). Field seismograms are acquisited by vibroseis source and 120 channel receivers are used. The field data from 0 to 4 km on the section are used for inversion. We picked the first breaks of 7200 traces consisting of 60 shot records by using the picking software for crosshole tomography. The average velocity of surface layer is calculated using the time-distance curve of direct wave. After 16 iterations, we could obtain the velocity model shown in Fig. 7. In this seismic inversion, we used the 20 Hz low pass filtered seismogram. Note that our inversion results are in a good agreement with the results ob-

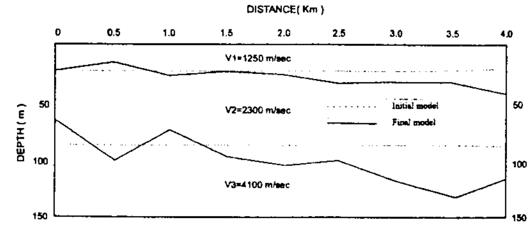

Fig. 7. Inverted velocity-interface model obtained from real data (Landa *et al.*, 1995) by pseudo waveform technique.

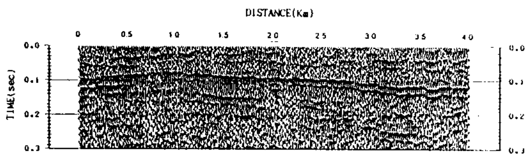

Fig. 8. Real data common shot refraction stacked section. Refraction events are shown about 0.05s an 0.1s at the right of the section.

tained by Landa *et al.* (1995) from the same data set shown in Fig. 8.

CONCLUSIONS

One of the advantage over other method is that, unlike the traveltime inversion, the inversion technique can be performed in the frequency domain using the damped Gauss Newton method. The shooting ray tracing allows us to calculate the travel time efficiently for the complex layered structure with irregular topography. Not only we can use the low frequency data when the initial model is far from the true model, but also we can refine the resolution limit with increasing the frequency band of the seismic data.

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유사파형역산에 의한 천부의 속도-경계면 모델 결정

정상용 · 신창수 · 양승진

요 약 : 본 논문에서는 탄성파 굴절법 탐사자료를 이용하여 천부지층의 속도와 심도를 결정하기 위한 새로운 접근방법을 소개한다. 굴절법 자료로부터 초동을 발췌한 후 실제 합성단면도를 이러한 초동의 시간이동에 해당하는 단위 델타 함수로 대치할 수 있다고 가정하였다. 주시의 계산은 발사법 파선추적을 이용하였다. 감쇠 최소자승법의 적용을 위한 편미분치의 계산은 이론주시의 계산과 동시에 해석적으로 구하였다. 본 역산법은 합성자료와 현장자료에 적용하여 성공적인 결과를 가져왔으며, 초기 가정 모델이 실제 모델과 많이 다르더라도 저주파수 대역에서 매우 양호한 결과를 보여주는 장점을 지닌다.