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# Application of Adaptive Control Theory to Nuclear Reactor Power Control

**Man Gyun Na**
Chosun University
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# 적응제어 기법을 이용한 원자로 출력제어

나만균
조선대학교
(1994. 11. 29 접수)

## Abstract

The Self Tuning Regulator(STR) method which is an approach of adaptive control theory, is applied to design the fully automatic power controller of the nonlinear reactor model. The adaptive control represents a proper approach to design the suboptimal controller for nonlinear, time-varying stochastic systems. The control system is based on a third-order linear model with unknown, time-varying parameters. The updating of the parameter estimates is achieved by the recursive extended least square method with a variable forgetting factor. Based on the estimated parameters, the output (average coolant temperature) is predicted one-step ahead. And then, a weighted one-step ahead controller is designed so that the difference between the output and the desired output is minimized and the variation of the control rod position is small. Also, an integral action is added in order to remove the steady-state error. A nonlinear PWR plant model was used to simulate the proposed controller of reactor power which covers a wide operating range. From the simulation results, the performances of this controller for ramp input (increase or decrease) are proved to be successful. However, for step input this controller leaves something to be desired.

## 요 약

적응제어의 한 방식인 자기동조제어(STR) 방식이 비선형 노심 모델의 출력 조정에 적용된다. 적응제어는 비선형, 시변 및 확률(Stochastic) 시스템을 위한 준최적 제어기를 설계하기 위한 적절한 제어 방식이다. 제어계통은 미지의 시변 파라메타를 갖는 3차 선형 모델에 기초한다. 파라메타는 가변 망각계수를 도입한 확장 최소자승법에 의하여 매시간(Time Step) 순환적으로 평가된다. 평가된 파라메타를 이용하여 한 스텝 먼저 냉각재 평균온도가 예측되고 이 예측된 값과 Setpoint 값과의 차이를 최소화함은 물론, 제어봉의 움직임을 막고자 가중(Weighted) One-step-ahead 제어기가 설계된다. 또한 적분동작이 첨가되어 정상상태 에러가 제거된다. 넓은 운전영역을 포괄하는 비선형 PWR 모델이 원자로 출력 조정을 위한 본 제어기를 시뮬레이션하는데 이용되었다. 시뮬레이션 결과로부터 본 제어기의 성능이 우수한 것으로 판명되었다.

## 1. Introduction

Power plants are highly complex, nonlinear, time-varying, and constrained systems. For example, the plant characteristics vary with the operating power level. Ageing effects in plant performance and changes in nuclear core reactivitv with fuel burnup generally degrade system performance. Also, If load-following operation is desired, daily load cycles can noticeably change plant performance. Recently, the problem of controlling uncertain dynamical systems has been of considerable interest to control engineers.

The conventional reactor control system consists of temperature deviation channel (the difference between the programmed coolant temperature and the average coolant temperature) and power mismatch channel (difference between the turbine load and the nuclear power). The conventional control method drives the control rods by compensating and filtering these two channels. However, it is difficult to optimally design compensators and filters for the control method because of variations in nuclear system parameters and nonlinear characteristics.

A digital processor offers flexibility because the control function can be altered by software and this facilitates provisions of sophisticated control. Also, instrumentation and control (I&C) technology has been improved rapidly. In spite of these positive aspects of using a digital controller, for many reasons modem systems have not been incorporated extensively in nuclear power plants. However, problems created by growing obsolescence of existing technology have stimulated interest in upgrading these systems [1].

Adaptive controllers are characterized by the gathering of information about an unknown process or the closed-loop during operation and by making changes in their control laws. According to the most widely accepted description, adaptive control systems adjust their behavior to the changing properties of the controlled processes and their signals [2]. The adaptive control represents a particular approach to design the suboptimal controller for nonlinear, time-varying stochastic systems. From their characteristics, the adaptive controllers are expected to solve the current control problems of the power plants.

The adaptive control method was applied by Park and Miley [3] in the control of the nuclear reactor power. In this control method, the control parameters are adjusted on-line to provide specified closed-loop poles and to satisfy a zero steady-state error. However, the collection ot the closed-loop poles has a direct relationship to the performance of the controller which is closely connected with the open-loop poles. Because the adaptive control method does not require the modeling of the plant, which means that the open-loop poles of the plant are unknown, it is difficult to properly choose the closed-loop poles.

In this paper, the controller parameters are estimated recursively and the estimated control parameters are used to calculate the control input. A weighted one-step ahead controller is designed in order to achieve a compromise between fast responses and the amount of control effort. Also, an integral control action is added to remove the steady-state error.

## 2. Design of an Adaptive Control System

### 2.1. Parameter Estimation

The ARMAX (AutoRegressive Moving Average with Auxiliary Input) model is applied for the mathematical descriptions used to design the control system, which is expressed as a linear combination of past output, past input, measurable disturbance and an independent noise in terms of difference operators [4].

The general discrete-time model of the process to be controlled is described by

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + q^{-d'}C(q^{-1})v(k) + D(q^{-1})w(k), \quad (1)$$

where $y(k)$, $u(k)$, $v(k)$ and $w(k)$ denote the output, input, measurable disturbance and stochastic noise sequences, respectively. In Eq. (1), $A(q^{-1})$, $B(q^{-1})$, $C(q^{-1})$, and $D(q^{-1})$ are process polynomials in the backward shift operator $q^{-1}$ with $a_0=d_0=1$, $b_0\neq 0$ and $C_0\neq 0$, and have orders $nA$, $nB$, $nC$, and $nD$, respectively. For example, the polynomial $A(q^{-1})$ is as follows:

$$A(q^{-1}) = a_0 + a_1 q^{-1} + a_2 q^{-2} + \dots + a_{nA} q^{-nA}. \quad (2)$$

The process time delays $d$ and $d'$ are associated with $u$ and $v$, respectively. The noise sequence, $w(k)$, is assumed to be white noise which is not measurable. Also, it is assumed that $D(q^{-1})$ is a stable polynomial of which the roots exist in the unit circle.

The main problem in the synthesis of an adaptive control system is the design of the adaptation mechanism for the process parameters of control parameters. In this paper, the direct method that the control parameters are estimated directly, is applied. Therefore, no additional calculations that convert the process parameters into the controller parameters in order to design the controller, are required.

The process output at time $k+d$ can be predicted from the measurements of the ouput, input and measurable disturbance up to time point $k$. The $d$-step ahead prediction of the filtered process output $R(q^{-1})\, y(k)$ $[=y_R(k)]$ can be derived below and the polynomial of the filter is expressed as follows:

$$R(q^{-1}) = 1 + r_1 q^{-1} + r_2 q^{-2} + \cdots + r_{nR} q^{nR} \quad (3)$$

Multiplying Eq. (1) by $E(q^{-1})$ gives

$$D(q^{-1})\,[R(q^{-1})y(k+d) - E(q^{-1})w(k+d)]$$
$$= E(q^{-1})B(q^{-1})u(k)$$
$$+ q^{d-d'}E(q^{-1})C(q^{-1})v(k) + G(q^{-1})y(k), \quad (4)$$

where $E(q^{-1})$ and $G(q^{-1})$ are the unique polynomials satisfying

$$D(q^{-1})R(q^{-1}) = E(q^{-1})A(q^{-1}) + q^{-d}G(q^{-1}), \quad (5)$$

$$E(q^{-1}) = 1 + e_1 q^{-1} + e_2 q^{-2} + \cdots + e_{d-1} q^{-d+1}, \quad (6)$$

$$G(q^{-1}) = g_0 + g_1 q^{-1} + g_2 q^{-2} + \cdots + g_{nG} q^{-nG}, \quad (7)$$

$$nG = \max\{nA-1,\ nD+nR-d\}. \quad (8)$$

Because it is assumed that $D(q^{-1})$ is stable, the optimal $d$-step ahead prediction of $y_R(k+d)$ satisfies

$$D(q^{-1})\,\hat{y}_R(k+d) = \alpha(q^{-1})y(k)$$
$$+ \beta(q^{-1})u(k) + q^{d-d'}\gamma(q^{-1})v(k), \quad (9)$$

where

$$\hat{y}_R(k+d) = E\{y_R(k+d)|F_k\}$$
$$= R(q^{-1})y(k+d) - E(q^{-1})w(k+d), \quad (10)$$

$$\alpha(q^{-1}) = G(q^{-1}), \quad (11)$$

$$\beta(q^{-1}) = E(q^{-1})B(q^{-1}), \quad (12)$$

$$\gamma(q^{-1}) = E(q^{-1})C(q^{-1}). \quad (13)$$

$F_k$ is the sigma algebra by sequences up to time $k$.

Equation (9) can be expressed in the usual format as

$$\hat{y}_R(k+d) = \hat{\theta}^T(k)\cdot\phi(k), \quad (14)$$

where

$$\hat{\theta}^T = [\ \hat{\alpha}_0(k) \cdots \hat{\alpha}_{nG}(k)\ \ \hat{\beta}_0(k) \cdots \hat{\beta}_{nB+d-1}(k)$$
$$\hat{\gamma}_0(k) \cdots \hat{\gamma}_{nC+d-1}(k)\ \ \hat{d}_1(k) \cdots \hat{d}_{nD}(k)\ ], \quad (15)$$

$$\phi^T = [y(k) \cdots y(k-nG)\ u(k) \cdots u(k-nB-d+1)$$
$$v(k+d-d') \cdots v(k-nC-d'+1)$$
$$-\hat{y}_R(k+d-1) \cdots -\hat{y}_R(k+d-nD)\ ], \quad (16)$$

The date vector, $\phi(k)$, consists of the output, input and measurable disturbance up to time point $k$. The parameter vector $\theta(k)$ can be estimated recursively from the extended least square method for minimizing the augmented error [5] defined as

$$\varepsilon(k) = [\theta - \hat{\theta}(k-d)]^T\phi(k-d)$$
$$+ [\hat{\theta}(k-d) - \hat{\theta}(k)]^T\phi(k-d)$$
$$= [\theta - \hat{\theta}(k)]^T\phi(k-d). \quad (17)$$

The recursive parameter estimation algorithm is as follows:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + F(k)\phi(k-d)\,[R(q^{-1})y(k)$$
$$- \hat{\theta}^T(k-1)\phi(k-d)], \quad (18)$$

$$F(k) = \frac{1}{\lambda(k)} \times$$

$$\left[ F(k-1) - \frac{F(k-1)\phi(k-d)\phi^T(k-d)F(k-1)}{\lambda(k) + \phi^T(k-d)F(k-1)\phi(k-d)} \right]. \quad (19)$$

The adaptation gain matrix $F(k)$ is a $(nG+nB+nC+nD+2d+1)\times(nG+nB+nC+nD+2d+1)$ dimensional symmetric matrix. The expectation of $F(k)$ is proportional to the covariance matrix of the parameter estimates [6]. The variable forgetting factor $\lambda(k)$ can be used to account for an exponetial decay of the past data in tracking a slow drift in parameters [7]. In order to keep constant a measure $[N(k)]$ of information content of the estimation, the forgetting factor is chosen to maintain a constant value $N_0$ for $N(k)$ in the following third-order equation [8]:

$$N(k) = \lambda(k)N(k-1)$$

$$+ \frac{\lambda^2(k)}{[\lambda(k) + \phi^T(k)F(k-1)\phi(k)]}[y(k) - \phi^T(k)\hat{\theta}(k-1)]^2. \quad (20)$$

The algorithm ensures the estimation stability (refer to [8])

### 2.2. Calculation of Control Input

An one-step-ahead controller is a very simple form of control law in which tracking performance is overemphasized. The basic principle behind this controller is that control input is determined at each point in time so as to bring the system output at a future time instant to a desired value. Potential difficulties with this approach are that an excessively large effort may be called for to bring the output to the desired value in one step and resonances may be excited in lightly damped systems. This will then lead us to consider weighted one-step-ahead controllers wherein a penalty is placed on excessive control effort.

Therefore, in order to achieve a compromise between fast responses and the amount of control effort expended, the following cost function is derived:

$$J = E\{[R(q^{-1})(y(k+d) - y_d(k+d))]^2$$

$$+ \omega[L(q^{-1})R(q^{-1})u(k)]^2 | F_k\}, \quad (21)$$

where $\omega$ is the weighting factor of the control input. In order to remove the steady-state error, $L(q^{-1})$ contains an integral action as follows:

$$L(q^{-1}) = 1 - q^{-1}. \quad (22)$$

From Eqs. (9) and (10),

$$J = E\{[R(q^{-1})y(k+d) - \hat{y}_R(k+d)$$

$$+ \hat{y}_R(k+d) - R(q^{-1})y_d(k+d)]^2$$

$$+ \omega[L(q^{-1})R(q^{-1})u(k)]^2 | F_k\}$$

$$= E\left\{\sum_{j=0}^{d-1} e_j^2\sigma^2 + [\hat{y}_R(k+d) - R(q^{-1})y_d(k+d)]^2\right.$$

$$\left. + \omega[L(q^{-1})R(q^{-1})u(k)]^2 \right| F_k \Big\}, \quad (23)$$

where

$$E\{w^2(k)|F_k\} = \sigma^2. \quad (24)$$

In order to minimize the above cost function, differentiating Eq. (23) with respect to $u(k)$ gives

$$[\hat{y}_R(k+d) - R(q^{-1})y_d(k+d)]\beta_0$$

$$+ \omega[L(q^{-1})R(q^{-1})u(k)] = 0. \quad (25)$$

By multiplying the polynomial $D(q^{-1})$ and using Eq. (9), the following control input is obtained:

$$u(k) = \frac{\beta_0}{\beta_0^2+\omega}\{D(q^{-1})R(q^{-1})y_d(k+d)$$

$$- \alpha(q^{-1})y(k) - q^{d-d'}\gamma(q^{-1})v(k)$$

$$- q\left[(\beta(q^{-1}) - 1) + \frac{\omega}{\beta_0}(D(q^{-1})R(q^{-1}) - 1)\right]u(k-1)$$

$$+ \frac{\omega}{\beta_0}D(q^{-1})R(q^{-1})u(k-1)\Big\}. \quad (26)$$

### 3. Application to Nuclear Power Plant

The nonlinear PWR plant model (six delayed neutron group nonlinear point kinetics equation and the lumped thermal-hydraulic balance equations) developed by Park, M.G. [9] is used to apply the proposed control method. The schematic diagram of

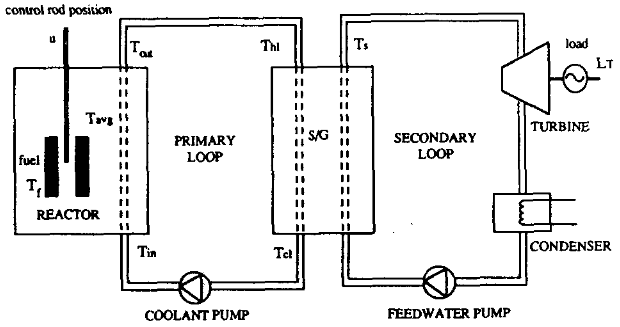


**Fig. 1. Schematic Diagram of PWR Plant.**

PWR plant is shown in Fig. 1. The process dynamics based on physical laws result in the following differential equations:

$$\frac{dP}{dt} = \frac{\rho - \beta}{l} P + \sum_{i=1}^{6} \lambda_i C_i, \tag{27}$$

$$\rho = \rho_0 + \alpha_f T_f + \alpha_c T_{avg} + bu, \tag{28}$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{l} P - \lambda_i C_i, \quad i = 1, \cdots, 6 \tag{29}$$

$$\frac{dT_f}{dt} = -\frac{UA}{M_f c_{pf}}(T_f - T_{avg}) + \frac{J}{M_f c_{pf}} P, \tag{30}$$

$$\frac{dT_{avg}}{dt} = \frac{UA}{M_c c_{pc}}(T_f - T_{avg}) - \frac{\dot{m}}{M_c}(T_{out} - T_{in}), \tag{31}$$

$$\frac{dT_{in}}{dt} = \frac{1}{\tau_{cl}}(T_{cl} - T_{in}), \tag{32}$$

$$\frac{dT_{hl}}{dt} = \frac{1}{\tau_{hl}}(T_{out} - T_{in}), \tag{33}$$

$$T_{out} = 2T_{avg} - T_{in}, \tag{34}$$

$$\frac{dT_s}{dt} = -\frac{1}{\tau_s}(T_s - T_{hl}) - D_1 L_T, \tag{35}$$

$$T_{cl} = D_2 T_s - D_3 T_{hl}. \tag{36}$$

The process is simulated using fifth-order Runge-Kutta method with adaptive time step sizes to deal with stiffness inherent in nuclear reactor dynamics. Nonlinearity in the heat transfer between fuel and coolant is considered from the heat transfer coefficient $U$ of Dittus-Boelter correlation [10]. The turbine load variation is performed by changing steam flow to the turbine.

The plant dynamics are approximated by the following model in order to apply the proposed control method:

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + q^{-d'}C(q^{-1})v(k) + D(q^{-1})w(k), \tag{37}$$

where $y(k)$ is the average coolant temperature, $u(k)$ the position of the control rods and $v(k)$ reactor power. The measurable disturbance $v(k)$ must be the steam flow to the turbine but the change of the steam flow brings that of the reactor power which has more close relationship to the average coolant temperature. Therefore, $v(k)$ is considered as the reactor power. The noise $w(k)$ is uncorrelated white $N(0, \sigma)$ noise sequence.

The nuclear reactor is controlled so that the average coolant temperature may track the programmed (desired) coolant temperature versus demand load, while an excessively large effort is not called for. In order to start the adaptive control algorithm, the following values have to be specified a priori:

orders; $nA=3$, $nB=2$, $nC=2$, $nD=1$, $nR=1$
delays; $d=1$, $d'=1$
input weighting factor; $\omega=0.01$
filtering polynomial; $R(q^{-1})=1-0.1q^{-1}$
initial conditions;
$\hat{\theta}^T(0)=[0.4\ 0.4\ 0.4\ 0.3\ 0.3\ 0.3\ 0.2\ 0.2\ 0.2\ 0.1]$,

$\phi^T(0)=[0.0, 0.0, \cdots, 0.0]$,

$F(0)=100000\times I$,

$N_0=0.5$

The delays $d=1$, $d'=1$ denote the inherent unit delay in discrete system representation [11]. The sampling period $h$ is 0.4 sec.

In the computer simulation, the operating condition of the process is an equilibrium point at a demand power 50% and a rod position 100 *steps*. The demand power for which the proposed control algorithm is tested is shown in Fig. 2. The demand power increases continuously from 0 to 1000 sec and approaches 90% power level at 1000 sec. And the power remains constant for 200 sec and decreases continuously from 1200 to 1800 sec. And then, the power remains constant at 75% power level for 200 sec and at 2000 sec, the 5% step increase of the demand power occurs.

The average coolant temperature tracks very well its setpoint for load as shown in Fig. 3. Its initial small oscillations are due to the uncertain estimation of parameters in the initial transients periods. And also, from Fig. 4 it is shown that the reactor power tracks the demand load very well. The position of control rods is shown in Fig. 5 and it follows the pattern similar to the power. The elements $F_{11}(k)$ and $F_{22}(k)$ of the adaptation gain matrix are shown in Fig. 6 and they have initial large values and decreases fast. The estimated controller parameters are shown in Fig. 7. It is known that the parameters are estimated fast but drift around in order to cope with the linear modeling of the nonlinear system and new circumstances when external disturbances are added. However, it does not degrade the system performances.

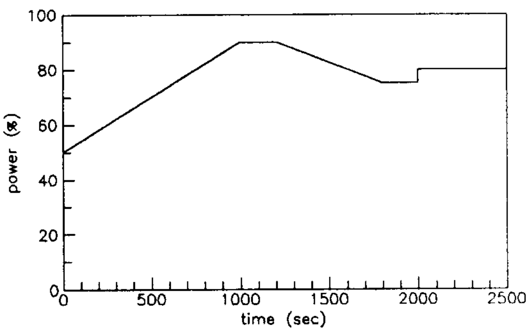


**Fig. 2. Variation of Demand Load.**

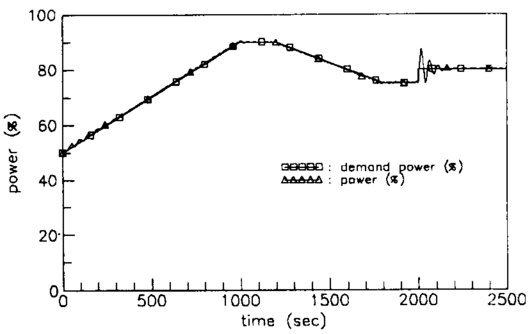


**Fig. 4. Reactor Power and Demand Load.**

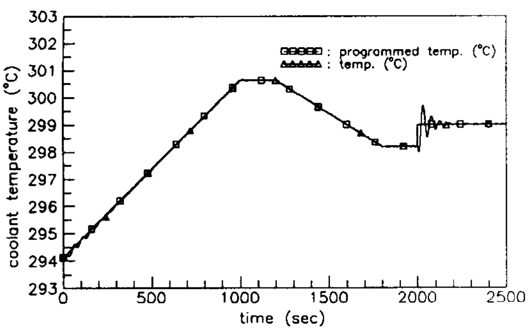


**Fig. 3. Average Coolant Temperature and Setpoint for Load.**

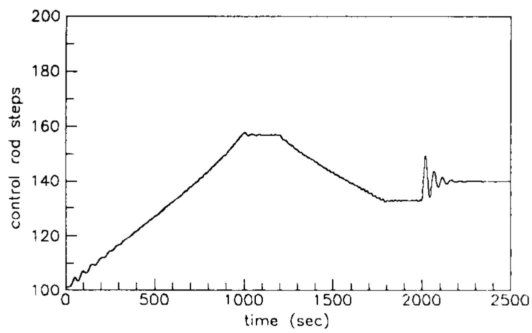


**Fig. 5. Position of Control Rod.**

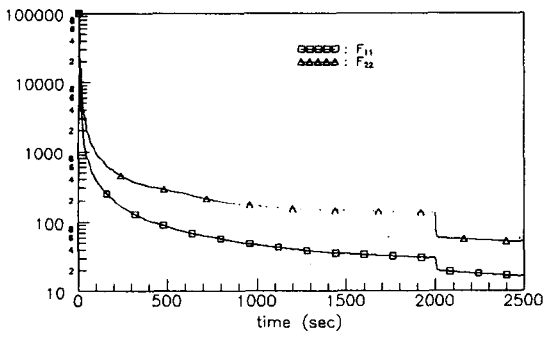


Fig. 6. Adaptation Gain Matrix.

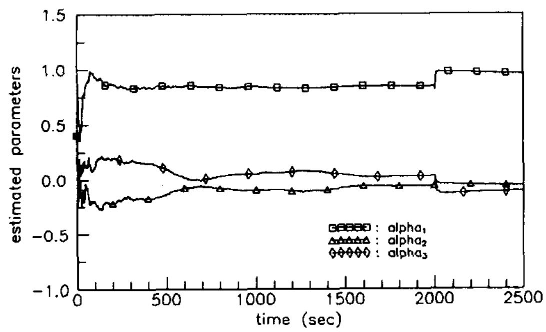


Fig. 7. Estimated Controller Parameters

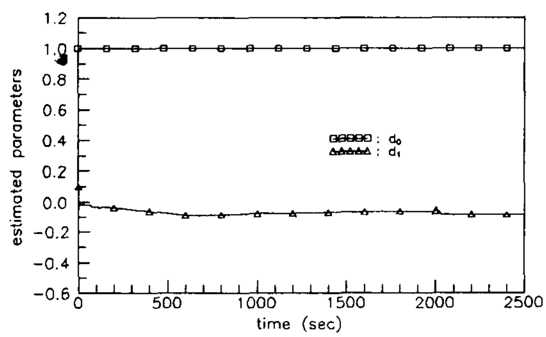


Fig. 7. Continued

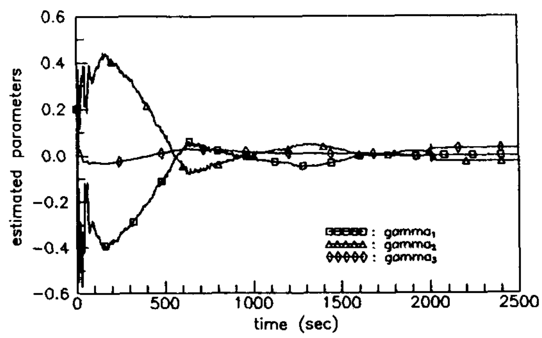


Fig. 7. Continued

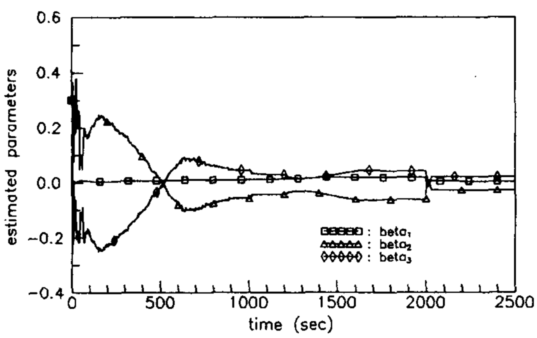


Fig. 7. Continued

## 4. Conclusions

An adaptive control system for a PWR plant is designed with the weighted one-step-ahead controller that a compromise between fast responses and the amount of control effort expended, is achieved. The control system is based on a third-order linear model with unknown, time-varying parameters. The controller parameters are estimated recursively and the estimated controller parameters are used to calculate the control input. The updating of the parameter estimates is achieved by the recursive extended least square method with a variable forgetting factor. Also, an integral action is added in order to remove the steady-state error. From the computer simulation results of a nonlinear reactor model, it is known that the proposed controller has a good performances of reference following and relatively small input variations for ramp input. However, for step input this controller leaves something to be desired.


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**Nomenclature**

| | |
|---|---|
| $A(q^{-1})$, $B(q^{-1})$ | Plant polynomials of output and input, respectively, |
| $C(q^{-1})$, $D(q^{-1})$ | Plant polynomials of measurable disturbance and noise, respectively, |
| $d$, $d'$ | Delays related to input and measurable disturbance, respectively, |
| $E(q^{-1})$ | Polynomial, |
| $F_k$ | Collection of all avaiable information up to time $k$, |
| $F(k)$ | Adaptation gain matrix (covariance matrix), |
| $h$ | Sampling time, |
| $I_n$ | $n \times n$ dimensional identity matrix, |
| $J$ | Cost function, |
| $k$ | Time step, |
| $L(q^{-1})$ | Input weighting polynomial, |
| $N(k)$ | Nominal memory length, |
| $q^{-1}$ | Backward difference operator, |
| $R(q^{-1})$ | Polynomial of filter, |
| $u(k)$ | Input (rod position), |
| $v(k)$ | Measurable disturbance (reactor power), |
| $w(k)$ | Noise, |
| $y(k)$ | Output (average coolant temperature), |
| $y_d(k)$ | Desired output (Programmed average coolant temperature), |
| $\alpha(q^{-1})$, $\beta(q^{-1})$, $\gamma(q^{-1})$ | Controller polynomials of output, input, and measurable disturbance, respectively, |
| $\varepsilon(k)$ | Augmented error, |
| $\theta$ | Parameter vector, |
| $\lambda(k)$ | Forgetting factor, |
| $\sigma^2$ | Noise variance, |
| $\phi(k)$ | Measurement vector, |
| $\omega$ | Input weighting factor. |

**References**

1. *Integrated Instrumentation and Upgrade Plan*, Rev. 3, EPRI NP-7343 (1992)
2. Isermann, R., Lachmann, K.-H., and Matko, D., *Adaptive Control System*, London, Prentice-Hall, London (1992).
3. Park, G.T., *Nucl. Sci. Eng.*, **94**, 145 (1986).
4. Goodwin, G.C. and Sin, K.S., *Adaptive Filering, Prediction and Control*, Prentice-Hall, Engelwood Cliffs (1984).
5. Landau, I.D. and Lozano, R., *Automatica*, **17**, 593 (1981).
6. Isermann, R., *Proc. 5th IFAC Symp. Darmstadt*, Pergamon Presss, Oxford (1979).
7. Fortescue, T.R., *et al.*, *Automatica*, **17**, 831 (1981).
8. Na, M.G. and No, H.C., *Nucl. Eng. Des.*, **122**, 301 (1990).
9. Park, M.G., *Robust Nonlinear Control of Nuclear Reactors Under Model Uncertainty*, Ph.D. Thesis, KAIST (1993).
10. Rust, J.H., *Nuclear Power Plant Engineering*, Haralson, Georgia, (1979).
11. Seborg, D.E., *et. al.*, *AIChE J.*, **32**, 881 (1986).