A Novel Approach to Improving the Performance of Randomly Perturbed Sensor Arrays

불규칙하게 흔들리는 센서어레이의 성능향상을 위한 새로운 방법

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ABSTRACT

The effects of random errors in array weights and sensor positions on the performance of a Linearly constrained linear sensor array is analyzed in a weight vector space. It is observed that a nonorthogonality exists between an optimum weight vector and the steering vector of an interference direction due to random errors. A novel approach to improving the nulling performance by compensating for the nonorthogonality is proposed. Computer simulation results are presented.

요 약

어레이 계수나 센서 위치에 일어나는 불규칙 오류가 선형조건이 주어진 센서어레이의 성능에 미치는 영향을 계수 벡터 공간에서 분석한다. 불규칙 오류에 의하여 최적계수 벡터와 방해신호 방향의 벡터사이에 비직교성이 존재한다는 것이 발견되었다. 이비직교성을 보상함으로써 영점화 성능을 개선하는 방법이 제안되었다. 컴퓨터 실험결과를 제시하였다.

I. Introduction

The random variations of array weight, element (i.e., sensor) position, or incoming signal wavefront generally result in degradation of array performance. If there exists no random variation in array parameters, the array performance will be the same as that of an ideal array [1]. In the past three decades, the effects of variations in array parameters on the array performance have been widely investigated in the literature [2-5]. It was shown that in an arbitrary array with directional elements which is subject to random variations of array weight and element position, the average power pattern results in a nominal power pattern (i.e., without random variations) superimposed by a power level which is proportional to the power pattern of directional elements. Thus the proportionality approximately depends on the product of the sum of variances of relevant errors and the sum of the squared average weights [2]. Constrained power minimization was discussed in a beamformer subject to random variations of

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input signal wavefront [3]. The average output power was shown to be the sum of the nominal power and the variances of the random errors of amplitude and phase multiplied by the total squared value of average weights. Even though the performance of randomly perturbed arrays has been widely investigated, the problem of reducing the degradation of array performance due to random errors in eliminating the interference signals has received relatively little attention.

In this paper, the effects of the random errors on the nulling performance of a linearly constrained narrowband linear array in the presence of random variations in array weight and element position are discussed and analyzed in the weight vector space. A novel approach to improving the array performance by compensating for the effect of random variations is proposed. Without loss of generality, it is assumed that : the average element positions are confined to a one-dimensional space ; the directions of incoming signals are confined to a two-dimensional space ; and the element positions vary randomly in a three-dimensional space.

Consider a narrowband linear array with N equispaced isotropic elements on the x-axis in a three-dimensional space as shown in Fig. 1 in which each element is followed by a complex weight. It is assumed that the array is subject to independent random variations of array weight and element position. Then the perturbed array weight vector $\boldsymbol{w}=[w_0\ w_1\ \cdots\ w_{N-1}]^T$ and element position vector $\boldsymbol{b}=[b_0\ \ b_1\ \ \cdots\ \ b_{N-1}]^t$ can be represented as

$$\boldsymbol{w}=\boldsymbol{c}+\boldsymbol{x} \tag{1}$$

and

$$\boldsymbol{b}=\boldsymbol{d}+\rho, \tag{2}$$

respectively, where

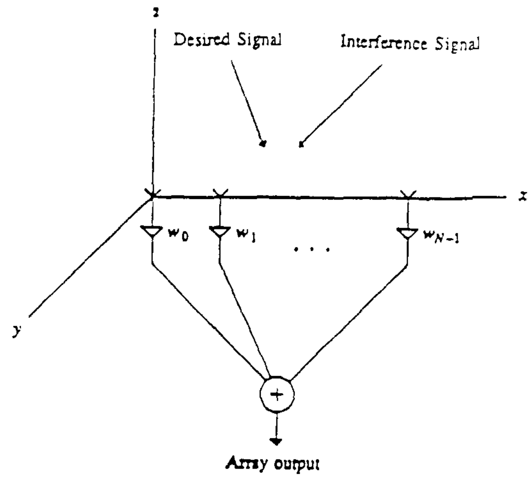

Fig. 1 Narrowband linear array.

$$\boldsymbol{c}=[c_0\, c_1\, \cdots\, c_{N-1}]^T, \tag{3}$$

$$\chi=[\chi_0\, \chi_1\, \cdots\, \chi_{N-1}]^T, \tag{4}$$

$$\boldsymbol{d}=[\,0\ d\boldsymbol{u}_x\ \cdots\ (N-1)d\ \boldsymbol{u}_x], \tag{5}$$

and

$$\rho=[\rho_0\, \rho_1\, \cdots\, \rho_{N-1}{}^T. \tag{6}$$

In (1)-(6), χ and ρ are the random error vectors for array weight and element position, respectively, c_n are the average values of perturbed complex weights w_n, *nd* $\boldsymbol{u}_x$ denote the average element positions, d is the spacing between neighboring elements, $\boldsymbol{u}_x$ is a unit vector on the x-axis, χ_n are independent complex random variables of weight errors with mean zero, ρ_n are the position error vectors whose cartesian components are denoted by random variables ρ_{nx}, ρ_{ny}, and ρ_{nz}, and n is index for vector component. It is to be noted that $\boldsymbol{b}$, $\boldsymbol{d}$, and ρ are $N\times 3$ matricies.

The nominal array factor of a narrowband linear array which is free of random variations is given by

$$H_n(u)=\boldsymbol{p}^H\,\boldsymbol{c}, \tag{7}$$

where $\boldsymbol{p}$ is a steering vector given by

$$\boldsymbol{p}=[1\ e^{jkdu}\ \cdots\ e^{jk(N-1)du}]^T, \tag{8}$$

$\boldsymbol{u}=\cos\theta$, θ is the angle from the array axis, $\boldsymbol{k}=2\pi/\lambda$, λ is the wavelength of incoming signals, $j=\sqrt{-1}$, and T and H denote the transpose and complex conjugate transpose, respectively. It can be shown that the power of the nominal array factor is expressed as

$$|H_n(\boldsymbol{u})|^2=\boldsymbol{c}^H\boldsymbol{P}\boldsymbol{c}, \tag{9}$$

where $\boldsymbol{P}$ is the autocorrelation matrix of $\boldsymbol{p}$.

If random variations in array parameters exist due to array imperfections resulting from manufacturing process or external circumstances [4, 5], the actual array performance becomes different from that originally designed. To analyze the effects of the random variations with respect to array weight and element position, we will investigate the array factor and corresponding power response in the presence of random variations.

Ⅱ. Effects of Random Errors

The array factor affected by random errors can be expresed as

$$H(u)=\boldsymbol{p}_r^H\boldsymbol{w}, \tag{10}$$

where $\boldsymbol{p}_r$ is a perturbed steering vector given by

$$\boldsymbol{p}_r=[e^{jk(\rho_{0x}u+\rho_{0z}\sqrt{1-u^2})}\ e^{jk(du+\rho_{1x}u+\rho_{1z}\sqrt{1-u^2})}$$

$$\cdots\ e^{jk((N-1)du+\rho_{(N-1)x}u+\rho_{(N-1)z}\sqrt{1-u^2})}], \tag{11}$$

If the distribution of the position error vectors is assumed to be spherically symmetric, jointly normal, independent , and identical for all elements, the average of the perturbed array factor in (10) is given by [2]

$$E[H(u)]=\frac{1}{\sqrt{1+\delta^2}}H_n(\boldsymbol{u}), \tag{12}$$

where

$$\delta^2=e^{\frac{k^2\sigma^2}{3}}-1, \tag{13}$$

and σ^2 is the variance of ρ_c, $c=x, y, z$.

It can be shown that the average output power is given by

$$E[|H(u)|^2]=\boldsymbol{c}^H\hat{\boldsymbol{P}}\boldsymbol{c}, \tag{14}$$

where

$$\hat{\boldsymbol{P}}=\frac{1}{1+\delta^2}(r\boldsymbol{I}+\boldsymbol{P}), \tag{15}$$

$$r=\delta^2+\gamma^2+\delta^2\gamma^2, \tag{16}$$

$$\sigma_{\chi_n}^2=\gamma^2|c_n|^2,\ \text{for } 0\le n\le N-1, \tag{17}$$

$\boldsymbol{I}$ is the identity matrix, and $\sigma_{\chi_n}^2$ is the variance of χ_n. The r in (16) will be called the error factor in this paper because it comprehensively represents the power of relevant errors. If a desired signal is incident on a randomly perturbed sensor array with interference signals whose power is to be nulled (i.e., minimized), the nulling performance will be different from that with no random errors as shown in the above analysis. In the following sections, the array performance in minimizing the interference power with a unit gain constraint in the look direction (i.e., the direction of a desired signal) will be discussed.

Ⅲ. Performance with Sinusoidal Interference

If a sinsoidal interference is coming from a direction different from that of a desired sinusoid, an optimum weight vector that yields a minimum power at the interference direction with a unit response at the look direction can be obtained by solving the following constrained optimization problem,

$$\min_{c} E[|H(u)|^2] \quad (18)$$
$$\text{subject to } E[\boldsymbol{p}_{rc}^H \boldsymbol{w}] = 1,$$

where $\boldsymbol{p}_{rc}$ is a perturbed steering vector for the look direction. It can be shown that the constraint in (18) is equivalent to

$$\boldsymbol{p}_c^H \boldsymbol{c} = \sqrt{1+\delta^2}, \quad (19)$$

where $\boldsymbol{p}_c$ is a nominal steering vector for the look direction. Using the method of Lagrange multipliers [6], the optimum weight vector is given by

$$\boldsymbol{c}_{opt} = \frac{\sqrt{1+\delta^2}\, \hat{\boldsymbol{P}}^{-1} \boldsymbol{p}_c}{\boldsymbol{p}_c^H \hat{\boldsymbol{P}}^{-1} \boldsymbol{p}_c} \quad (20)$$

Evaluating the inverse of $\hat{\boldsymbol{P}}$ using a matrix inversion lemma [7], we have

$$\hat{\boldsymbol{P}}^{-1} = \frac{1+\delta^2}{r} (\boldsymbol{I} - \boldsymbol{J}), \quad (21)$$

where

$$\boldsymbol{J} = \frac{N}{N+r} \boldsymbol{q}\boldsymbol{q}^H, \quad (22)$$

$\boldsymbol{q}$ is a normalized eigenvector of $\hat{\boldsymbol{P}}$ corresponding to an eigenvalue of $(N+r)/(1+\delta^2)$, i.e., $\boldsymbol{q} = \boldsymbol{p}/\sqrt{N}$. Substituting (21) into (20), we have the optimum weight vector as

$$\boldsymbol{c}_{opt} = \frac{\sqrt{1+\delta^2}\,(N+r)}{N(N+r) - |\boldsymbol{p}^H \boldsymbol{p}_c|^2} (\boldsymbol{I} - \boldsymbol{J})\boldsymbol{p}_c \quad (23)$$

Since the $\boldsymbol{J}$ is a nonorthogonal projection matrix for nonzero r, the vector $\boldsymbol{p}_c$ is projected onto the vector $\boldsymbol{p}$ nonorthogonally so that the optimum weight vector is not orthogonal to $\boldsymbol{p}$. Thus, the nulling performance is inversely proportional to the extent of nonorthogonality of $\boldsymbol{J}$ in such a way that the nulling performance improves as the random error factor r decreases or the number of elements N increases. It can be shown that if $\boldsymbol{p}$ and $\boldsymbol{p}_c$ are orthogonal, the array response at the interference direction is not affected by the relevant random errors in an average sense. This implies that $\boldsymbol{p}$ is on a subspace S_o which is parallel to the constrained surface S_c given by

$$S_c = \{\boldsymbol{c} + \boldsymbol{c}_t : \boldsymbol{c} \in S_o\}, \quad (24)$$

where $\boldsymbol{c}_t$ is a translation vector between S_o and S_c and is given by

$$\boldsymbol{c}_t = \frac{\sqrt{1+\delta^2}}{N} \boldsymbol{p}_c. \quad (25)$$

It is to be noted that the $\boldsymbol{c}_t$ becomes an optimum weight vector when $\boldsymbol{p}_c$ and $\boldsymbol{p}$ are orthogonal. The geometry in the weight vector space is shown in Fig. 2.

To reduce the effects of the random errors on the array performance, we form a weight vector translated by $\mu\boldsymbol{z}$ on the constrained surface, i.e.,

$$\boldsymbol{c}_o = \boldsymbol{c}_{opt} + \mu\boldsymbol{z}, \quad (26)$$

$$\boldsymbol{z} = \boldsymbol{c}_{opt} - \boldsymbol{c}_t \quad (27)$$

and a scal factor μ is a positive real number. Solving the following orthogonal condition for μ

$$\boldsymbol{p}^H \boldsymbol{c}_o = 0, \quad (28)$$

we get a scale factor which yields a weight vector which is orthogonal to $\boldsymbol{p}$ as

$$\mu_o = \frac{Nr}{N^2 - |\boldsymbol{p}^H \boldsymbol{p}_c|^2}. \quad (29)$$

Substituting (29) into (26), we find the corresponding weight vector as

$$\boldsymbol{c}_o = \frac{\sqrt{1+\delta^2}\,(N\boldsymbol{I} - \boldsymbol{P})\boldsymbol{p}_c}{N^2 - |\boldsymbol{p}^H \boldsymbol{p}_c|^2}. \quad (30)$$

which is the optimum weight vector orthogonal to the steering vector $\boldsymbol{p}$ and thus yields the best nulling performance in the presence of random errors. It turns out that $\boldsymbol{c}_o$ is shifted by $\mu_o \boldsymbol{z}$ to become $\boldsymbol{c}_{opt}$ on the constranied surface due to the nonorthogonal projection. Thus to compensate for the random errors, we shift the $\boldsymbol{c}_o$ by $\mu_o \boldsymbol{z}$ to the direction opposite to $\boldsymbol{c}_{opt}$. Therefore, the compensated weight vector is given by

$$\boldsymbol{c}_c = \boldsymbol{c}_o + \mu_o \boldsymbol{z} \tag{31}$$

or

$$\boldsymbol{c}_c = \frac{\sqrt{1+\delta^2}\,[(N-r)\boldsymbol{I}-\boldsymbol{P}]\boldsymbol{p}_c}{N^2 - |\boldsymbol{p}^H \boldsymbol{p}_c|^2} \tag{32}$$

If $\boldsymbol{c}_c$ is set as an actual array weight, the best nulling performance will be achieved.

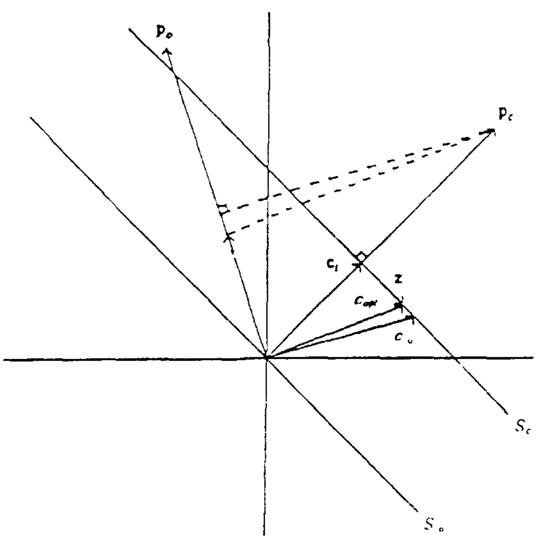

Fig. 2 Geometry in the weight vector space.

Ⅳ. Performance with Narrowband Interference

To eliminate a spatially narrowband interference, we need to form a narrowband null around the angular region of the interference. If the lower and upper limits of the angular region are u_l and u_u, respectively, the optimum weight vector can be found by solving the following constrained optimization problem.

$$\min_{\mathbf{c}} \quad E\left[\int_{u_l}^{u_u} |H(u)|^2\, du\right]$$
$$\text{subject ot } \boldsymbol{P}_c^H \boldsymbol{c} = \sqrt{1+\delta^2}. \tag{33}$$

It can be shown that

$$E\left[\int_{u_l}^{u_u} |H(u)|^2\, du\right] = \boldsymbol{c}^H \,\tilde{\tilde{\boldsymbol{P}}}\, \boldsymbol{c}, \tag{34}$$

where

$$\tilde{\tilde{\boldsymbol{P}}} = \frac{1}{1+\delta^2}\,(\Delta u\, r\boldsymbol{I} + \tilde{\boldsymbol{P}}), \tag{35}$$

$$\tilde{\boldsymbol{P}} = \begin{cases} \Delta u & \text{for } n = m \\ \dfrac{e^{jk(n-m)du_u} - e^{jk(n-m)du_l}}{jk(n-m)d} & \text{for } n \neq m \\ & 1 \le n,\ m \le N, \end{cases} \tag{36}$$

and $\Delta u = u_u - u_l$. Assuming that the angular region Δu is reasonably narrow, $\tilde{\tilde{\boldsymbol{P}}}$ can be approximated as

$$\tilde{\tilde{\boldsymbol{P}}} = \frac{\Delta u}{1+\delta^2}\,(r\,\boldsymbol{I} + \tilde{\boldsymbol{p}}\,\tilde{\boldsymbol{p}}^H), \tag{37}$$

where $\tilde{\boldsymbol{p}}$ is a steering vector which lies between the two steering vectors corresponding to the two limits of the angular region. The $\tilde{\boldsymbol{p}}$ is given by

$$[\tilde{\boldsymbol{p}}]_n = \Delta u\, e^{jknd(u_l + 0.5\Delta u)}. \tag{38}$$

It can be shown that the optimum weight vector is given by

$$\tilde{\boldsymbol{c}}_{opt} = \frac{\sqrt{1+\delta^2}\,(N+r)}{N(N+r) - |\tilde{\boldsymbol{p}}^H \boldsymbol{p}_c|^2}\,(\boldsymbol{I} - \tilde{\boldsymbol{J}})\boldsymbol{p}_c, \tag{39}$$

where

$$\tilde{J} = \frac{N}{N+r} \tilde{q}\, \tilde{q}^{H} \tag{40}$$

and $\tilde{q}$ is a normalized eigenvector of $\tilde{\tilde{P}}$ corresponding to an eigenvalue of $\Delta u(N+r)/(1+\delta^2)$. Also, the compensated weight vector is given by

$$\tilde{c}_c = \frac{\sqrt{1+\delta^2}\,[(N+r)I - \tilde{p}\,\tilde{p}^{H}]p_c}{N^2 - |\tilde{q}^{H} p_c|^2} \tag{41}$$

If $\tilde{c}_c$ is set as a wight vector, a narrowband null with a deeper depth will be achieved compared to that by $\tilde{c}_{opt}$. It is to be noted that a set of point nulls may be synthesized to find an equivalent optimum weight vector for the narrowband case.

V. Simulation Results

For the case of sinusoidal interference, a 3-element linear array is experimented with interelement spacing half the wavelength for incoming signals. The magnitude response at the look direction is assumed to be $1/\sqrt{1+\delta^2}$ for convenience so that the nominal magnitude response is one at the look direction. It is assumed that the incident angles of the desired and interference signals are 90° and 60° from the array axis, respectively. Then the optimum weight vector is given by

$$c_{opt} = \frac{1}{8+3r} \begin{bmatrix} 3+r+j \\ 2+r \\ 3+r-j \end{bmatrix} \tag{42}$$

The best optimum and compensated weight vectors are given by

$$c_o = \frac{1}{8} \begin{bmatrix} 3+j \\ 2 \\ 3-j \end{bmatrix} \tag{43}$$

and

$$c_c = \frac{1}{8} \begin{bmatrix} 3-r+j \\ 2-r \\ 3-r-j \end{bmatrix}, \tag{44}$$

respectively. The beam pattern for a broadside uniform linear array free of random errors is shown in Fig. 3. The beam pattern with error factor $r=0.1$ is shown in Fig. 4 in which the power response at 60° is -38.8 dB. If the errors are compensated by using the c_c as the actual weight, the resulting beam pattern is shown in Fig. 5 sense which corresponds to the pattern by c_o. It is observed that the power response at the interference direction is -149.5 dB which is about 110 dB lower than that for the uncompensated array.

A 15-element linear array is employed for a narrowband interference whose angular region is assumed to be from 59.5° to 60.5°. It is assumed that the error factor r is 0.001. Figs. 6(a) and 6 (b) display the beam patterns by the optimum and compensated weight vectors respectively. From the figures, it is not easy to compare the nulling performances of the two weight vectors. To find out the difference in nulling performance more exactly, the beam patterns around 60° are shown in Figs. 7(a) and 7(b). It is observed that the compensated array yields a deeper and narrower null around 60° than the uncompensated one does.

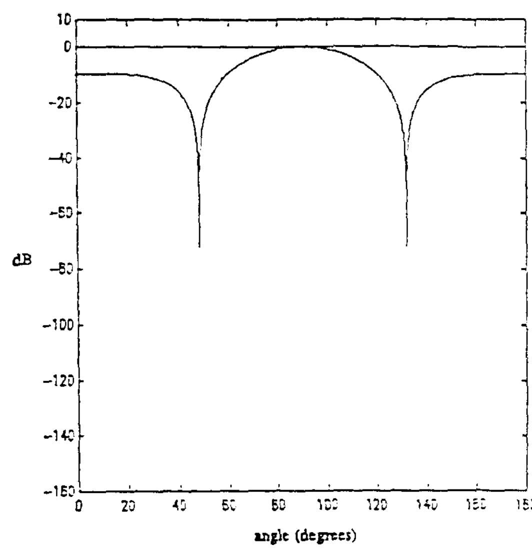

Fig. 3 Beam pattern for a 3-element broadside uniform linear array.

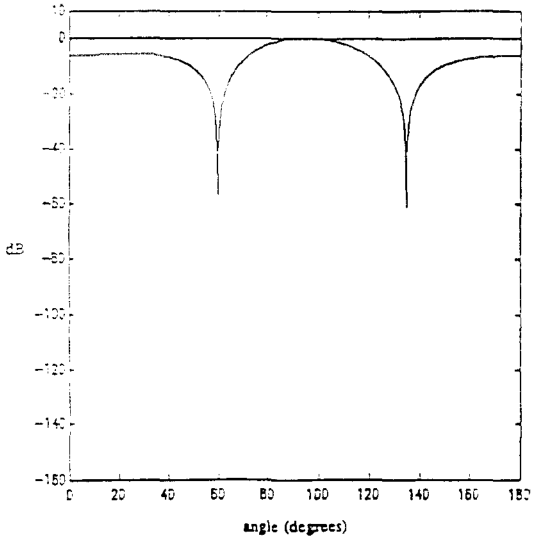

Fig. 4 Beam pattern with a point null at 60° and $r=0.1$.

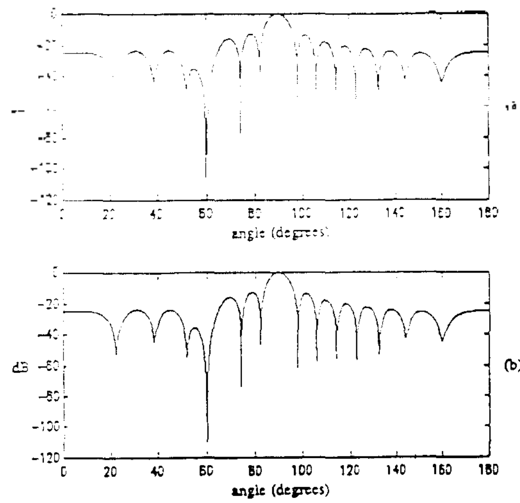

Fig. 6 Beam patterns of a 15-element linear array for narowband interference with $r=0.001$.

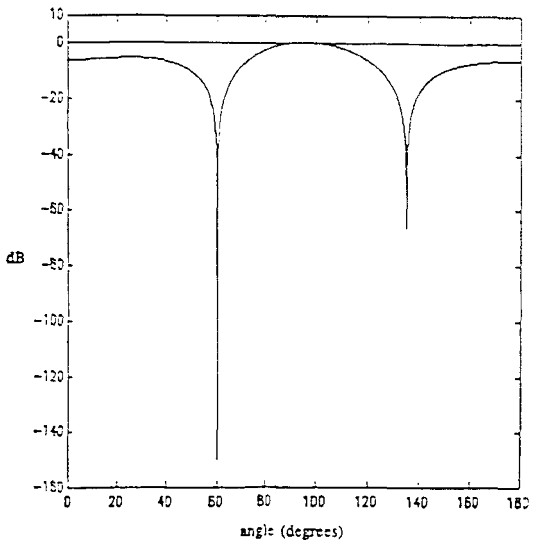

Fig. 5 Beam pattern by the best optimum weight vector.

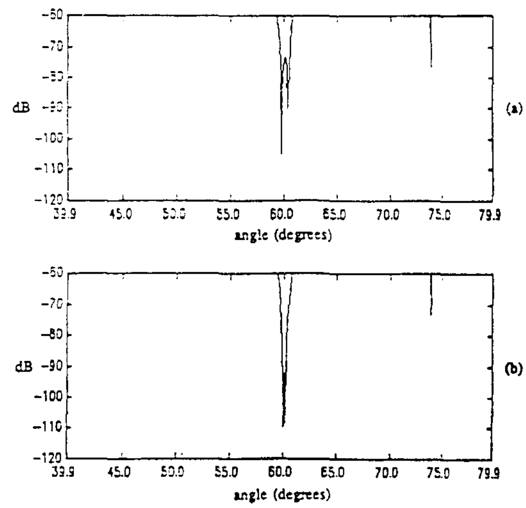

Fig. 7 Beam patterns around 60°.

Ⅵ. Conclusions

Linearly constrained optimization problem for a narrowband linear array which is subject to random variations in array weight and element position was discussed. The nulling performance was analyzed in the weight vector space with respect to sinusoidal and narrowband interference signals. It was found that a nonorthogonality condition exists between the optimum weight vector and the steering vector of the interference direction due to the relevant random errors. A novel approach to reducing the effects of random errors by compensating for the nonorthogonality was proposed to improve the array performance. It was shown that the nulling performance at the interference direction was remarkably improved with the compensated weight vector. The proposed approaches may be applied in practical sensor arrays to improve the performance in estimating a desired signal corrupted by undesired interferences in the presence of random errors with respect to array weight and sensor position.

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