Performance Analysis of Highly Effective Proposed Direction Finding Method
제안된 최적전파 도래방향각 예측기법 실현을 위한 성능분석

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※ This paper was supported by NON DIRECTED RESEARCH FUND, Kerea Research Foundation, 1993.

ABSTRACT

The main purpose of this paper is to show the realizability of the proposed highly effective direction finiding method which performs extremely well under the circumstances like low signal-to-noise ratio (S/N), very closely located signal sources, and so on.

In order to achieve the purpose, the degree to which the proposed method is superior to the MUSIC(multiple signal classification) with respect to the S/N is discussed, and the result is analyzed in terms of the S/N and the number of sample data.

요 약

본 논문의 주된 목적은 주위 환경의 영향등으로 인하여 신호대잡음비(S/N)가 저하된 상황, 다중신호원들이 근접하여 위치한 상황등의 특수한 상황들에서도 고해상도를 지니는 도래방향각 예측방법의 개념적 S/N의 향상도를 이론적으로 증명하고 기존의 도래방향각 예측방법과 비교하므로서 제안된 방법의 실용화를 위한 이론적 타당성을 입증하는데 있다.

Ⅰ. Introduction

Array signal processing is concerned as a field of analyzing and processing procedure of received data from a spatially distributed sensor array in noise. One of the essential goals of passive sonar or radar array signal processing is to estimate the direction-of-arrival (DOA) of distant source signals [1-3]. Compared with existing spectral estimation techniques [4-6], the MUSIC (MUltiple SIgnal Classification) [7-8] developed by Schmidt in 1981 is based on the eigenstructure algorithm and gives better resolution with less complexity of computation in DOA estimation. Since the advent of the MUSIC which is still used as a benchmark for the subsequently developing DOA estimation methods, there has been an epoch-making development in the area of DOA estimation and have been introduced lots of high resolution DOA estimation methods based on the eigenstructure algorithm [9-11]. However, the eigenstructure

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algorithm has crucial requirements that the additive sensor noises are spatially white Gaussian and uncorrelated from sensor to sensor. Due to those requirements, in the cases of correlated noise fields, non-Gaussian noises, and/or low S/N, the MUSIC often deteriorates its DOA estimation performance.

Recently, some other eigenstructure-based methods [12, 13, 16] have been suggested to compensate partially for the MUSIC's disadvantages stated above. In [14], a highly effective modified eigenstructure-based DOA estimation method has been proposed and compared with the previously developed method, under the circumstances such as correlated sensor noises, very closely located signal sources, and low S/N. The proposed method uses new data sequence obtained by auto-convolution operation on the original data sequence from sensor of array, different from the existing DOA estimation methods which direclty use the received original data sequence [1-13]. The key to the proposed method is to retain information on all the other data points in correlation operations with a constant lag and consequently improve its resolution for DOA estimation.

One of the main aims at this paper is to derive theoretically the S/N improvement of the proposed method over the MUSIC taken as a benchmark for the methods using original data sequence directly, and the other is to verify the superiority of the proposed method to the MUSIC in terms of the number of data and the S/N.

To achieve these goals, the derivation and DOA estimation procedure of the proposed method are briefly described in Section II. Section III verifies and analyzes the theoretical improvement of the S/N for the proposed method over the MUSIC. Lastly, this study is summarized and concluded with closing remarks in Section IV.

II. Signal and Noise Model

With the M narrow-band source signals incident to a uniform linear array of Q sensors from directions $\{\theta_1, \theta_2, \ldots, \theta_M\}$, the signals recevied at the i th sensor can be written as

$$r_i(t) = \sum_{m=1}^{M} s_m(t-(i-1)(D/c)\sin\theta_m) + x_i(t), \qquad (1)$$

where

$s_m(t)$ = the m th source signal,
D = the sensor spacing,
c = the wave propagation velocity,
θ_m = the DOA of the m th source,
$x_i(t)$ = the additive noise at the i the sensor, with independent and identically distributed (i.i.d) $x(1), x(2), \ldots, x(N)$.

The complex envelope representation [11] can be applied to the m th narrow-band source signal $s_m(t)$ with center frequency ω_0, for $r_i(t)$ in (1) to obtatin:

$$r_i(t) = \sum_{m=1}^{M} s_m(t) \exp[-j\,\omega_0\,\tau_{mi}] + x_i(t), \qquad (2)$$

where

$$\tau_{mi} = (i-1)(D/c)\sin\theta_m. \qquad (3)$$

The received signals on the Q sensors can be expressed in the vector form:

$$\mathbf{r}(t) = \sum_{m=1}^{M} \mathbf{a}(\theta_m) s_m(t) + \mathbf{x}(t), \qquad (4)$$

or

$$\mathbf{r}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{x}(t), \qquad (5)$$

where

$\mathbf{r}^T(t) = [r_1(t), r_2(t), \ldots, r_Q(t)]$,
$\mathbf{s}^T(t) = [s_1(t), s_2(t), \ldots, s_M(t)]$,
$\mathbf{x}^T(t) = [x_1(t), x_2(t), \ldots, x_Q(t)]$,

and the columns of the $Q \times M$ direction matrix $\mathbf{A}(\theta)$ are composed of the directional-vectors expressed as

$$\mathbf{a}^T(\theta_m) = [1, \exp[-j\,\omega_0\,\tau_{m2}],$$
$$\exp[-j\,\omega_0\,\tau_{m3}], \ldots, \exp[-j\,\omega_0\,\tau_{mQ}]]. \tag{6}$$

Now, we briefly describe the signal and noise model for the proposed DOA estimation method and its processing procedure [14]. Assuming that the additive noises are zero-mean and uncorrelated with the source singals, the Fourier transform of the received signal vector in (4) or (5) has the form of

$$\mathbf{F} = \mathbf{AS} + \mathbf{X}, \tag{7}$$

where

$\mathbf{F} = \mathcal{F}[\mathbf{r}]$, $\mathbf{S} = \mathcal{F}[\mathbf{s}]$, $\mathbf{X} = \mathcal{F}[\mathbf{x}]$,
$\mathcal{F}$ denotes the Fourier transform operator,

$\mathbf{F}^T = [F_1(\omega), F_2(\omega), \ldots, F_Q(\omega)]$,
and $F_i(\omega) = \mathcal{F}[r_i(t)]$.

Then, the spectral density matrix $\mathbf{L}_{(1)}$ of $\mathbf{r}$ in (5) is obtained as [15]

$$\mathbf{L}_{(1)} = E[\mathbf{F}\,\mathbf{F}^*]$$
$$= \mathbf{APA}^* + \mathbf{D}_{X1}, \tag{8}$$

where

$\mathbf{P} = E[\mathbf{S}\,\mathbf{S}^*]$ and $\mathbf{D}_{X1} = E[\mathbf{X}\,\mathbf{X}^*]$.

Now, the new signals for the proposed method are built by auto-convolution operations on the received signals as the i th sensor as

$$r_{(2)i}(t) = r_i(t) \otimes r_i(t), \tag{9}$$

where

$\otimes$ represents the convolution operator.

These new signals $r_{(2)i}(t)$ can be expressed, in the frequency domain, as the corresponding vector :

$$\mathbf{F}_{(2)}{}^T = [F_1{}^2(\omega), F_2{}^2(\omega), \ldots, F_Q{}^2(\omega)]. \tag{10}$$

In order to efficiently handle those matrices a new matrix operator Δ, called a "delta product", performing a component to component multiplication, is introduced as

$$\mathbf{A}\,\Delta\,\mathbf{B} = \mathbf{C} \longleftrightarrow a_{ij} \cdot b_{ij} = c_{ij}, \tag{11}$$

where

matrices **A**, **B**, and **C** are of identical dimensions.

Then, by definition, the spectral density matrix of the new signal $r_{(2)i}(t)$ is expressed as

$$\mathbf{L}_{(2)} = E[\mathbf{F}_{(2)}\,\mathbf{F}_{(2)}{}^*]$$
$$= E[(\mathbf{FF}^*)^{\Delta 2}]$$
$$= E\{[(\mathbf{AS}+\mathbf{X})(\mathbf{AS}+\mathbf{X})^*]^{\Delta 2}\}$$
$$= E[(\mathbf{AS\,S^*\,A^*} + \mathbf{ASX^*} + \mathbf{XS^*A^*} + \mathbf{XX^*})^{\Delta 2}], \tag{12}$$

where

$\Delta 2$ is the operator derived from (11) such that $\mathbf{A}^{\Delta 2} = \mathbf{A}\,\Delta\,\mathbf{A}$.

Note that in (12), each element inside the expectation bracket is the same as an element of $\mathbf{L}_{(1)}$, but is raised to the power of 2.

Using the characteristics of Δ product and conditions given, (12) can be simplified as [15]

$$\mathbf{L}_{(2)} = \mathbf{A}_2\mathbf{P}_2\mathbf{A}_2{}^* + 2[(\mathbf{APA}^*)^{\Delta 2} - \mathbf{A}_2\mathbf{P}_2\mathbf{A}_2{}^*] + 2(\mathbf{APA}^*)\,\Delta\,E[\mathbf{XX}^*]$$
$$+ 2(\mathbf{APA}^*)\,\Delta\,E[\mathbf{XX}^*] + E[(\mathbf{XX}^*)^{\Delta 2}], \tag{13}$$

where

$\mathbf{A}_2 = \mathbf{A}^{\Delta 2}$ and $\mathbf{P}_2 = E[(\mathbf{SS}^*)^{\Delta 2}]$.

Furthermore, from (8),

$$\mathbf{APA}^* = \mathbf{L}_{(1)} - \mathbf{D}_{X1}. \tag{14}$$

Combining (13) with (14) and arranging the result by way of Δ product yield :

$$\mathbf{L}_{(2)} = \mathbf{A}_2\mathbf{P}_2\mathbf{A}_2^* + 2(\mathbf{L}_{(1)} \quad \mathbf{D}_{X1})^{\Delta 2} - \mathbf{A}_2\mathbf{P}_2\mathbf{A}_2^*$$
$$+ 4(\mathbf{L}_{(1)} \quad \mathbf{D}_{X1})\, \Delta\, \mathbf{D}_{X1} + E[(\mathbf{XX}^*)^{\Delta 2}]$$
$$= -\mathbf{A}_2\mathbf{P}_2\mathbf{A}_2^* - 2\mathbf{D}_{X1}^{\Delta 2} + E[(\mathbf{XX}^*)^{\Delta 2}] + 2\mathbf{L}_{(1)}^{\Delta 2}. \quad (15)$$

If we define the spectral density matrix of the proposed method as

$$\mathbf{L}_{(2)R} = 2\mathbf{L}_{(1)}^{\Delta 2} - \mathbf{L}_{(2)} \quad (16)$$

and combine with (15), we get :

$$\mathbf{L}_{(2)R} = \mathbf{A}_2\mathbf{P}_2\mathbf{A}_2^* + (2\mathbf{D}_{X1}^{\Delta 2} - E[(\mathbf{XX}^*)^{\Delta 2}]). \quad (17)$$

Now, (9) for the orthogonality of [16] is applied to (17) by squaring each element of the directional-vector in (6) [14], that is

$$\hat{\mathbf{D}}_S = \left[\mathbf{a}_s^*(\theta) \left(\sum_{k=1}^{Q} \mathbf{v}_k \mathbf{v}_k^* \right) \mathbf{a}_s(\theta) \right]^{-1}, \quad (18)$$

where

$$\mathbf{a}_s^T(\theta) = [1, \exp[-2j\,\omega_0\,k_{x2}],$$
$$\exp[-2j\,\omega_0\,k_{x3}], \ldots, \exp[-2j\,\omega_0\,k_{xQ}]] \quad (19)$$

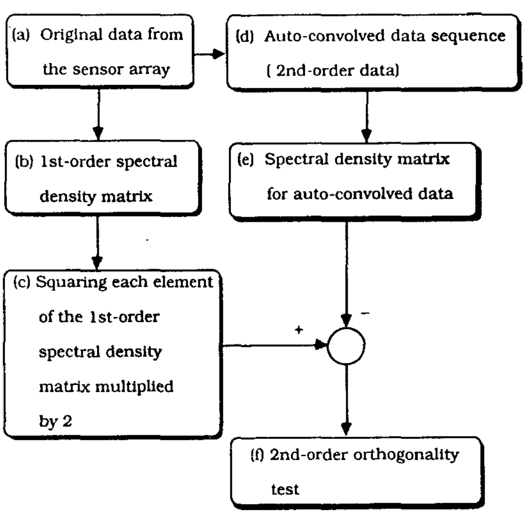

Fig 1. Block diagram for realization of the proposed method.

and, $\mathbf{v}_k$, $k = 1, 2, \ldots, Q$, are the eigenvectors corresponding to the eigenvalues of the covariance matrix obtained by inverse Fourier transform [15] of $\mathbf{L}_{(2)R}$ in (17).

Fig. 1 shows the procedure of peforming the proposed method DOA estimation, in the form of block diagram.

We now describe the procedures, shown in Fig. 1, to carry out DOA estimation using the proposed method in detail as follows :

(a) Obtain the received data from a sensor array in the presence of background additive noise (refer to (11).
(b) Get the spectral density matrix from the received original data obtained in (a) (see (8)).
(c) Square and then double each element value of the spectral density matrix in (8).
(d) On the other hand, produce the new data sequences by auto-convolution of the original data sequences taken from (a) (confer (9)).
(e) Obtain the spectral density matrix using the auto-convolved data sequences of (d) (refer to (12)).
(f) Finally, perform DOA estimation by applying the spectral density matrix, which is obtained in consequence of subtraction of the matrix in (c) from the one in (e), to (18) and (19).

Ⅲ. S/N Improvement of Proposed Method

In this section we compare and discuss the performances of the MUSIC which is considered as a benchmark for the subsequently developing DOA estimation methods and the proposed method described above, based on the theoretically derived S/N.

For the relative comparison of the MUSIC and the proposed method in terms of the S/N, consider a deterministic source signal $s(t) = A\exp(j\omega t)$, $t = 1, 2, \ldots, N$, with zero-sample mean and complex stationary random noise $x(t)$ with zero-mean and variance $2\sigma_x^2$. When the source signal and noise are uncorrelated with each other, the S/N of the MUSIC can be obtained as

$$[S/N]_{MUSIC} = A^2/(2\sigma_x^2). \quad (20)$$

Considering a single source for simplicity, a received signal with additive noise, i.e., $r(t) = s(t) + x(t)$, can be auto-convolved to obtain the new signal for the proposed method :

$$r_s(t) = r(t) \otimes r(t)$$
$$= \{s(t) + x(t)\} \otimes \{s(t) + x(t)\}$$
$$= s_s(t) + x_s(t) + 2\{s(t) \otimes x(t)\}, \quad (21)$$

where

$s_s(t) = s(t) \otimes s(t)$ and $x_s(t) = x(t) \otimes x(t)$.

For $s(t)$ and $x(t)$ to be independent, $E_s[r_s(t)]$, where $E_s[\cdot]$ denotes the sample mean through $t = 1, 2,..., 2N-1$, must be equal to zero. Therefore, the variance of $r_s(t)$ can be obtained as

$$V_s[r_s(t)] = E_s[r_s(t) r_s^*(t)]$$
$$= E_s[s_s(t)s_s^*(t)] + E_s[x_s(t)x_s^*(t)] + E_s[s_s(t)x_s^*(t)]$$
$$+ E_s[x_s(t)x_s^*(t)] + 2E_s[s_s(t)\{s(t) \otimes x(t)\}^*]$$
$$+ 2E_s[\{s(t) \otimes x(t)\}x_s^*(t)] + 2E_s[\{s(t) \otimes (t)\}s_s^*(t)]$$
$$+ 2E_s[x_s(t)\{s(t) \otimes x(t)\}^*]$$
$$+ 4E_s[\{s(t) \otimes x(t)\}\{s(t) \otimes x(t)\}^*].$$

Given that each odd moment of the zero-mean process is zero and the independence of $s(t)$ and $x(t)$, the remaining terms of the above equation become :

$$E_s[r_s(t)r_s^*(t)] = E_s[s_s(t)s_s^*(t)] + E_s[x_s(t)x_s^*(t)]$$
$$+ 4E_s[\{s(t) \otimes x(t)\}\{s(t) \otimes x(t)\}^*]. \quad (22)$$

Thus the value of $E_s[\{s(t) \otimes x(t)\}\{s(t) \otimes x(t)\}^*]$ in (22) is approximated in the finite discrete case by

$$E_s[\{s(t) \otimes x(t)\}\{s(t) \otimes x(t)\}^*]$$

$$\approx E_s[\sum_{k=1}^{N} \sum_{m=1}^{N} s(k)\, x(t-k)\, s^*(m)\, x^*(t-m)]. \quad (23)$$

Note that $x(t)$ takes the form as

$x(t) = a(t) + j\, b(t),\ t = 1, 2,..., N,$

where

$a(t)$ and $b(t)$ are independent and real-valued random processes.

If $a(t)$ and $b(t)$ have the same distributions with zero-mean and variance σ_x^2, then
$E[x(t)] = E[a(t) + j\, b(t)] = 0,$
and $E[x(t)x^*(t)] = E[a^2(t) + b^2(t)] = 2\,\sigma_x^2$.

Using those facts, (23) can be rewitten as follows :

i) <u>When $k = m$,</u>

$$E_s[\{s(t) \otimes x(t)\}\{s(t) \otimes x(t)\}^*]$$

$$\approx E_s[(\sum_{k=1}^{N} s(k)\, s^*(k))\, (x(t-k)x^*(t-k)]$$

$$= \sum_{k=1}^{N} (s(k)\, s^*(k))\, E_s[x(t-k)x^*(t-k)]$$

$$\approx (N A^2)\, ((N/(2N-1))(2\sigma_x^2))$$

$$= (N^2 A^2/(2N-1))(2\sigma_x^2)\ ; \quad (24)$$

ii) <u>Otherwise,</u>

$E_s[\{s(t) \otimes x(t)\}\{s(t) \otimes x(t)\}^*] = 0$, since all the terms of the above equation $E_s[\{s(t) \otimes x(t)\}\{s(t) \otimes x(t)\}^*]$ consist of odd moments of $s(t)$ and $x(t)$.

The valuses of $E_s[s_s(t)s_s^*(t)]$ and $E_s[x_s(t)x_s^*(t)]$, are then obtained, one-by-one. The new source signal $s_s(t)$ can be obtained by taking the auto-convolution of the original source signal as

$$s_s(t) = s(t) \otimes s(t) = \sum_{k=1}^{N} s(k)\, s(t-k).$$

Since

$$s_s(t) = \begin{cases} A^2 \exp(j\omega) & \text{when } t = 1, \\ 2A^2 \exp(j2\omega) & \text{when } t = 2, \\ \quad \vdots & \\ (N-1)A^2 \exp(j-1)\omega) & \text{when } t = N-1, \\ N A^2 \exp(j N \omega) & \text{when } t = N, \\ (N-1)A^2 \exp(j (N+1)\omega) & \text{when } t = N+1, \\ (N-2)A^2 \exp(j (N+2)\omega) & \text{when } t = N+2, \\ \quad \vdots & \\ A^2 \exp(j (2N-1)\omega) & \text{when } t = 2N-1, \\ 0 & \text{otherwise,} \end{cases}$$

where

$s_s(t)$ has maximum value of magnitude at $t = N$.

The variance of $s_s(t)$, i.e., $E_s[s_s(t)s_s^*(t)]$, is obtained as

$$E_s[s_s(t)s_s^*(t)]$$

$$\approx (1/2N-1)) \sum_{k=1}^{2N-1} s_s(k)s_s^*(k)$$

$$= (A^4/(2N-1)) \ (2 \ (\sum_{k=1}^{N} k^2) - A^2)$$

$$= (A^4/(2N-1)) \ \{(2N(N+1)(2N+1)/6) - N^2\}$$

$$= \{A^4(2N^3+N)\}/\{3(2N-1)\}. \tag{25}$$

The variance of $x_s(t)$, i.e., $E_s[x_s(t)x_s^*(t)]$, in (22) can be also expressed as

$$E_s[x_s(t)x_s^*(t)]$$

$$\approx E_s[\sum_{k=1}^{N} \sum_{m=1}^{N} x(k) \ x(t-k) \ x^*(m) \ x^*(t-m)]$$

$$= (1/(2N-1)) \sum_{k=1}^{2N-1} x_s(k) \ x_s^*(k), \tag{26}$$

where

$$x_s(t) = \sum_{k=1}^{N} s(k) \ s(t-k)$$

Then, (26) can be successfully analyzed by considering the following cases :

i) <u>When $k \neq m$ and $t = k+m$,</u>

$$E_s[x_s(t)x_s^*(t)]$$

$$= E_s[\sum_{k=1}^{N} \sum_{m=1, m\neq k}^{N} x(k) \ x^*(m) \ x(m) \ x^*(k)]$$

$$= (1/(2N-1))[\sum_{k=1}^{N} \sum_{m=1, m\neq k}^{N} x(k) \ x^*(k) \ x(m) \ x^*(m)]$$

$$= (1/(2N-1))\{\sum_{k=1}^{N} x(k) \ x^*(k)\}\{\sum_{m=1, m\neq k}^{N} x(m) \ x^*(m)\}, \tag{27}$$

and for large N,

$$E[x(t)x^*(t)] = 2\sigma_x^2 = (1/N) \sum_{m=1}^{N} x(m)x^*(m). \tag{28}$$

Thus, (27) can be rewritten as the following (29) :

$$E_s[x_s(t)x_s^*(t)]$$

$$= (1/(2N-1))\{4\sigma_x^4 N^2 - \sum_{m=1}^{N} x^2(m) \ (x^*(m))^2\}$$

$$= (1/(2N-1))\{4 \ \sigma_x^2 N^2 - 2(K_e+1)\sigma_x^4 N\} \tag{29}$$

where

$$K_e = \{E[a^4(t)]/E^2[a^2(t)]\}. \tag{30}$$

Note that for $x(t) = a(t) + j \ b(t)$, $t = 1, 2, \ldots, N$, where $a(t)$ and $b(t)$ are independent and real-valued random processes,

$$E[x^2(t)] = E[(a(t) + jb(t))^2]$$

$$= E[(a^2(t) - b^2(t) + j \ 2a(t)b(t)]$$

$$= 0, \tag{31}$$

since $a(t)$ and $b(t)$ are independent and have the same distributions with zero-mean and variance σ_x^2.

In the same manner,

$$E[(x^*(t))^2]=0. \quad (32)$$

Futhermore,

$$E[x^2(t)(x^*(t))^2]$$
$$=E[(a(t)+j\,b(t))^2\,(a(t)-j\,b(t))^2]$$
$$=E[a^4(t)]+E[b^4(t)]+2E[a^2(t)]E[b^2(t)]$$
$$=K_e E^2[a^2(t)]+K_e E^2[b^2(t)]+2E[a^2(t)]E[b^2(t)]$$
$$=2K_e\,\sigma_x^4+2\sigma_x^4$$
$$=2(K_e+1)\sigma_x^4. \quad (33)$$

ii) When $k \neq m$ and $t \neq k+m$,

Since (26) is composed of all odd moments and each odd moment of a symmetrically distributed zero-mean process is zero, the result is :

$$E_s[x_s(t)x_s^*(t)]=0. \quad (34)$$

iii) When $k=m$,

$$E_s[x_s(t)x_s^*(t)]$$
$$=E_s[\sum_{k=1}^{N} x(k)x^*(k)x(t-k)x^*(t-k)]$$
$$=[\sum_{k=1}^{N} x(k)x^*(k)]E_s[x(t-k)x^*(t-k)]$$
$$=(N^2/(2N-1))(4\sigma_x^4), \quad (35)$$

where

$$k+1 \leq t \leq N+k.$$

Now, adding (29), (34), and (35) yields :

$$E_s[x_s(t)x_s^*(t)]$$
$$=(4N-K_e-1)((2N\,\sigma_x^4)/(2N-1)). \quad (36)$$

When we define the S/N for the proposed method as

$$[S/N]_s=E_s[s_s(t)s_s^*(t)]/E_s[x_s(t)x_s^*(t)], \quad (37)$$

the value in (24) causes an error in the S/N of the proposed method. However, the value in (24) can be englected if $A^2 \ll 2\sigma_x^2$, since, under this condition,

$$E_s[\{s(t)\otimes x(t)\}\{s(t)\otimes x(t)\}^*] \ll E_s[x_s(t)x_s^*(t)].$$

Therefore, from (25) and (36), the S/N of the proposed method can be expressed as

$$[S/N]_s=\{A^4(2N^2+1)\}/\{6\sigma_x^4(4N-K_e-1)\}, \quad (38)$$

or, using expression of the $[S/N]_{MUSIC}$ in (20), we have :

$$[S/N]_s=([S/N]_{MUSIC})^2\{2(2N^2+1)\}/\{3(4N-K_e-1)\}. \quad (39)$$

Note that (39) always bocomes :

$$[S/N]_s/[S/N]_{MUSIC} \geq 1 \quad (40)$$

as long as

$$[S/N]_{MUSIC} \geq \{3(4N-K_e-1)\}/\{2(2N^2+1)\}, \quad (41)$$

which indicates that the proposed method provides better resolution than the MUSIC so long as the inequality in (41) is maintained.

For example, if both $a(t)$ and $b(t)$ are Gaussian random processes, then $K_e=3$ from (30) and thereby the minimum values of the $[S/N]_{MUSIC}$ satisfying (40) can be obtained, with respect to the number of sample data N, as in Table 1.

Table 1 demonstrates that the minimum value of the $[S/N]_{MUSIC}$ decreases as N increases, for astisfying the improved S/N of the proposed method compared to the S/N of the MUSIC.

Table 1. Minimum $[S/N]_{MUSIC}$ for Gaussian random noises in order to hold eq.(41).

N	Minimum $[S/N]_{MUSIC}$ [dB]
32	-10.42
64	-13.36
128	-16.33
256	-19.32
512	-22.37
10.24	-25.38

Even taking small N of 32, the proposed method provides better S/N than the MUSIC, as long as the S/N from the original data becomes larger than approximately -10 dB, which practically indicates very poor circumstances.

On the other hand, with fixed values of $N=$ 128,256 and 512, the graphs for the $[S/N]_s$ are obtained as a function of the $[S/N]_{MUSIC}$ (Fig. 2). From Fig. 2, we can easily see that the proposed method provides better S/N than the MUSIC's S/N satisfying the minimum $[S/N]_{MUSIC}$ for N given in Table 1.

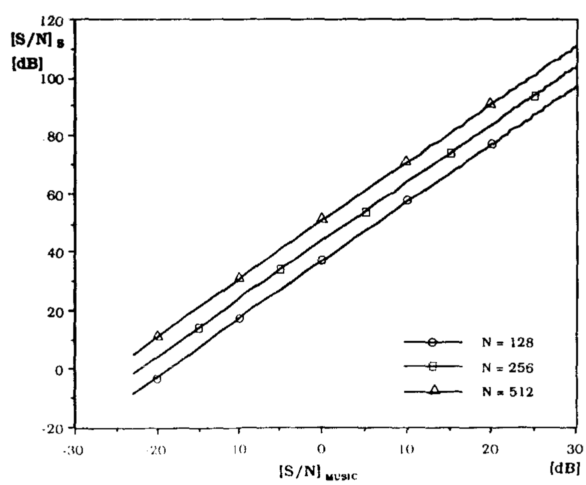

Fig 2. $[S/N]_s$ expressed in terms of $[S/N]_{MUSIC}$, with $N=128,256$, and 512.

Fig. 3 shows the three graphs for the $[S/N]_s$ drawn in terms of N, when the values of $[S/N]_{MUSIC}$ are given as -3.01 dB, 0 dB, and 3.01 dB, respectively. As shown in the figure, all the values of $[S/N]_s$ continuously increase as N becomes larger. Furthermore, Fig. 3 indicates that the proposed method improves the S/N regardless of N if the $[S/N]_{MUSIC}$ has the value of 0 dB or 3.01 dB, and that $[S/N]_s/[S/N]_{MUSIC} \geq 1$ holds as long as $N \geq 8$ if the $[S/N]_{MUSIC}$ has the value of -3.01 dB. Overall the example described so far represents that the proposed method offers better performance than the MUSIC even in very poor circumstances.

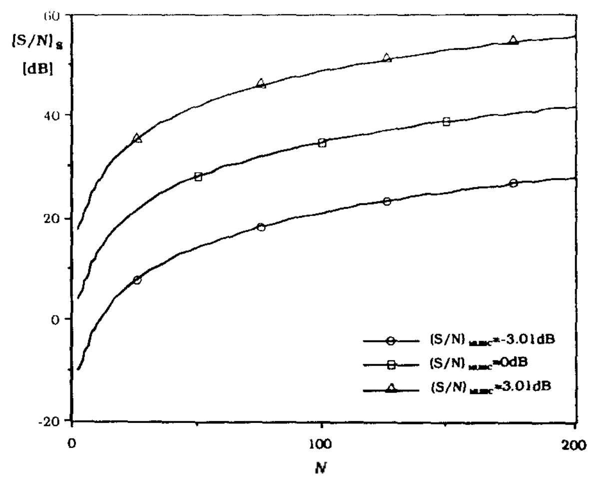

Fig 3. $[S/N]_s$ expressed in terms of N, with $[S/N]_{MUSIC}$ = -3.01 dB, 0 dB, and 3.01 dB.

Ⅳ. Conclusion

Different from various versions of DOA estimation methods based on the eigenstructure algorithm, the key to proposed method is the use of the new data sequence by auto-convolution of the original received data on each sensor of the array, where each auto-convolved data point provides information on all the other data points in correlation operations with a constant lag.

Thus, the S/N for the new data sequence is improved and thereby the proposed method can be effectively used to accommodate troublesome cases such as closely located multiple sources, limited number of sensors, and/or low S/N[14].

In this paper, not only the theoretical derivation of the S/N for the proposed method that performs high resolution DOA estimation even in poor circumstances described above, but also the conditions for the proposed method to improve the S/N compared to the MUSCIC have been made. Furthermore, throughout a concrete example considering a practical point of view, the superior performance of the proposed method to that of the MUSIC has been discussed from S/N and number of data points of view. It is therefore expected that these efforts may lead to new insight into realization of the proposed method for high resolution DOA estimation.

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