An Improved Parametric Estimation Method of High-Resolution Bispectrum

고해상도의 바이스펙트럼을 추정하기 위한 개선된 매개변수 방법

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ABSTRACT

The maximum entropy method is a well-known parametric estimation method of the power spectrum with high-resolution for short-time signals. Although a parametric estimation method for the bispectrum was proposed in recent years, it is not easy to estimate the bispectrum with high resolution for relatively short-time signals of which the total length is about 1000 data points. In this paper, a bispectrum estimation method is proposed to estimate the high-resolution bispectrum even for the relatively short-time signals.

요 약

측정된 신호의 길이가 짧은 경우, 높은 해상도의 전력스펙트럼을 추정하는 매개변수 방법으로 최대 엔트로피 방법이 있다. 바이스펙트럼을 추정하기 위한 매개변수 방법은 최근에 제안 되었는데 표본화한 데이타의 길이가 약 1000 정도의 비교적 짧은 신호에 적용할 경우 높은 해상도를 얻기 어렵다. 이 논문에서는 위와 같은 비교적 짧은 신호에 대해서도 높은 해상도의 바이스펙트럼을 추정할 수 있는 방법을 제안하였다.

I. Introduction

Many techniques have been developed to estimate power spectra[1, 2]. The maximum entropy method (MEM) is one of the well-known parametric methods to obtain high-resolution power spectra[2, 3]. The MEM has been widely used to improve the frequency resolution of the power spectrum for short time duration data records. Unfortunately, it is somewhat difficult to apply the concept of the MEM to estimate higher-order spectra[4]. The classical FFT method has been widely used to estimate higher-order spectra. However, it is limited when analyzing short duration random signals because of its relatively low frequency resolution[5, 6].

Although the parametric approach to estimate higher-order spectra has not been extensively developed, a parametric bispectrum estimation method by using an autoregressive model was proposed in[7, 8]. The algorithm is useful in estimating a high-resolution bispectrum for random data. It is the objective of this paper to describe an im-

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proved parametric bispectrum estimation method and demonstrate its improved performance under relatively poor signal-to-noise ratio conditions and relatively short data lengths.

In the next section, some properties of the bispectrum will be described. In Section Ⅲ, the proposed method will be presented. In Section Ⅳ, the performances of the proposed parametric bispectral estimator of this paper and the parametric bispectral estimator proposed in [7, 8] will be presented. In Section Ⅴ, advantages and limitations of the various bispectrum estimation methods will be summarized.

Ⅱ. Properties of Bispectrum

Consider a real zero-mean random process $x(t)$ which is stationary to order three, then the triple correlation function, namely, bicorrelation function, of the random process is given by

$$R_{xxx}(\tau_1, \tau_2) = E[x(t)x(t-\tau_1)x(t-\tau_2)] \quad (1)$$

where E denotes the expected value. As shown in Eq. (1), the bicorrelation function is a function of two time delays, τ_1 and τ_2. As the correlation function is the 1-D inverse Fourier transform of the power spectrum, the bicorrelation function is given by the 2-D inverse Fourier transform of the bispectrum, $S_{XXX}(f_1, f_2)$,

$$R_{xxx}(\tau_1, \tau_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{XXX}(f_1, f_2) e^{j2\pi(f_1\tau_1 + f_2\tau_2)}\, df_1\, df_2. \quad (2)$$

Ⅲ. Model and Estimation

We start with the same model as proposed in[8]. Consider a real r-th order autoregressive (AR) process $x(n)$,

$$x(n) + \sum_{i=1}^{r} a_i x(n-i) = e(n) \quad (3)$$

where $e(n)$'s are i.i.d. with $E[e(n)] = 0$, $E[e^2(n)] = Q$, $E[e^3(n)] = \beta \neq 0$ and $x(m)$ is independent of $e(n)$ for $m < n$. Note that $e(n)$ is zero mean non-Gaussian.

Since $e(n)$ is third-order stationary it follows that $x(n)$ is also third-order stationary. Multiplying both sides of Eq. (3) by $x(n-j)x(n-k)$ and then taking the expectation and using the symmetry property of the bicorrelation functions, we obtain

$$R(j, k) + \sum_{i=1}^{r} a_i R(j-i, k-i) = \beta\delta(j, k) \quad k \geq j \geq 0 \quad (4)$$

where $R(m, n)$ is the third-order moment sequence of the AR process and $\delta(j, k)$ is the 2-D unit impulse function, $j = 0, 1, 2, \cdots, r$, $k = 0, 1, 2, \cdots, r$ and $j \leq k$. In Eq. (4), $a_1, a_2, \cdots, a_r$, and β are unknowns. Since $j = 0, 1, 2, \cdots, r$, $k = 0, 1, 2, \cdots, r$, and $j \leq k$, the number of independent AR bicorrelation equations is $\frac{(r+1)(r+2)}{2}$. Since the number of equations is greater than the number of unknowns, it is overdetermined.

Since there are $r+1$ unknowns, only $r+1$ equations in which $j = k$ in Eq. (4) are chosen among $\frac{(r+1)(r+2)}{2}$ equations in order to calculate the unknowns (i.e., parameters) in the method proposed in [8]. Generally speaking, however, it is not easy to state that the parameters which are obtained by using the former method also satisfy the other equations in which $j \neq k$. Therefore, we utilize a multiple regression method to provide a best-fitting plane to the data, that is, we obtain the least squares estimates of the a_i's, which are optimum in a least squares sense. In other words, we use all $\frac{(r+1)(r+2)}{2}$ equations, in order to calculate the overdetermined $r+1$ unknowns by using the least squares method[9, 10].

We may thus rewrite Eq. (4) in matrix form as follows :

$$\mathbf{R}\,\mathbf{a} = \mathbf{b} \tag{5}$$

where

$$\mathbf{R} = \begin{bmatrix} R(0,0) & R(-1,-1) & \cdots & R(-r,-r) \\ R(0,1) & R(-1,0) & \cdots & R(-r,1-r) \\ & \cdots & \cdots & \\ R(0,r) & R(-1,r-1) & \cdots & R(-r,0) \\ R(1,1) & R(0,0) & \cdots & R(1-r,1-r) \\ R(1,2) & R(0,1) & \cdots & R(1-r,2-r) \\ & \cdots & \cdots & \\ R(r,r) & R(r-1,r-1) & \cdots & R(0,0) \end{bmatrix}, \tag{6}$$

$$\mathbf{a} = [1 \;\; a_1 \;\; a_2 \;\; \cdots \;\; a_r]^T, \tag{7}$$

$$\mathbf{b} = [\beta \;\; 0 \;\; 0 \;\; \cdots \;\; 0]^T, \tag{8}$$

and $\mathbf{T}$ denotes the transpose.

Thus, by solving the following matrix equation, one can obtain the least squares estimates of the a_i parameters [9, 10],

$$\mathbf{R}^T\,\mathbf{R}\,\mathbf{a} = \mathbf{R}^T\,\mathbf{b}. \tag{9}$$

Finally, the bispectrum is given by [8]

$$S_{XXX}(f_1, f_2) = \beta\, H(f_1)\, H(f_2)\, H^*(f_1+f_2) \tag{10}$$

where

$$H(f) = \left[\, 1 + \sum_{i=1}^{r} a_i \exp(-j2\pi f i) \right]^{-1}. \tag{11}$$

Ⅳ. Experimental Results

We show experimental tests which compare their ability to resolve two closely spaced bispectrum peaks in the bispectral plane. Consider a signal given by

$$x(t) = \sum_{i=1}^{6} \cos(2\pi g_i t + \theta_i) + n(t). \tag{12}$$

The frequency components of the signal are given by $g_1 = 0.34$ Hz, $g_2 = 0.14$ Hz, $g_3 = 0.48$ Hz, $g_4 = 0.35$ Hz, $g_5 = 0.15$ Hz and $g_6 = 0.50$ Hz. The phase relations of the signal are given by $\theta_1 + \theta_2 = \theta_3$ and $\theta_4 + \theta_5 = \theta_6$, that is, the former three sinusoids are correlated and the latter three sinusoids are also correlated. Therefore, two bispectrum peaks of the signal given in Eq. (12) will appear at (0.34 Hz, 0.14 Hz) and (0.35 Hz, 0.15 Hz) on the bispectrum plane, that is, the two bispectra frequency coordinates are very close. The Nyquist frequency is 0.64 Hz in these experiments.

The bispectrum is obtained by using the following two techniques: (1) the parametric method proposed in this paper (Method Ⅰ), (2) the parametric method proposed in [8] (Method Ⅱ). The experiments are carried out for data lengths (L's) of 8192 (i.e., 8k), 2048 (i.e., 2k) and 1024 (i.e., 1k) and signal-to-noise ratios (SNR's) of 30, 10 and 3 dB. A sample function, of which the length is L, of a random process is generated and is divided into M segments for ensemble averaging in Eq. (4) since the random process is stationary. Let the number of data points in each segment be N, that is, L = M × N. When we use the AR methods, the maximum model order we can choose is N-1, that is, the maximum r in Eq. (3) is N-1. For example, the maximum model order is 63, when N = 64. Since there is not a method to determine the optimum model order in order to estimate the bispectrum parametrically, experiments are carried out for all model orders ranging from 3 to N-1.

If the SNR is 30 dB and the data length of the signal is 8k (we choose M = 128 and N = 64 for convenience), Method Ⅰ and Method Ⅱ are able to resolve these two closely spaced bispectra. If the SNR of the signal of which the data length is 8k is reduced from 30 dB to 3 dB, Method Ⅰ can resolve the two peaks in the noisy signal with model order 30, as shown in Fig. 1(a). Method Ⅱ can also separate the two peaks with model order 61. However, one of the two peaks is much sm-

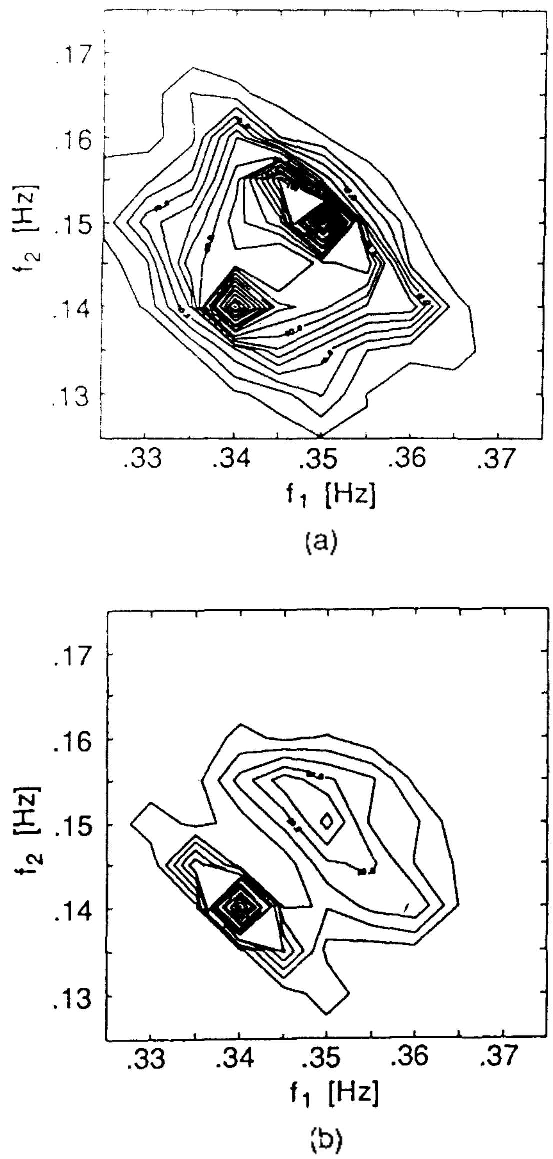


Fig. 1. Estimated two closely bispectrum peaks (number of data length = 8k, $M=128$, $N=64$, SNR = 3 dB) ; (a) Method I, (b) Method II

aller than the other peak as shown in Fig. 1(b). In this example, Method II can separate the two peaks only for model orders 53, 54, 56, 57, 61 and 62, while Method I can resolve the two peaks for all model orders from 30 to 63.

Next, consider a worse case in which the data length is reduced to 2k, in this case, the number of ensemble averaging to calculate the autocorrelation function in Eq. (4), M, is reduced from 128 to 32 and the SNR is 3 dB. Method I can separate the closely spaced two peaks with model orders 34 to 50. However, Method II can not resolve the closely spaced two peaks for all model orders, ranging from 3 to 63. Contour plots using Method I with model order 35 and using Method II with model order 57, are shown in Fig. 2(a) and (b), respectively.

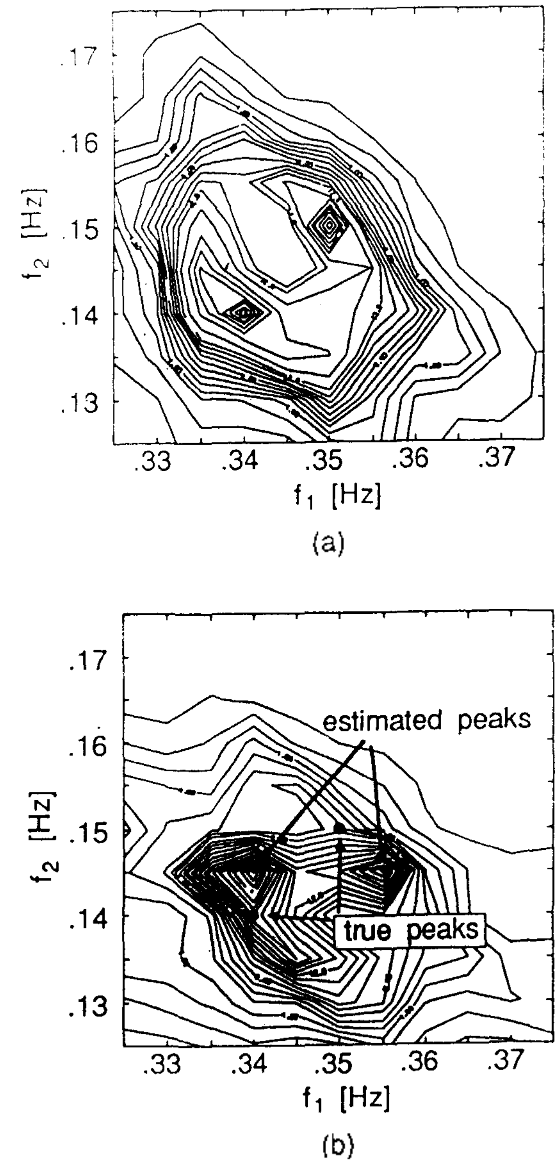
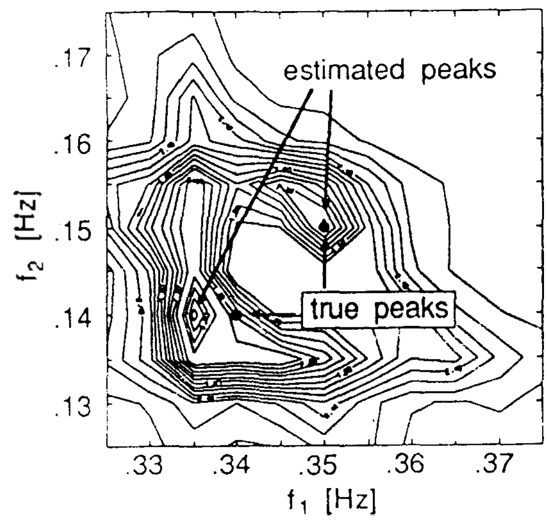
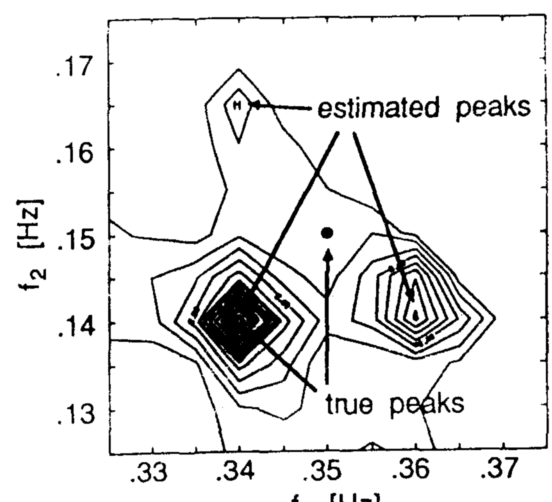


Fig. 2. Estimated two closely bispectrum peaks (number of data length = 2k, $M=32$, $N=64$, SNR = 3 dB) ; (a) Method I, (b) Method II

We consider an even worse case where the data length is reduced to 1k. In this case, we choose 16 as M and, therefore, N is 64, at a SNR of 3 dB. In Fig. 3(a) and (b) are shown contour plots for Method I (model order = 48) and Method II (model order = 44), respectively. Note that Method I does resolve the closely spaced bispectra, but with a slight error in the frequency coordinates of the peak. That is, the estimated coordinates are (0.335 Hz, 0.14 Hz) and (0.35 Hz, 0.15 Hz), while the true coordinates are (0.34 Hz, 0.14 Hz) and (0.35 Hz, 0.15 Hz). In the case of Method II, three peaks are apparent. There is considerable error between the estimated coordinates and the true coordinates.



(a)



(b)

Fig. 3. Estimated two closely bispectrum peaks (number of data length = 1k, $M = 16$, $N = 64$, SNR = 3 dB) : (a) Method I, (b) Method II

In Table 1 we summarize our experiments in resolving two closely spaced peaks in the bispectrum using Method I and Method II. Table 1 indicates that Method I and Method II have somewhat comparable resolving powers for relatively long data records (say, 8k). Moreover, as data length decreases (say, 2k) the two approaches have comparable performances at fairly good SNR's (say, greater than 10dB). However, for smaller SNR's or for shorter data records Method I, proposed in this paper, clearly offers the better performance.

Table 1. Resolution of two methods for two very closely spaced bispectra

data length	M	N	SNR	Method I	Method II
8k	64	128	30dB	YES	YES
			10dB	YES	YES
			3dB	YES	YES
2k	32	64	30dB	YES	YES
			10dB	YES	NO
			3dB	YES	NO
1k	16	64	30dB	YES	NO
			10dB	YES*	NO
			3dB	YES*	NO

*The location of one of the two estimated bispectrum peaks is with a slight error(0.005Hz) in the frequency coordinates of the peak as shown in Fig. 3(a).

V. Conclusions

The results represented in Section IV are not general since they are based on specific simulation experiments. However, they are useful in terms of providing some insight into the advantages and limitations of the bispectral estimator discussed in this paper. On the basis of this initial investigation, it appears that Method I is a particularly useful approach for estimating bispectra when dealing with relatively low SNR (about 3 dB) and short duration data records (about 1k data points). On the other hand, when the data record is not so short and the SNR is not so low, Method I and Method II yield comparable and acceptable results.

One of the problems of the parametric bispectrum estimation methods is how to determine the best model order. This is a particularly challenging issue when dealing with field test data since the true bispectrum is not known beforehand. Fortunately, the bispectral estimation of Method I appears to be less sensitive to the model order. The determination of the best model order is one of the limitations of the parametric bispectrum estimation method and should be investigated in the future.

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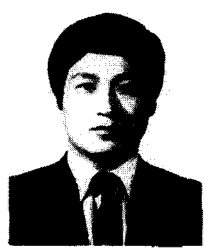

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