Diffraction-Limited Beam for One Dimensional Array in Ultrasonic Imaging

초음파 영상에서 선형어레이를 이용한 제한회절빔의 발생

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ABSTRACT

A new solution to the two-dimensional scalar wave equation is presented, which describes a diffraction-limited beam maintaining the lateral field response expressed by the sinc function. Physically, it is a superposition of plane waves having different wavelengths traveling in different directions. The beam can attain a line focus with one-dimensional array transducer in ultrasonic medical imaging.

요 약

측방향의 음장 모양이 sinc 함수 형태를 갖고 제한 깊이까지 그 모양을 유지하는 새로운 빔의 수학적인 해석방법을 제안하였다. 물리적으로 이러한 음장의 형성은 각각 다른 파장을 갖는 파가 다른 방향으로 진행할때 그 파장들이 합성되면서 나타난다. 이러한 제한회절빔은 초음파 영상에서 선형 어레이로 구현될 수 있으며 제한 깊이 내에서 균일한 측방향 해상도를 얻을 수 있다.

Ⅰ. Introduction

Of recent years, a great deal of interest has been drawn to finding nondiffracting solutions to the wave equation that describe new classes of beams whose lateral field intensity profile is sharply peaked along the axis of propagation and independent of the propagation depth [1]-[3]. Durnin [1] discovered a wave equation solution for a theoretical nondiffraction beam in two-dimensional array. He reported that the non-diffraction beam is of the form of zero-order Bessel function, and Lu and Greenleaf [2] showed that the nondiffraction beam can be approximately realized by using a finite aperture of annular array transducer. In this paper, we present a new solution to the two-dimensional wave equation for the wave propagation in a two-dimensional medium assuming the wave is generated by a one-dimensional array transducer. The new beam described by this solution consists of plane waves traveling in different directions with different wavelengths. By a proper superposition of these plane waves in such a way that all the plane waves have the same phase only on the z-axis, we obtain a new beam that has approximately nondiffracting properties along the lateral direction. This beam is physically realizable with a rectangular aperture and its lateral field response can be expressed by a sinc function.

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접수일자 : 1995년 5월 30일

Ⅱ. Derivation of the sinc wave

An array transducer which has a number of ele-

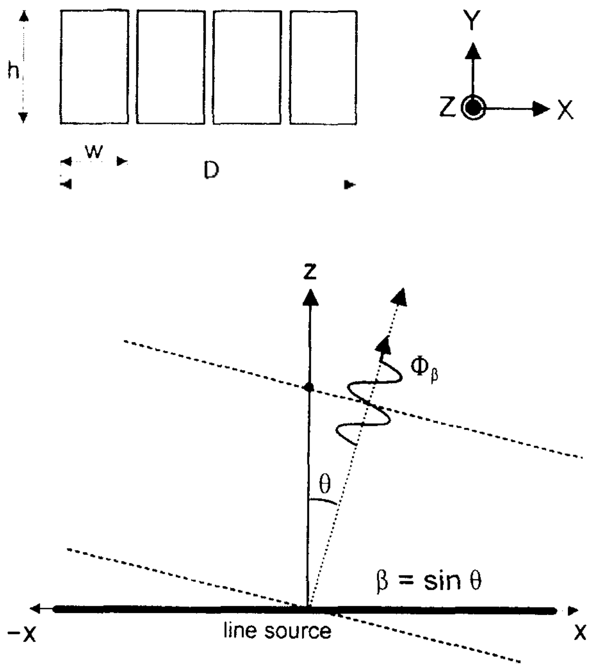

Fig. 1 (a) One-dimensional array model, and (b) the plane wave propagating at an angle $\beta=\sin\theta$ in the x-z plane. Dashed line is wave front of plane wave.

ments arranged in the x dimension within a rectangular aperture shown in Figure 1(a) is widely used currently to focus and steer an ultrasonic beam. Any field insonified by such a finite transducer is subject to diffraction spreading, reducing depth of field, as it propagates away from the transducer. If the element height h is chosen properly such that the Fresnel depth given by h^2/λ, within which the insonified ultrasonic beam remains approximately collimated in the y direction, is greater than the depth of interest, we need only to concern about the lateral field response along the x axis. Under this assumption, the lateral field response of the rectangular aperture can be analyzed by the following two-dimensional wave equation [4] that describes the two-dimensional wave propagation on the x-z plane from a line source as shown in Figure 1(b) :

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi=0. \qquad (1)$$

A simple solution of Eq. (1) is a plane wave traveling at an angle $\theta=\sin^{-1}\beta=\cos^{-1}\nu$ to the z axis :

$$\phi_\beta=e^{-i\omega_\beta t}\,e^{ik_\beta\beta x}\,e^{ik_\beta\nu z} \qquad (2)$$

where β and ν are real numbers satisfying $\beta^2+\nu^2=1$, $k_\beta=\omega_\beta/c$, ω_β is the angular frequency of Φ_β, and c is the ultrasonic velocity. If the angular frequency is dependent on the propagation angle and the relation is $\omega_\beta=\omega_0/\nu$, Φ_β can be represented by a function of ω_0 of Φ_0, the wave for $\beta=0$, then Eq. (2) becomes

$$\phi_\beta=e^{-i\omega_0 t/\nu}\,e^{ik_0\beta x/\nu}\,e^{ik_0 z} \qquad (3)$$

Then, superposition of the plane waves having *different* frequencies along *different* traveling angles from $-\beta_m$ to β_m with respect to the z axis yields

$$\Phi(x,z,t)=\int_{-\beta_m}^{+\beta_m}\phi_\beta(x,z,t)\,d\beta \qquad (4)$$

$$=\int_{-\beta_m}^{+\beta_m} e^{-i\frac{\omega_0 t}{\sqrt{1-\beta^2}}}\,e^{i\frac{k_0\beta x}{\sqrt{1-\beta^2}}}\,e^{ik_0 z}\,d\beta \qquad (5)$$

which also satisfies Eq. (1), where β_m is the maximum angle of β. The plane waves of Eq. (5) are constructively superposed on the z axis (x=0) but are destructive elsewhere at any depth as shown in Figure 2.

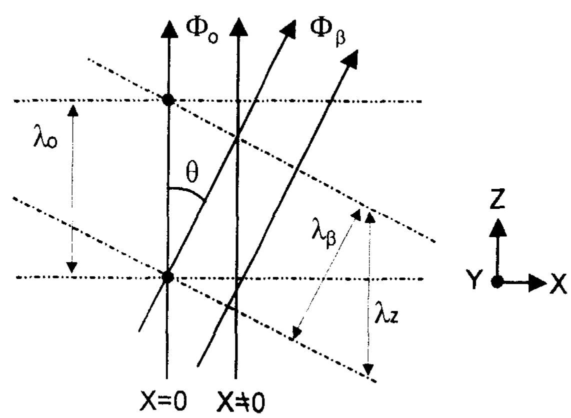

Fig. 2 Superposition of plane waves propagating with different angles with different frequencies, where $\lambda_0=\lambda_z=\lambda_\beta/\nu$; wave front of Φ_0 (dash-dot-dotted line) and wave front of Φ_β (dash-dotted line).

Eq. (5) can not be expressed in a closed form. However, when $\beta_m \ll 1$,

$$\frac{1}{\sqrt{(1-\beta^2)}}\approx 1. \qquad (6)$$

Thus, Eq. (5) can be approximated by

$$\Phi(x,z,t)\approx\int_{-\beta_m}^{+\beta_m} e^{ik_0\beta x}\,e^{i(k_0 z-\omega_0 t)}\,d\beta \qquad (7)$$

$$=2\beta_m\, sinc\left(\frac{2\beta_m x}{\lambda_0}\right)\cdot e^{i(k_0 z-\omega_0 t)} \qquad (8)$$

where $\lambda_o = 2\pi c/\omega_o$. So the lateral field response is expressed by the sinc function, which is not a function of depth z, and is the nondiffracting solution if the array size D is infinite. If the array size is finite, the maximum depth of field which preserve the sinc wave (L) is restricted by the array size D and maximum angle β_m and is

$$z_{max} = D/2 \tan(\sin^{-1}\beta_m) \quad (9)$$

In conventional focused one-dimensional array transducer, focal plane response is also the sinc function. Thus, the above method is all depth focusing scheme by superposing plane waves of different frequencies.

Ⅲ. Simulation

In Figure 2, the wavelength λ_z of Φ_β regarded in the z direction is the same as that of Φ_o, which means that at $x \neq 0$ the slanted plane waves are superposed in the z direction with the same wavelength but with different phases. So, the sinc wave can be generated by superposing monochromatic continuous waves with different phases in an infinite line source. In principle, the sinc wave can be exactly generated by an infinite aperture. However, since the energy between two consecutive zeros in the sinc function decays with the x coordinate, we can approximately generate the sinc wave by simply truncating the source distribution with a finite aperture as far as the energy of the truncated portion is negligible. Figure 3 shows the source distribution ($z=0$) of the sinc wave when $f_o = 3.5MHz$ and $\beta_m = 0.05$, and the simulated lateral field pattern in Figure 4. The solid curve is the nondiffracting sinc function for an infinite aperture and the dashed curve is the field pattern driven by a finite aperture of $D = 50mm$ at depth $z = 50mm$ and dotted curve at depth $100mm$. We see that the sinc wave is diffracted for a finite aperture but the mainlobe beam width remains almost the same within a finite depth.

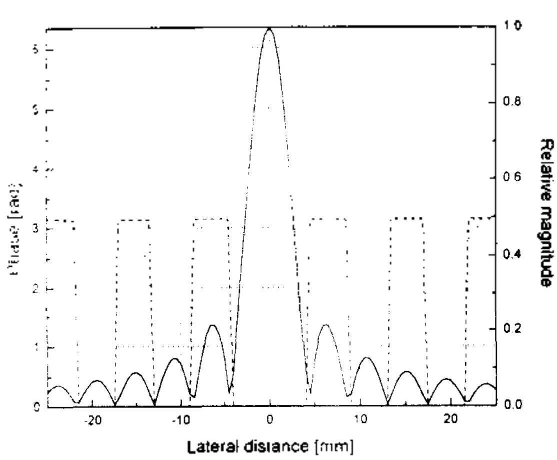

Fig. 3 Source distribution of the sinc wave, for $f_o = 3.5$ MHz, $\beta_m = 0.05$: (a) magnitude(solid line), (b) phase (dashed line).

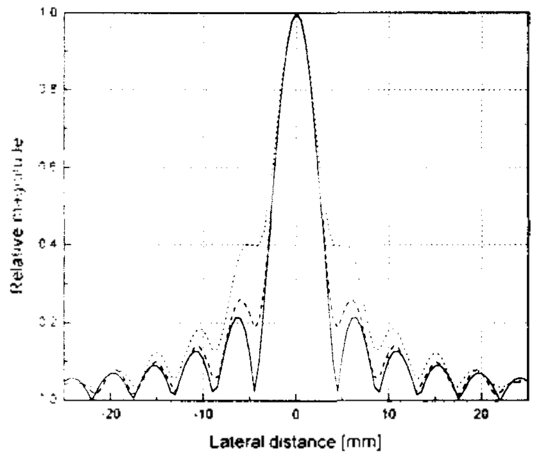

Fig. 4 Lateral field responses of a sinc wave with infinite line source(solid line) and that of 50 mm aperture at depth $z = 50$ mm(dashed line), $z = 100$ mm(dotted line) : $f_o = 3.5MHz$ and $\beta_m = 0.05$.

Ⅳ. Conclusion

In this paper, theoretical solution of diffraction-limited beam for one-dimensional array in ultrasonic imaging is suggested, which field pattern is sinc function. The degree of uniformity of the lateral field pattern with depth becomes better for a smaller β_m and larger array size.

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The Journal of the Acoustical Society of Korea, Vol. 14. No. 2E (1995)

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