◉ 論 文

不確實한 環境에 接觸하는 매니퓰레이터의 强靭制御

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Robust Control of a Manipulator Contacting the Uncertain Environment

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Key Words : Manipulator, Contact, Control, Robust, Stability, Performance

초 록

본 논문은 불확실한 환경에 접촉하는 매니퓰레이터의 강인 제어법을 제시하였다. 이 제어법은 고차수 보상기를 응용한 표적특성에 H_∞ 강인제어 설계법을 적용하여 얻어졌다.

접촉중의 위치와 힘에 대한 강인 안정성과 강인 성능 조건을 유도하였다. 결과에 의하면, 위치제어기는 강인 안정성과 강인 성능 조건을 동시에 향상시킬 수 있으나, 힘 제어기는 그 둘 사이에 최적화가 요구되었다. 강인 성능 제어기를 얻기 위한 최적화 설계기법은 변형 해석 기법을 사용하였으며, 결과의 예를 제시하였다. 이 예에서는 힘제어기의 강인 성능이 설계될 수 있음을 보였다.

1. Introduction

During execution of the contact tasks of robots, the end-effector changes its contact location and orientations. The mechanical properties of the environment at the contact also changes. The change of the mechanical properties can cause performance degradation and even instability. In practice, the contact environment is poorly known, or is just an estimation with uncertainties. The controller of a manipulator must guarantee stability and performance robust to the uncertainties.

Robust control with respect to the

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environmental change is achieved in two ways; First, controllers adapt to the new environment by sensing the contact states and identifying the environment. Second, a fixed controller is used that guarantees robust stability, and possibly performance, in the presence of the bounded uncertainties of the environment. The former uses the estimation of mechanical properties and geometry of the environment to evaluate the second derivative of force[1], to adjust impedance[2], or to cancel out the contact dynamics[3]. The reliability of these controls, however, depends on the effectiveness of adaptive algorithms, which is hard to achieve in the rapid changing environment. The latter uses variable structure control[4], robust control gains[5,6], or force compensators[7]. In the variable structure control or the robust control gains, the bound of the uncertainty is quantified and considered in the design of the controllers. The force compensators, however, is lacking in adequate reflection of the uncertainty of the environment. While improving performance in transient force responses[3,8], the force compensators evoke stability problems[9]. Consequently, the application of compensators requires tradeoff between performance and stability in the presence of the environmental uncertainty. The present research focuses on the force compensators in the target dynamics that guarantee robust stability and performance to the uncertainty.

For the robust design of the controllers, the bound of the uncertainty as well as stability and performance must be quantified. Quantified measures for robust stability and performance are developed in $\mathbf{H}_\infty$ control theory[10,11]. Relying on the computed torque linearization, this chapter aims at applying the $\mathbf{H}_\infty$ control theory to the robust design of force compensators.

The $\mathbf{H}_\infty$ control theory is ideal for handling the uncertainties in the frequency response of systems, since the multiplcative property of the $\mathbf{H}_\infty$ norm holds[12]. Using $\mathbf{H}_\infty$ norm properties, the robust stability and performance conditions are defined. The two conditions, one for stability and the other for performance condition, result in multi-objective optimization problems[10]. In that case, optimization methods, such as U-parameter design[13], coupled Riccati Equation approach[14] can be used. In case of force control, the two conditions can be merged into one equivalent condition, called robust performance condition[11,15].

The content in this paper begins with the brief description of the target dynamics that uses the high order compensator. Subsequent contents focus on the design of the compensator. The environment model is represented in linear mechanical system, and sensitivity functions for position and force control are introduced. The uncertainty for a SISO system is defined, the SISO version for $\mathbf{H}_\infty$ synthesis is applied. The robust stability and performance conditions to the environmental uncertainties are derived in terms of the parameterized sensitivity functions. The design problem for the compensator is transformed into a mixed-sensitivity problem that guarantees a solution.

2. Target dynamics model

A model of the target dynamics[16] is suggested that uses state error feedbacks and compensated force errors. It is shown that the method can be applied to contact position and force tracking control by the proper design of the compensator. The present paper extends the application to robust control. The control method based on the target dynamics is briefly described.

Assume that an articulated manipulator is composed of n simply connected links, i.e., each joint which connects the links has one relative degree of freedom. A global coordinate system O-xyz is fixed on the ground. The position and orientation of the end-effector are $\mathbf{x} = [x, y, ...,]^T \in \mathbf{R}^{n\times 1}$ in the global coordinates. The joint coordinates are also used to express the relative translational or rotational motion as $\mathbf{q} = [q_1, q_2, ..., q_n]^T \in \mathbf{R}^{n\times 1}$ in vector form, and corresponding joint torque (including force for translational joint) $\boldsymbol{\tau} = [\tau_1, \tau_2, ..., \tau_n]^T \in \mathbf{R}^{n\times 1}$. The symbol $\mathbf{x}_0$ is the environmental position before contact, and $\mathbf{f}$ is the force applied to the environment by the manipulator. For simplicity, it is assumed that the degree of the manipulator, n, is same as the number of Cartesian coordinates used.

Manipulator dynamics can be derived, using the Lagrangian or variational principles, in the joint coordinates as,

$$\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} + \mathbf{v}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau}_c + \mathbf{J}^T \mathbf{f}_{ext} \qquad (1)$$

where $\mathbf{v}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbf{R}^{n\times 1}$ is the gravitational, Coriolis, and centrifugal force; $\boldsymbol{\tau}_c \in \mathbf{R}^{n\times 1}$ is the control torque applied at joint actuators; $\mathbf{J}$ is the Jacobian of the Cartesian coordinates with respect to the joint coordinates; $\mathbf{f}_{ext} \in \mathbf{R}^{n\times 1}$ is the external force due to contact; and $\mathbf{M} \in \mathbf{R}^{n\times n}$ is the generalized mass matrix.

The target dynamics is represented in Cartesian formulation as,

$$\mathbf{G}(s)\mathbf{x}_e = \mathbf{H}(s)\mathbf{f}_e \qquad (2)$$

where $\mathbf{x}_e = \mathbf{x}_d - \mathbf{x}$, position error vector, $\mathbf{x}_d$ = desired trajectory $\in \mathbf{R}^{n\times 1}$, $\mathbf{x}$ = present Cartesian position and orientation vector $\in \mathbf{R}^{n\times 1}$, $\mathbf{f}_e = \mathbf{f}_s - \mathbf{f}_d$, force error vector, $\mathbf{f}_d$ = the desired force or torque $\in \mathbf{R}^{n\times 1}$, $\mathbf{f}_s$ = the sensed force or torque ($\approx \mathbf{f}_{ext}$) $\in \mathbf{R}^{n\times 1}$, $\mathbf{G}(s) = (\mathbf{I}s^2 + \mathbf{K}_v s + \mathbf{K}_p)$ impedance for free motion control $\in \mathbf{R}^{n\times n}$, $\mathbf{H}(s)$ = force compensator $\in \mathbf{R}^{n\times n}$, and s is the Laplace transformation. The constants $\mathbf{K}_v$ and $\mathbf{K}_p$ are diagonal matrices of derivative and proportional position feedback gains, respectively.

For simplicity, $\mathbf{H}(s)$ and $\mathbf{G}(s)$ are chosen as diagonal matrices which decouple the control dynamics. The joint torque is derived from the control algorithm. Feedback and feedfowards of states and force are used according to the target dynamics. The joint driving torques based on the target dynamics Eq.(2) are obtained as,

$$\boldsymbol{\tau}_c(t) = \hat{\mathbf{v}}(\mathbf{q}, \dot{\mathbf{q}}) + \hat{\mathbf{M}}(\mathbf{q})\mathbf{u}(t) - \mathbf{J}^T\mathbf{f}_s \qquad (3)$$

where

$$\mathbf{u}(t) = \mathbf{J}^{-1}[\dot{\mathbf{x}}_d + \mathbf{K}_v \dot{\mathbf{x}}_e + \mathbf{K}_p \mathbf{x}_e - \mathbf{H} * (\mathbf{f}_s - \mathbf{f}_d) - \dot{\mathbf{J}}\dot{\mathbf{q}}] \quad (4)$$

and the symbol $*$ is a time convolution and $\hat{\mathbf{v}}(\mathbf{q}, \dot{\mathbf{q}})$ and $\hat{\mathbf{M}}(\mathbf{q})$ are the estimates of nonlinear force $\mathbf{v}$ and inertia $\mathbf{M}$, respectively. In practice, it is possible that the sensed force $\mathbf{f}_s$ may contain errors, and the estimates, $\hat{\mathbf{v}}(\mathbf{q}, \dot{\mathbf{q}})$ and $\hat{\mathbf{M}}(\mathbf{q})$, which results from on-line computation of the manipulator model dynamics, may have estimation errors. However, we assume, for simplicity in developing control algorithm, that the sensing and the estimations are so accurate that the measurement and estimation errors are negligible.

3. Sensitivity Functions

The stability and performance of the controller depends on the interaction between the end-effector and the environment. The sensitivity functions for position and force errors are introduced to evaluate the stability and performance. Parameterization of the compensators, in general, simplifies the design process. The compensators are parameterized, and the design procedure is applied to the parameterized variables.

In the present control, the plant consists of the interaction between the end-effector and the environment. For analysis purpose, it is assumed that the environment can be modelled as a simple passive mechanical system that consists of inertia, damping, and stiffness as,

$$\mathbf{f}_s = \mathbf{E}(s)(\mathbf{x} - \mathbf{x}_0) + \mathbf{f}_0 \approx \mathbf{f}_{ext} \tag{5}$$

where

$$\mathbf{E}(s) = \mathbf{M}_E\, s^2 + \mathbf{C}_E\, s + \mathbf{K}_E \tag{6}$$

and the symbol $\mathbf{f}_0$ is the force disturbance from static load. The coefficients matrices; inertia $\mathbf{M}_E$, damping $\mathbf{C}_E$, and stiffness $\mathbf{K}_E$ of the environment; are diagonal matrices. These matrices as well as the environmental geometry $\mathbf{x}_0$ may vary according to contact position.

A simplified linear control loop can be drawn as in Fig.1 by substituting Eq.(5) to Eq.(2). The closed loop can be viewed as a system with two-input ($\mathbf{x}_d$, $\mathbf{f}_d$) and two-output ($\mathbf{x}$, $\mathbf{f}_s$) with position disturbance $\mathbf{x}_o$ and force disturbance $\mathbf{f}_0$. Viewing the control loop in Fig.2 as a multi-input and multi-output system, each component of sensitivity functions is obtained. The force error $\mathbf{f}_e = \mathbf{f}_d - \mathbf{f}_s$ is written as,

$$\mathbf{f}_e = -\mathbf{E}(s)(\mathbf{x} - \mathbf{x}_0) + \mathbf{f}_d - \mathbf{f}_0$$
$$= \mathbf{E}(s)\,\mathbf{x}_e - \mathbf{E}(s)(\mathbf{x}_d - \mathbf{x}_0) + \mathbf{f}_d - \mathbf{f}_0 \tag{7}$$

From Eq.(2) and (7), the output errors are derived in terms of sensitivity functions as,

$$\begin{bmatrix} \mathbf{x}_e \\ \mathbf{f}_e \end{bmatrix} = \begin{bmatrix} S_{xx}(s) & S_{xf}(s) \\ S_{fx}(s) & S_{ff}(s) \end{bmatrix} \begin{bmatrix} \mathbf{x}_d - \mathbf{x}_o \\ \mathbf{f}_d - \mathbf{f}_o \end{bmatrix} \tag{8}$$

where the sensitivity functions are defined as,

$$S(s) := \begin{bmatrix} S_{xx}(s) & S_{xf}(s) \\ S_{fx}(s) & S_{ff}(s) \end{bmatrix}$$
$$= \begin{bmatrix} G^{-1}H(I+E\,G^{-1}H)^{-1}E & -G^{-1}H(I+E\,G^{-1}H)^{-1} \\ (I+E\,G^{-1}H)^{-1}E & (I+E\,G^{-1}H)^{-1} \end{bmatrix} \tag{9}$$

In Equation (9), $S_{xx}(s)$, $S_{xf}(s)$, $S_{fx}(s)$ and $S_{ff}(s)$ denote the sensitivity functions of position-position, position-force, force-position, and force-force, respectively. Equations (7) shows that the output force and position error are dependent. Since the output vector Eq.(8) has twice the dimension of the system output, either the position or force error can be exclusively chosen in design, depending on control task.

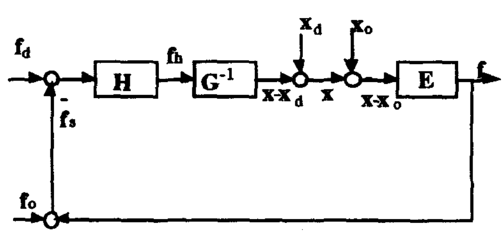

Fig. 1 The control system

It is desirable to reduce design specifications by parameterizing the compensators. Since the plant, i.e., the contact environment in the present research, is stable, the closed-loop dynamics can be parameterized by a family of stable, proper, and rational polynomials[11,17]. The design problem is then simplified to picking up a polynomial from that family, instead of checking broad potential compensators. Moreover, the sensitivity functions can be expressed as affine functions (linear plus constant) of the parameterization, and further derivation of $\mathbf{H}_\infty$ control problems is simplified.

The stabilization parameter $\mathbf{Q}(s)$ is the transfer function from $\mathbf{f}_d$ to $\mathbf{f}_h$ as,

$$\mathbf{Q}(s)=\mathbf{H}\left[\mathbf{I}+\mathbf{E}\,\mathbf{G}^{-1}\,\mathbf{H}\right]^{-1} \tag{10}$$

The sensitivity functions Eq.(9) can be written in terms of the stabilization parameter Eq.(10) as,

$$\mathbf{S}(s)=\begin{bmatrix} \mathbf{S}_{xx}(s) & \mathbf{S}_{xf}(s) \\ \mathbf{S}_{fx}(s) & \mathbf{S}_{ff}(s) \end{bmatrix} = \begin{bmatrix} \mathbf{G}^{-1}\mathbf{QE} & -\mathbf{G}^{-1}\mathbf{Q} \\ (\mathbf{I}-\mathbf{EG}^{-1}\mathbf{QE} & (\mathbf{I}-\mathbf{EG}^{-1}\mathbf{Q}) \end{bmatrix} \tag{11}$$

The sensitivity functions can be defined in another way as the response variations with respect to the plants[11], i.e., the environments in the present problem. This definition is useful in dealing with the uncertainties. Robust control design requires that the response variation due to the uncertainties be suppressed within a given bound. It implies that the sensitivity functions are bounded in the presence of the uncertainties. It is also useful to define complementary sensitivities for representation of robust stability. The complementary sensitivity function for force control is defined as $\mathbf{T}_{ff} := \mathbf{I} - \mathbf{S}_{ff}$. Once the uncertainties and the sensitivity functions are quantified, the well-established control theory is available for robust control design.

4. Robust Stability and Performance Condition

Robust stability requires that the controller is stable under the uncertainties of the environment, and the robust performance requires that the controller achieve the prescribed performance under the uncertainties. These conditions are defined applying the $\mathbf{H}_\infty$ synthesis technique.

For simplicity, the robust condition is derived for a single degree of freedom manipulator. After linearizing the dynamics of the multi-degree of freedom manipulator, the linearized equation has no basic difference to the single degree of freedom. The subsequent section uses plain characters to denote scalar quantities.

The contact environment has uncertainties in mechanical properties, such as stiffness, inertial effects, or unmodeled higher order dynamics. These uncertainties are quantitatively defined for analysis. It is assumed that the environment in the multiplicative uncertainty is perturbed around the nominal environment, $E(s) = M_E s^2 + C_E s + K_E$, as,

$$\tilde{E}=(1+\Delta(s)W_2(s))E(s),\quad \|\Delta(s)\|_\infty\leq 1 \tag{12}$$

where the weighting function $W_2(s)$ is stable, proper, real-rational, and minimum phase. Typically, the magnitude of the weighting function is monotonically increasing, since uncertainties increase with increasing frequency.

Before discussing the robust stability and performance, it is necessary to describe a nominal performance condition, which applies to the nominal environment. The condition requires that the force or position sensitivity functions be suppressed within a desired bound in the frequency bandwidth. The performance condition is written for contact position control,

$$\| W_1(s) S_{xx}(s) \|_\infty < 1 \tag{13}$$

and for contact force control,

$$\| W_1(s) S_{ff}(s) \|_\infty < 1 \tag{14}$$

The weighting function $W_1(s)$ is stable, proper, real-rational, and minimum phase. The weighting functions in Eqs.(13) and (14) need not be the same one. In Eqs. (13) and (14), only the magnitude of $W_1(s)$ is relevant. Typically,the magnitude $|W_1(s)|$ is monotonically decreasing, since good tracking is required at low frequencies.

The robust stability condition[18] is that the Nyquist plot of the closed-loop characteristics equation does not change the number of encirclements of the origin under the bounded uncertainties. The condition is expressed in terms of the complementary force sensitivity function $T_{ff}(s)$ and the weighting function $W_2(s)$ that satisfy the following inequality,

$$\| T_{ff}(s) \Delta W_2(s) \|_\infty \leq \| T_{ff}(s) W_2(s) \|_\infty < 1 \tag{15}$$

The robust stability condition is applied to both position control and force control.

Robust performance requires that the controlled system meets the robust stability and the nominal performance condition in the presence of the uncertainties of the environment. After a few steps[11], the robust performance Eq.(13) to the uncertainties Eq.(12) is derived as,

$$\left\| W_1 S_{xx} \frac{1 + |W_2|}{(1 - |W_2 S_{xx}|)} \right\|_\infty < 1 \tag{16a}$$

The stability as well as the performance must be guaranteed. From Eq.(9) and Eq.(15), the robust stability is

$$\| W_2(s) S_{xx}(s) \|_\infty < 1 \tag{16b}$$

The robust performance condition for position control is Eq.(16a) and (16b). The position-position sensitivity $S_{xx}(s)$ in both the inequality conditions Eq.(16) is a common factor. When the position-position sensitivity function approaches zero, the left hand side of both inequalities in Eq.(16) approach zero. It implies that robust performance as well as stability can be improved by reducing the position-position sensitivity without any restriction. Such a compensator for robust position control can be easily designed.

Contrary to position control, robust force control needs trade-off between robust stability and performance. The robust performance condition requires that the performance condition Eq.(14) and robust stability condition Eq.(15) be satisfied in the presence of the uncertainties. The two conditions are dependent upon each other and are merged into one condition[11,15,19] as,

$$\Psi := \| |W_1(s) S_{ff}(s)| + |W_2(s) T_{ff}(s)| \|_\infty < 1 \tag{17}$$

The performance weighting W_1 cannot be chosen arbitrarily for the given stability bound W_2 in order to satisfy Eq.(17). A necessary condition for robust performance[11] is that the weighting functions satisfy $\min\{ | W_1(j\omega) |, | W_2(j\omega) | \} < 1, \forall \omega$. It implies that if the given perturbation of $| W_2(j\omega) |$ is greater than 1, the performance specification $| W_1(j\omega) |$ should be less than 1.

5. Design by Solving a Modified Problem

When the uncertainties of the environment are given as bounded and unstructured, the robust performance controller must satisfy the inequality conditions, Eq.(16) or (17). This section uses the conventional mixed-sensitivity problem to design a robust controller that satisfies the robust performance condition for force control.

General solution methods for the inequality problem Eq.(17) are not yet developed. Thus the inequality problem is approximated by loop shaping[11] or a modified problem[11,20]. Depending on the approximation, compensators varying in order, magnitude, and phase, can be designed. In design by the modified problem, the modified criterion approximates the inequality condition by a quadratic form, and transforms it into a model-matching problem. The solution of the model-matching problem leads to a compensator, which is optimal with respect to the modified criterion.

The performance measure Ψ in Eq.(17) is a function of the sum of two absolute values. This is not easily trackable. The performance measure is modified into a quadratic form using the relationship

$$\frac{1}{\sqrt{2}}\{|W_1(s)S_{ff}(s)|+|W_2(s)T_{ff}(s)|\}$$

$$\le \{|W_1(s)S_{ff}(s)|^2+|W_2(s)T_{ff}(s)|^2\}^{1/2} \qquad (18)$$

Instead of solving the problem Eq.(17), attempts can be made to solve the quadratic form in the right-hand side in Eq.(18). A sufficient condition for robust performance condition Eq.(17) is modified into a trackable one as,

$$\Gamma := \| |W_1(s)S_{ff}(s)|^2 + |W_2(s)T_{ff}(s)|^2 \|_\infty < \frac{1}{2} \quad (19)$$

The modified problem, Eq.(19), is posed and solved by Verma and Jonckheere[20] and Kwakernaak[21]. Verma and Jonckheere analytically found an optimal value of Γ in Eq.(19). A more efficient and simple suboptimal controller can be designed using the simplified algorithm[11]. This algorithm transforms the quadratic equation (19) into a model-matching problem. The solution procedure for the modified problem is briefly described.

Equation (19) is rewritten using the sensitivity functions Eq.(11) in matrix notation as,

$$\left\| \begin{bmatrix} W_1(s) \\ 0 \end{bmatrix} - \begin{bmatrix} W_1(s)\,B(s)\,/\,G(s) \\ W_2(s)\,B(s)\,/\,G(s) \end{bmatrix} Q(s) \right\|_\infty^2 < \frac{1}{2} \quad (20)$$

The left side of Eq.(20) is the identical form of the model-matching problem. This modified problem can be treated using the solution algorithm described[14,22]. Difference is that the right side should be less than 1/2 to satisfy the inequality condition in Eq.(20). To avoid iterative procedure, Doyle et al.[11] suggest to use equality($\gamma = 1/2$).

For simplicity, the following terms are defined,

$$P(s) := \frac{B(s)}{G(s)} \qquad (21a)$$

$$R_1(s) := W_1(s) \qquad (21b)$$

$$R_2(s) := W_1(s)P(s) \qquad (21c)$$

$$S_1(s) := 0 \qquad (21d)$$

$$S_2(s) := -W_2(s)P(s) \qquad (21e)$$

Then the modified equation is transformed as,

$$\| |R_1(s)-R_2(s)Q(s)|^2 + |S_1(s)-S_2(s)Q(s)|^2 \|_\infty < \frac{1}{2} \qquad (22)$$

The inequality Eq.(22) involves the sum of the two quadratic terms in the stabilization parameter Q(s). The equation (22) is further rearranged so that the model matching technique is applied as,

$$\| |U_1(s)-U_2(s)Q(s)|^2 + U_3(s) \|_\infty < \frac{1}{2} \qquad (23)$$

where

$$U_3(s) := \frac{W_1(-s)W_1(s)W_2(-s)W_2(s)}{W_1(-s)W_1(s) + W_2(-s)W_2(s)} \qquad (24)$$

The function $U_3(s)$ satisfies $U_3(-s) = U_3(s)$. The functions $U_1(s)$ and $U_2(s)$ are the solutions of the following two equations,

$$R_2(-s)R_2(s)+S_2(-s)S_2(s)=U_2(-s)U_2(s) \quad (25a)$$

$$R_2(-s)R_1(s)+S_2(-s)S_1(s)=U_2(-s)U_1(s) \quad (25b)$$

By putting $F(s) := R_2(-s)R_2(s) + S_2(-s)S_2(s)$, a spectral factor $F\_(s)$, which has stable zeros and poles and satisfies $F(s) = F\_(-s)F\_(s)$, is computed. To remove unstable poles in $U_1(s)$, an all-pass function $V(s)$ is chosen such that

$$U_1(s)=\frac{R_2(-s)R_1(s) + S_2(-s)S_1(s)}{F\_(-s)}V(s) \in \mathbf{RH}_\infty \quad (26)$$

$$U_2(s)=F\_(s)V(s) \quad (27)$$

The all-pass function $V(s)$ satisfies $|V(s)| = 1$. Rearranging the inequality equation (23) yields a model-matching problem as,

$$\| T_1(s) - T_2(s)Q(s) \|_\infty < 1 \quad (28)$$

where $T_1(s) = U_4^{-1}(s)U_1(s)$, $T_2(s) = U_4^{-1}(s)U_2(s)$, and $U_4(s)$ is a spectral factor of $(\frac{1}{2} - U_3(s))$. The solution is given as,

$$Q(s)=\frac{T_1(s) - \lambda_{opt}Z(s)}{T_2(s)} \quad (29)$$

where $Z(s)$ and λ_{opt} is a solution to the Nevanlinna's Pick problem[11,23]. When $T_2(s)$ has a unstable single zero in $\mathbf{Re}\ s > 0$ at s_0, the solution yields $Z = 1$ and $\lambda_{opt} = T_1(s_0)$. When $T_2(s)$ has multiple zeros in $\mathbf{Re}\ s > 0$, the Nevanlinna's Pick algorithm can be used. If $\lambda_{opt} \geq 1$, the modified robust performance problem is not solvable. If $\lambda_{opt} < 1$, the compensator is, then, determined as,

$$H(s)=\frac{Q(s)}{1 - E(s)\, G^{-1}(s)\, Q(s)} \quad (30)$$

As an example, a compensator is designed by solving the modified problem. The bounds of environmental uncertainty, performance weighting functions, and the free motion controller $G(s)$ are given as,

$$W_1(s)=\frac{0.5s+0.5}{s+0.01} \quad (31a)$$

$$W_2(s)=\frac{1.2s+1}{s+10} \quad (31b)$$

$$E(s)= K_E = 10^5 \quad (31c)$$

$$G(s) = s^2+80s+1600 \quad (31d)$$

The performance weight Eq.(31a) is chosen such that it is large at the low frequencies and low in the high frequencies. The environment with stiffness is chosen such that its uncertainty Eq.(31b) is low at the low frequencies and high at the high frequencies. The free motion controller Eq.(31d) is designed as a critically damped system with stable zeros at the $s = -40$. The modified problem is solved by following the procedure. The optimum solution with $\gamma = 1/2$ is obtained formally as,

$$Q(s)=\frac{2.78\text{x}10^{-6}(s+40)(s+40)(s+10)(s+1.38)}{(s+3.81)(s+1.01)} \quad (32)$$

The formal solution, $Q(s)$, of the modified problem results in an improper polynomial. To avoid the improper compensator, the solution is rolled off by introducing

$$Q_i(s)=Q(s)\frac{1}{(\tau s+1)^k} \quad (33)$$

where k is the difference of the order of the numerator to the denominator, and τ is a small number so that it may not affect the infinite norm. The small number is chosen as $\tau = 0.001$ with $k = 2$. The proper solution is

$$Q_i(s)=\frac{2.78\text{x}10^{-6}(s+40)(s+40)(s+10)(s+1.38)}{(0.001s+1)(0.001s+1)(s+3.81)(s+1.01)} \quad (34)$$

By substituting Eq.(34) into Eq.(30), the compensator is obtained as,

$$H(s)=\frac{2.7914(s+40)^2(s+10)(s+1.3849)}{(s+1527)(s+425.3)(s+2.284)(s-0.0113)} \tag{35}$$

The compensator in Eq.(35) is proper. It is noted that the compensator by the modified problem is in high order, and the compensator uses the right-plane pole. The sensitivity and complementary sensitivity functions are shown in Figs. 2 and 3, respectively. The force sensitivity function shows the desired property that its value is low at low frequencies, while it become large at high frequencies. The magnitude and phase of the compensators Eq.(35) are shown in Figs. 4 and 5, respectively.

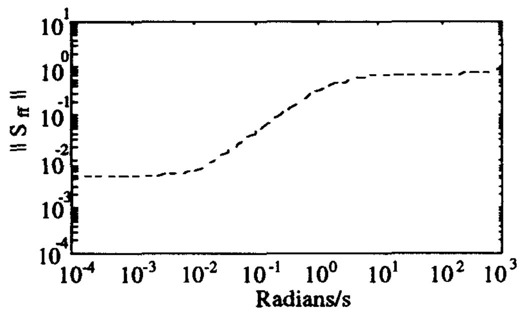

Fig. 2 Sensitivity function

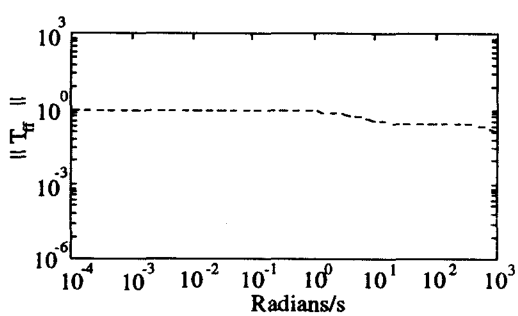

Fig. 3 Complementary sensitivity function

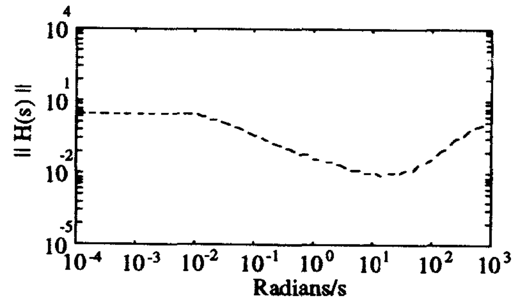

Fig. 4. Magnitude of the compensators

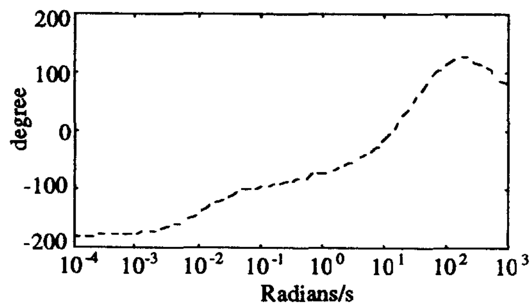

Fig. 5 The phase angle of the compensator

6. Conclusion

It has been shown that the target dynamics can be applied to position and force tracking[14]. This paper presents theoretical bases to extend the application of the target dynamics to robust control. To theoretically show the applicability, it is assumed that the nonlinear dynamics of a manipulator is exactly cancelled, and the environmental uncertainty is expressed in multiplicative form. Introducing sensitivity functions for position and force output errors, the robust stability and performance conditions are derived using H_∞ control synthesis. The result shows that the robust performance compensator can be easily designed by suppressing the position sensitivity function. Robust force control, however, requires trade-off between the stability and performance. The design example for robust force control by solving the modified problem shows the robust performance of the force controller.

References

1) Komada, S., Ohnishi, K. and Hori, T., 1991, "Hybrid Position/Force Control of Robot Manipulators Based on Acceleration Controller," IEEE International Conference on Robotics and Automation, Vol. 1, pp. 48-55.

2) Lin, S.T., 1991, "Control Design for Robot Tasks Involving Free and Contact Motion," Ph.D Dissertation, The University of Iowa.

3) Payandeh, S. and Goldenberg, A.A., 1991, "A Robust Force Controller : Theory and Experiments," IEEE International Conference on Robotics and Automation, Vol. 1, pp. 36-41.

4) Slotine, J.J., 1985, "The Robust Control of Robot Manipulators," International Journal of Robotics Research, Vol. 4, No. 2, pp. 49-64.

5) Kazerooni, H., Sheridan, T.B., and Houpt, P.K., 1986a, "Robust Compliant Motion for Manipulators, Part 1: The Fundamental Concepts of Compliant Motion," IEEE Journal of Robotics and Automation, RA-2, pp. 83-92.

6) Kazerooni, H., Sheridan, T.B., and Houpt, P.K., 1986b, "Robust Compliant Motion for Manipulators, Part 2: Design Method," IEEE Journal of Robotics and Automation, RA-2, pp. 93-105.

7) Kazerooni, H., 1987, "Robust, Non-linear Impedance Control for Robot Manipulator," IEEE International Conference on Robotics & Automation, Vol. 2, pp. 741-750.

8) An, C.H. and Hollerbach, J.M., 1987, "Dynamic Stability Issues in Force Control of Manipulators," IEEE International Conference on Robotics and Automation, Vol. 2, pp. 890-896.

9) Kazerooni, H. and Kim, S., 1989, "Contact Instability of the Direct Drive Robot When Constrained by a Rigid Environment," Proceedings of Robotics Research, Minneapolis.

10) Dorato, P., Fortuna, L., and Muscato, G., 1992, Robust Control for Unstructured Perturbations - An Introduction, Springer-Verlag, New York.

11) Doyle, J.C., Francis, B.A. and Tannenbaum, A.R., 1992, Feedback Control Theory, Macmillan Publishing Company, New York.

12) Francis, B.A. and Zames, G., 1984, "On H^{∞}-optimal Sensitivity Theory for SISO Feedback Systems," IEEE Transactions on Automatic Control, Vol. AC-29, No. 1, January, pp. 9-16.

13) Dorato, P. and Li, Y., 1991, "U-Parameter Design of Robust Single-Input Single-Output Systems," IEEE Transactions on Automatic Control, Vol. AC-36, No. 8, August, pp. 971-975.

14) Bernstein, D.S. and Haddad, W.M., 1989, "LQG Control with an H_{∞} Performance Bound: A Riccati Equation Approach," IEEE Transactions on Automatic Control, Vol. 34, No. 3, March, pp. 293-305.

15) Holohan, A.M. and Safonov, M.G., 1992, "Neoclassical Control Theory: A Functional Analysis Approach to Optimal Frequency Domain Controller Synthesis," Control and

Dynamic Systems, Vol. 50, pp. 297-329.

16) Lee, S.M., 1993, "Robust Position and Force Tracking Control for a Robot in Contact", Ph.D Dissertation, The University of Iowa.

17) Zames, G. and Francis, B.A., 1983, "Feedback, Minimax Sensitivity, and Optimal Robustness," IEEE Transactions on Automatic Control, Vol. AC-28, No. 5, May, pp. 585-600.

18) Chen, M.J. and Desoer, C.A., 1982, "Necessary and Sufficient Condition for Robust Stability of Linear Distributed Feedback Systems," International Journal on Control, Vol. 35, No. 2, pp. 255-267.

19) Cho, C.H., 1990, "H-infinity Approach to Manipulator Control," Ph.D. Dissertation, The University of Iowa.

20) Verma, M. and Jonckheere, E., 1984, "L^{∞}-compensation with Mixed Sensitivity as Broadband Matching Problem," System & Control Letters, Vol. 4, pp. 125-129.

21) Kwakernaak, H., 1985, "Minimax Frequency Domain Performance and Robustness Optimization of Linear Feedback Systems," IEEE Transactions on Automatic Control, Vol. AC-30, No. 10, October, pp. 994-1004.

22) Francis, B.A., 1987, "A Course in H^{∞}-Control Theory," in Lecture Notes in Control and Information Sciences Spring_Verlag, New York.

23) Young, N.J., 1986, "The Nevalinna-Pick Problem for Matrix-Valued Functions," Journal of Operator Theory, Vol. 15, pp. 239-265.