Journal of the Korean Nuclear Society
Volume 27, Number 6, December 1995

〈Technical Report〉

Study on the Relationship Between Turbulent Normal Stresses in the Fully Developed Bare Rod Bundle Flow

Kye Bock Lee and Byung Jin Lee
Korea Atomic Energy Research Institute
(Received July 31, 1995)

완전히 발달된 맨봉주위의 난류 유동장에서 난류 응력사이의 상관 관계에 대한 연구

이계복 · 이병진
한국원자력연구소
(1995. 7. 31 접수)

Abstract

The turbulence structure for fully developed flow through the subchannels formed by the bare rod array depends on the pitch to rod diameter ratio. For fairly open spaced bare rod arrays, the distributions of the three components of the turbulent normal stresses are similar to those measured in circular pipe. However, for more closely spaced arrays, the turbulence structure, especially in the gap region, departs markedly from the pipe flow distribution.

A linear relationship between turbulent normal stresses and turbulent kinetic energy for fully developed turbulent flow through regularly spaced bare rod arrays has been developed. This correlation can be used in connection with various theoretical analyses applied in turbulence research.

요 약

맨봉 배열에 의해 형성되는 부수로를 흐르는 난류 유동장의 구조는 피치 대 봉직경의 비에 따라 변하게 된다. 피치 대 봉직경 비가 큰 경우에는 난류 응력 분포가 관 유동의 분포와 유사하다. 그러나 피치 대 봉직경 비가 작은 경우에는 특히 간극 영역에서 난류 특성이 관 유동의 분포와는 달라진다.

완전히 발달된 맨봉 주위의 난류 유동장에서 난류 응력과 난류 운동 에너지 사이의 선형 관계가 개발되었다. 개발된 상관 관계식은 난류 연구에 응용되는 여러 이론적 분석에 연관지어 사용될 수 있다.

1. Introduction

A similarity in the distribution of turbulent normal stresses has been observed in the duct flow. They are all minimum at the centerline, increasing steadily as the wall is approached, until they start to decrease as the wall is approached further. Furthermore, the three components of turbulence intensities have been shown to vary in a similar way with the Reynolds number[1].

The Prandtl mixing theory assumes that the turbulent fluctuations are proportional to the product of a local mixing length and a velocity gradient. This implies that the turbulent fluctuations are directly proportional to each other. Practically, turbulent isothermal flow in the fully developed regions of smooth pipes and two dimensional duct has been examined for a wide range of Reynolds number and it was found that any two components of turbulent intensities are very nearly linearly related[2].

The structure of axially developed turbulent flow through regularly spaced bare rod arrays is of considerable importance to the design and analysis of the thermohydraulics of nuclear reactor cores. Of particular significance are features that promote intersubchannel heat and momentum transfer. Experimental investigations on the structure of turbulent flow through rod bundles have been reviewed. The data clearly show that turbulent flow through rod bundles differs greatly from turbulent flow through circular tube. The distributions of turbulence kinetic energy usually receive special attention because it is regarded as a transferable quantity that reflects the effect of mean convection and also perhaps the effect of large scale turbulent convection. Experimental investigations yielded mean flow and turbulence kinetic energy distribution with distortions. These characteristics are much influenced by the turbulence-driven secondary flows that occur in the cross plane of all non-circular passages[3, 4, 5].

Detailed velocity and temperature distributions within rod bundles have recently been predicted by solving the basic differential equations of turbulent flow and energy. The most widely used procedure has been based on $k-\varepsilon$ turbulence model for the effective viscosity, together with a model for the cross plane Reynolds stress. However, the lacks of high quality experimental measurements of the turbulence structure within rod bundles have inhibited the development of the turbulence models. Thus, the systematic data package of the turbulence characteristics is required.

The objective of this study is to show the relationship between the turbulent normal stresses and turbulent kinetic energy as a possible alternative that can be used in connection with various theoretical analyses applied in turbulence research.

2. Relationship Between Turbulent Normal Stresses

The relationship between the turbulent normal stresses and turbulent kinetic energy for fully developed pipe and two dimensional duct flows is presented in Figs. 1−3 respectively for a wide range of Reynolds number. These figures are based on the data presented in references 1 and 2 with $1.23\times 10^4 \le Re \le 5\times 10^5$. Alshamani[1, 2] found that the turbulent normal stresses normalized by the local wall friction velocity were shown to be related by simple linear relationship and that the curvature in the k^+ relationship implied by these equations was found to be small and k^+ could also be expressed as a linear function.

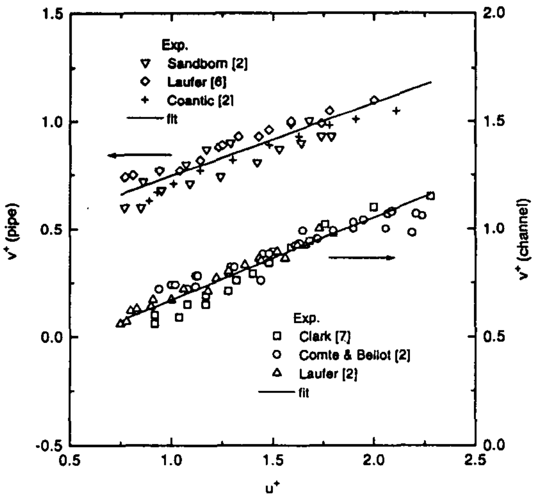

Fig. 1. Relationship Between Radial and Axial Turbulent Stresses in Pipe and Channel Flow

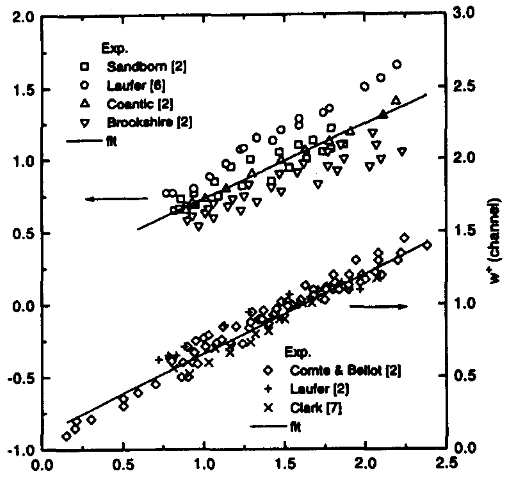

Fig. 2. Relationship Between Azimuthal and Axial Turbulent Stresses in Pipe and Channel Flow

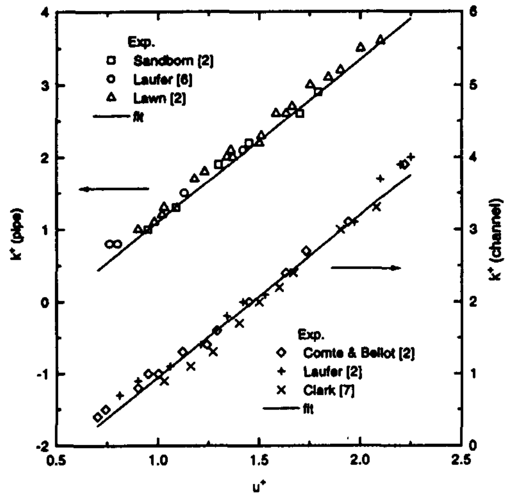

Fig. 3. Turbulent Kinetic Energy Plotted Against the Axial Turbulent Stress in Pipe and Channel Flow

For fully developed pipe flow[6], the relationship between the turbulent normal stresses and turbulent kinetic energy is as follows :

$$v^+ = 0.339u^+ + 0.408 \quad (1)$$

$$\omega^+ = 0.515u^+ + 0.217 \quad (2)$$

$$k^+ = 2.24u^+ - 1.13 \quad (3)$$

for fully developed channel flow[7],

$$v^+ = 0.383u^+ + 0.287 \quad (4)$$

$$\omega^+ = 0.55u^+ + 0.11 \quad (5)$$

$$k^+ = 2.25u^+ - 1.3 \quad (6)$$

where $v^+ = \sqrt{\overline{v'^2}}/u_\tau$, $u^+ = \sqrt{\overline{v'^2}}/u_\tau$, $\omega^+ = \sqrt{\overline{\omega'^2}}/u_\tau$ and $k^+ = k/u_\tau^2$

u_τ is local wall friction velocity.

Alshamani[1, 2] examined a wide range of published experimental data and found that the fit using the coefficients of the linear equations could predict the data within ±15% for fully developed pipe and channel flow.

In order to propose the relationship between the turbulent normal stresses in the fully developed bare rod bundle flow, all available turbulent normal stress data, which include the local wall friction velocity, have been accumulated for the triangular and square array rod bundles. The turbulence structure for fully developed flow through the subchannels formed by the rod array depends on the pitch to rod diameter ratio. For fairly open ducts (P/D≥1.2) the distributions of the three components of the turbulent normal stresses are similar to those measured in circular pipes. However, for more closely spaced arrays the turbulence structure, especially in the rod gap region, departs markedly from the pipe flow distributions. This behaviour is generally attributed to the increasing strength of secondary flows as the rod gap spacing is reduced.

The turbulent normal stresses in the current study have been poltted in the way suggested by Alshamani. The relationship between v^+ and u^+ for triangular(P/D = 1.2) and square(P/D = 1.194) array with $2\times10^4 \le Re \le 1.6\times10^5$ is presented in Fig 4[4, 5]. The turbulent normal stresses v^+ and u^+ are related linearly by the following linear form :

$$v^+ = 0.634u^+ + 0.121 \quad \text{for triangular array}$$

$$0.318u^+ + 0.494 \quad \text{for square array} \quad (7)$$

The relationship between ω^+ and u^+ for triangular (P/D = 1.2) and square(P/D = 1.194) array is shown

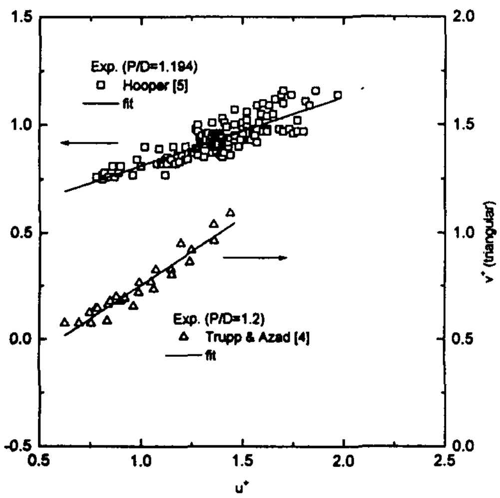

Fig. 4. Relationship Between Radial and Axial Turbulent Stresses in Fairly Open Spaced rod Arrays

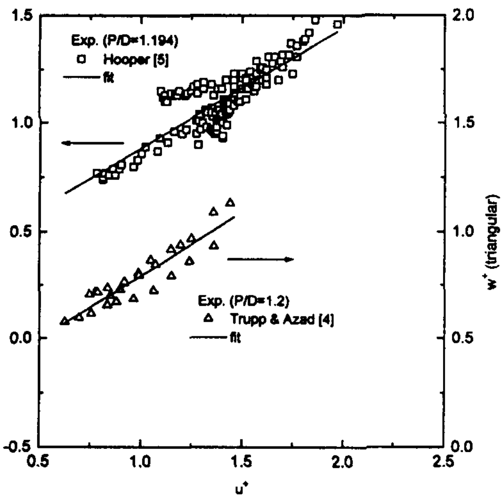

Fig. 5. Relationship Between Azimuthal and Axial Turbulent Stresses in Fairly Open Spaced rod Arrays

in Fig. 5 and the following equation is obtained :

$$\omega^+ = 0.595u^+ + 0.199 \quad \text{for triangular array}$$
$$0.557u^+ + 0.329 \quad \text{for square array} \qquad (8)$$

The higher turbulent intensity was measured in the rod-to-rod gap region.

In Fig. 6, the k^+ and u^+ relationship is presented for the triangular(P/D = 1.2) and square(P/D = 1.194)

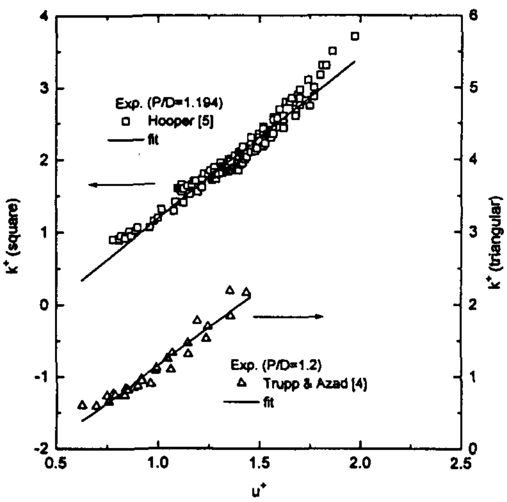

Fig. 6. Turbulent Kinetic Energy Plotted Against the Axial Turbulent Stress in Fairly Open Spaced rod Arrays

array. It is clear from this figure that linearity between k^+ and u^+ is well established.

$$k^+ = 2.073u^+ + 0.915 \quad \text{for triangular array}$$
$$2.239u^+ + 1.055 \quad \text{for square array} \qquad (9)$$

Within the level of uncertainty associated with the measurement of the turbulence quantities(about $\pm$ 13%) the results from all the traverses could be represented by single equations. These coefficients are different from those found by Alshamani.

For more closely spaced triangular(P/D = 1.123) and square(P/D = 1.107) array with $1.2 \times 10^4 \leq Re \leq 1.6 \times 10^5$, the relationships between v^+, ω^+, k^+ and u^+ are presented in Figs. 7, 8 and 9[3, 8, 9]. In this case, the scatter of the data is higher than for fairly open array. Thus, it is evident that the shape of the gap affects the turbulence structure.

The turbulent normal stresses and turbulent kinetic energy are related linearly by the following linear form :

$$v^+ = 0.522u^+ - 0.065 \quad \text{for triangular array}$$
$$0.218u^+ + 0.503 \quad \text{for square array} \qquad (10)$$

$$\omega^+ = 0.543u^+ + 0.099 \quad \text{for triangular array}$$

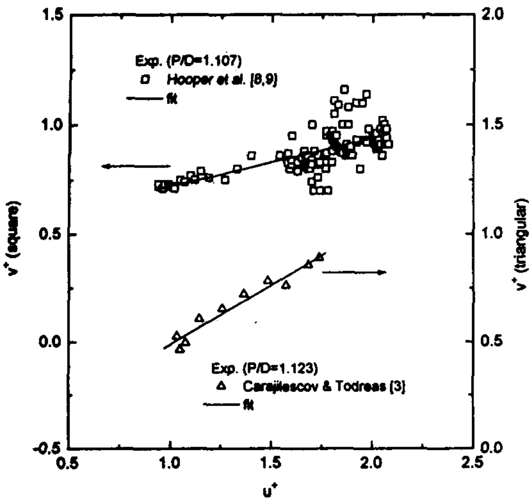

Fig. 7. Relationship Between Radial and Axial Turbulent Stresses in Closely Spaced rod Arrays

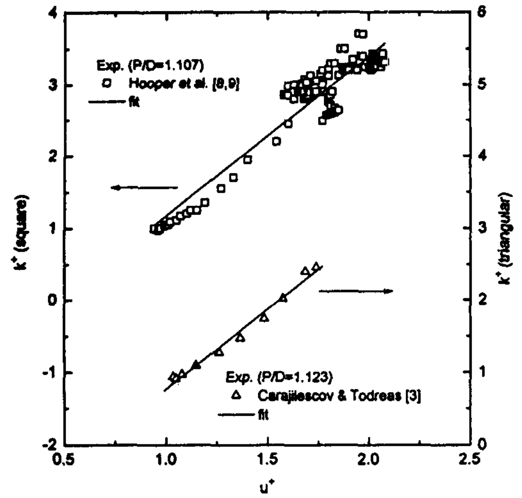

Fig. 9. Turbulent Kinetic Energy Plotted Against the Axial Turbulent Stress in Closely Spaced rod Arrays

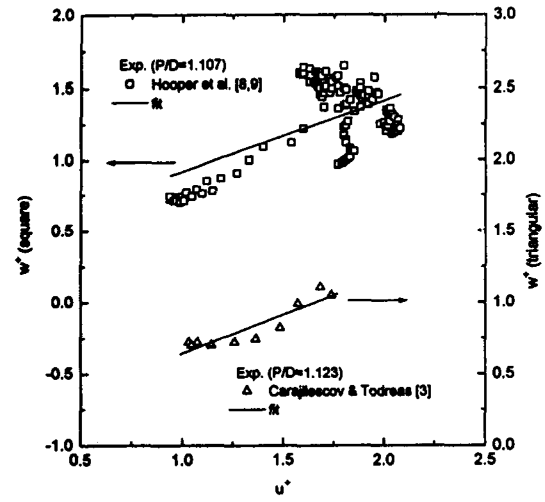

Fig. 8. Relationship Between Azimuthal and Axial Turbulent Stresses in Closely Spaced rod Arrays

$$0.494u^+ + 0.428 \quad \text{for square array} \qquad (11)$$

$$k^+ = 2.223u^+ - 1.460 \quad \text{for triangular array}$$
$$2.219u^+ - 1.041 \quad \text{for square array} \qquad (12)$$

Despite the considerable scatter, the results appear to be capable of a linear relationship. The predictive errors of the fit for the turbulent normal stresses remain within $\pm 23\%$. The azimuthal turbulence intensity is considerably higher than the pipe flow result. This effect is due to a macroscopic flow process in addition to the diffusion process[10]. The azimuthal turbulent intensity generally increases with decreasing P/D ratio. Thus, the structure of turbulent flow through rod bundles is different from turbulent flow through circular tubes, at least for P/D ratio less than 1.2. It is not surprising that attempts fail to compute turbulent flow through rod bundles using turbulence models adjusted to pipe flow.

The ability to express the turbulence quantities in this way emphasizes the essential similarity of the turbulence structure in the non-circular subchannel with the turbulence structure in a circular pipe.

3. Conclusions

The experimental data on the turbulent normal stresses and turbulent kinetic energy between subchannels of rod bundles have been reviewed.

The distributions of the normal Reynolds stresses and the turbulent kinetic energy for fairly open array (P/D>1.2) are similar to those observed in a number of pipe and two dimensional channel flows. However, the level of all the turbulent normal stresses and of turbulent kinetic energy for more closely spaced array($P/D<1.2$) is significantly higher around the gap region than in pipe flow. In addition, the narrow gap causes a significantly different distribution of turbulent kinetic energy in the gap region to that in pipe flow.

A linear relationship between turbulent normal stresses and turbulent kinetic energy has been developed which may be used as a possible alternative in connection with various theoretical analyses applied in turbulence research.

To represent the complete relationship between turbulent normal stresses, more experimental data should be accumulated.

Nomenclature

D_h	*Hydraulic diameter*
k	Turbulent kinetic energy $=\frac{1}{2}(u'^2+v'^2+\omega'^2)$
k^+	Dimensionless kinetic energy $=k/u_\tau^2$
Re	Reynolds number, $U_b\ D_h/\nu$
U_b	Mean or bulk velocity
u'	Fluctuating turbulent velocity component in the axial direction
v'	Fluctuating turbulent velocity component in the radial direction
ω'	Fluctuating turbulent velocity component in the azimuthal direction
u^+	Dimensionless velocity $=\sqrt{\overline{u'^2}}/u_\tau$
v^+	Dimensionless velocity $=\sqrt{\overline{v'^2}}/u_\tau$
ω^+	Dimensionless velocity $=\sqrt{\overline{\omega'^2}}/u_\tau$
u_τ	Local wall friction velocity
ν	Kinematic viscosity

References

1. K.M.M. Alshamani, "Correlations among Turbulent Shear Stress, Turbulent Kinetic Energy, and Axial Turbulence Intensity," *AIAA Journal*, **16** (1978), pp. 859~861.
2. K.M.M. Alshamani, "Relationships between Turbulent Intensities in Turbulent Pipe and Channel Flows," *Aeronautical Journal*, **83**(1979), pp. 159~161.
3. P. Carajilescov and N.E. Todreas, "Experimental and Analytical Study of Axial Turbulent Flows in an Interior Subchannel of a Bare Rod Bundle," *Trans. ASME, J. Heat Transfer*, **98**(1976), pp. 262~268.
4. A.C. Trupp and R.S. Azad, "The Structure of Turbulent Flow in Triangular Array rod Bundles," *Nuclear Engineering and Design*, **32** (1975), pp. 47~84.
5. J.D. Hooper, "Developed Single Phase Turbulent Flow through a Square-Pitch Rod Cluster," *Nuclear Engineering and Design*, **60**(1980), pp. 365~379.
6. J. Laufer, "The Structure of Turbulence in Fully Developed Pipe Flow," NACA TN 2954, 1953
7. J.A. Clark, "A Study of Incompressible Turbulent Boundary Layers in Channel Flow," *Trans. ASME, J. Basic Eng.*, **90**(1968), pp. 455~468.
8. J.D. Hooper and D.H. Wood, "Fully Developed Rod Bundle Flow over a Large Range of Reynolds Number," *Nuclear Engineering and Design*, **83**(1984), pp. 31~46.
9. J.D. Hooper, D.H. Wood and W.J. Crawford, "Data Bank of Developed Single-Phase Flow through a Square-Pitch Rod Cluster for Four Reynolds Numbers," AAEC/E559, 1983.
10. K. Rehme, "The Structure of Turbulence in Rod Bundles and the Implications on Natural Mixing between the Subchannels," Int. J. Heat Mass Transfer, 35(1992), pp. 567~581.