

A Plastic Design Method of Grillages under a Lateral Point Load

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Abstract

The plastic collapse equations are obtained for regular grids under a central point load with two types of boundaries, i.e. simply-supported ends and fixed ends. General plastic collapse mechanisms are derived by applying upper bound theorem for assumed collapse mechanisms. The possible collapse mechanisms are divided into three types which allows to examine all possible collapse mechanisms systematically.

Then, those formulae are applied to the pontoon deck problem to find the minimum weight design.

1 Introduction

Ship and offshore structures consist of grillages with orthogonal stiffeners. In the case of ships, decks are reinforced with transverse beams and longitudinal girders, bulkheads of deep water tank and cargo hold consist of horizontal and vertical beams and double bottoms consist of a center girder, side girders and frames.

The application of plate design methods introduces the use of 'collapse' as a limiting condition of a structure. Therefore, the main emphasis of this study is placed on the development of a plastic design method of grillages under a lateral point load.

To find the plastic collapse equations, upper bound theorem is applied. Then this upper bound equations are compared with lower bound solution to find unique equations.

Finally, this collapse equations are applied to pontoon deck problem to find minimum weight design.

2 Grillages under a Point Load

2.1 Idealization of analysis model

To proceed with plastic analysis of grillages, various assumptions have to be introduced. Then it is assumed that;

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- (1) The grid models are considered to be regular in two orthogonal directions.
- (2) The material of the plates and the beams are elastic-perfectly plastic.
- (3) The deformations are small, therefore the equilibrium equations can be formulated for the undeformed structure.
- (4) The effects of shear and torsion in the beams are neglected, and no instability will occur before the attainment of the plastic limit load.
- (5) The plated grillages are idealized into the grids where the plate is assumed to provide an “effective flange” to each set of beams.

2.2 Analysis method

A simple way of plastic analysis is through the use of the kinematic theorem (or upper bound theorem). It is necessary first to find all possible collapse mechanisms, and then to evaluate the collapse load for each mechanism assumed.

In the case of complex grids with arbitrary beam end conditions and arbitrary loading conditions, the above procedure of finding the all with through an intuitive and systematic search for the possible mechanism. The procedure will be illustrated by reference to a 3 x 3 grid.

The grid in Fig. 1 has plastic bending moment M_{pl} in the longitudinal direction and M_{pt} in the transverse direction and the applied load P_c is at the central intersection. The beam ends are simply-supported. All the possible collapse mechanisms are shown in Fig. 1 (a)~(e) with their hinge positions.

The values of the collapse loads for each mode are as follows;

$$\begin{aligned}
 \text{Collapse mode (a)} : P_c &= \frac{8}{A} M_{pl} + \frac{8}{B} M_{pt} \\
 \text{Collapse mode (b)} : P_c &= \frac{12}{A} M_{pl} + \frac{12}{B} M_{pt} \\
 \text{Collapse mode (c)} : P_c &= \frac{16}{A} M_{pl} + \frac{16}{B} M_{pt} \quad \dots\dots(1) \\
 \text{Collapse mode (d)} : P_c &= \frac{4}{A} M_{pl} + \frac{32}{B} M_{pt} \\
 \text{Collapse mode (e)} : P_c &= \frac{32}{A} M_{pl} + \frac{4}{B} M_{pt}
 \end{aligned}$$

In the case of $\delta = 1.5$, eqns.(1) are plotted in the design space in Fig. 2.

The examination of the design space of Fig. 2 shows that collapse modes (b) and (c) are never active for any combination of β_l and β_t .

By following the same procedure with intuitive and systematic search for possible collapse modes, and general case of grid analysis and design equations can thus be formulated. The whole range of equations for the grids with simply-supported ends and fixed ends and a single point load at, or near, the central intersection point is shown in Appendix A. The collapse modes are grouped together according to the hinge positions. The three basic modes are:

(a) Mode A : the collapse mode with hinges at central intersections for both sets of beams.

(b) Mode B : the collapse modes with hinges at central intersections in longitudinal beams, and in other bays of transverse beams.

(c) Mode C : the collapse modes with hinges at central intersections in transverse beams, and in other bays of longitudinal beams.

The definitions for equations are ;

P_c = Plastic collapse load.

A = Overall longitudinal length of grid.

B = Overall transverse length of grid.

m = Number of longitudinal beams.

n = Number of transverse beams.

M_{pl} = Plastic moment of longitudinal beams.

M_{pt} = Plastic moment of transverse beams.

3 Application to Pontoon Deck Design

3.1 Design problem

A design is required for a pontoon deck structure for Ro-Ro traffic. The deck area to be designed is 11.55 x 9.3m as in Fig. 3. There is a central bulkhead supporting the deck, therefore the panel area for design can be considered 5.775 x 9.3m. The worst design case is due to a fork lift truck which can apply a total load of 66 tonnes over four wheels on one axle. The combined tire print may be considered 670 x 937mm, and the wheel spacing is 2400mm along the axle. The sketch of tire print dimensions is shown in Fig. 4. The structural material to be used is mild steel of $\sigma_y = 240N/mm^2$ and $\rho = 7850Kg/m^3$. The plate thickness may be determined by Lloyd's rules with minimum thickness of 5mm. The safety factor to be used in the design is a load factor of 2 against total collapse of the beams.

The originally proposed design was with a beam arrangement of 2 girders along the shorter span and 6 stiffeners along the longer span within the panel area for the present design study. The present design study, therefore, examines the possibility of weight saving in the above pontoon deck.

3.2 Plate design

According to L. R. 3-9-3.4.1, the required plate thickness under wheel loading is;

$$t = 4.6 \times (A \cdot P_w)^{1/2} + 1.5 \quad \dots\dots (2)$$

where, t = Deck plate thickness(mm)

P_w = Load on the tire print(tonnes)
 A = Stress factor obtained from Fig. 5

The tire print ratio(PR) in Fig. 5 is;

$$PR = \frac{937}{670} = 1.40 \quad \dots\dots(3)$$

The plate panel ratio is unlikely to be less than 2.5, therefore the full line in Fig. 5 with PR = 1.4 can be taken. This may be fitted to the equation of;

$$A = 0.335 \times \eta^{-0.776} \quad \dots\dots(4)$$

where, A = Stress factor.
 η = Ratio of width of tire print to length of plate panel.

3.3 Beam design

In the grid design method under single point load in chapter 2, a point load is assumed to act at the center or near the central intersection of each grid and the grid and the grid boundary condition was assumed simply-supported or fixed ends all along the edges.

In the case of a two point wheel loading, which is the present problem, the two point load is considered as a single point load at the center of the axle.

With the above assumptions, the design method of section 2.2 is directly applicable, and this design problem can be stated as follows;

Minimize $F = m \cdot A \cdot \{c_l \cdot Z_{pl}^{k_l}\} + n \cdot B \cdot \{c_t \cdot Z_{pt}^{k_t}\} \quad \dots\dots(5)$

Subject to $P_c \leq L_i \cdot M_{pl} + T_i \cdot M_{pt}$ [Overall collapse modes]
 $P_c \leq C \cdot M_{pl}$
 $P_c \leq D \cdot M_{pt}$ [Local collapse modes] ... (6)

where,
 F = Weight objective function(mm^3)
 P_c = Design load(N)
 m, n = Number of longitudinal and transverse beams
 A = Overall grid length(mm)
 B = Overall grid breadth(mm)
 c_l, k_l = Constants for Z_p and A_s relationship of longitudinal beams
 c_t, k_t = Constants for Z_p and A_s relationship of transverse beams
 M_{pl}, M_{pt} = Plastic moment of longitudinal and transverse beams(N-mm)
 L_i, T_i = Consists for overall collapse modes

C,D = Consists for local collapse modes or minimum size of beams

In eqn.(5), constants c and k equal to 0.198 and 0.718, which is mean values for T-sections. [1]

Overall collapse modes can be obtained by using method described in section 2.2, and two local collapse modes can be calculated by considering the local collapse of individual beams between intersections. The plastic collapse is considered when the constant line load is applied at the mid-span of a beam[see Fig. 6]. When the wheel length is longer than the beam span, the load is considered as uniform over the full span. The beam ends are considered fixed for local collapse. The derivation of the local collapse equation is also shown in Fig. 6.

Through the above procedure, Fig. 7 is obtained for 3 x 5 grid. Then, the optimal design point is shown by small circle.

Plastic section modulus are calculated by using the point, then it is observed that Z_{pl} and Z_{pt} include plastic modulus of the plate. To find only plastic modulus of beams, we can use the effective breadth for plate thickness which was calculated in section 3.2. Then, using the Table 1, it is possible to determine the dimensions of web and flange of beams.

Table 1: Standard beam data

Item	Girder	T - section	Flat - bar
Dw / Tw	150.0	50.0	18.0
Tf / Tw	2.5	1.5	-
Df / Tf	10.0	18.0	-
Min. Tw (mm)	-	5.0	5.0
Max. Tw (mm)	-	15.0	-
Min. Dw (mm)	-	150.0	-

Dw = Depth of web(mm), Tw = Thickness of web(mm)

Df = Width of flange(mm), Tw = Thickness of flange(mm)

3.4 Results and Discussions

The number of beams in each direction is limited to 1 to 5 for the longitudinal beams(girders) and 3 to 12 for transverse beams(stiffeners). The weights and dimensions of beams and plates for the simply-supported and fixed cases are shown in Tables 2 and 3 respectively. As can be noticed from these tables, the plastic thickness is only a function of stiffener spacing. Another important result is that, in a pontoon deck design problem under wheel loading only, the plate thickness is a dominant design factor where the weight of plating is 56 to 88% of total weight. The minimum weight design is certainly with a large number of girders and stiffeners to give thinner plating. This tendency is shown in Figs. 8 and 9. This fact suggests that, in grillage design under wheel loading only, the plate thickness should be reduced by increasing the number of beams in each direction. Therefore, if the girder numbers are given, the minimum weight design corresponds to a larger number of stiffeners.

The optimal design of pontoon deck in this study, within the beam numbers shown in Tables 2 and 3, is with 5 x 11 at simply-supported ends and 5 x 12 at fixed ends. The present optimum design with the above beam arrangement offers weight saving of about 6% at simply-supported ends and about 10% at fixed ends compared to the originally proposed beam arrangement of 2 x 6.

4 Conclusions

In the case of grillage structures under a concentrated load, some formulae are presented to obtain the limit load for each collapse mode with known plastic bending moment when the boundary conditions of beams are simply-supported and fixed ends.

Then, those formulae are applied to the pontoon deck problem to find the minimum weight design. It is observed that optimized design has more weight-saving effect than preliminary design.

These results can be applied to the design of car deck of car-carrier or heli-deck etc.

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Appendix A: Plastic Collapse Equations

1. Simply-supported grids under a point load

1.1 (m x n) = (odd x odd) grids

(a) Mode A

$$P_c = \frac{(n+1)}{A} \times \left[\frac{4}{(n+1)} + \frac{2}{(n+1)} \times (m-1) \right] \times M_{pt} \\ + \frac{(m+1)}{B} \times \left[\frac{4}{(m+1)} + \frac{2}{(m+1)} \times (n-1) \right] \times M_{pt}$$

(b) Mode B_j [max. $j = (m-1) / 2$]

$$P_c = \frac{(n+1)}{A} \times \left[\frac{4}{(n+1)} + \frac{4}{(n+1)} \times (j-1) \right] \times M_{pt} \\ + \frac{(m+1)}{B} \times \left[\frac{4}{j} + \frac{2}{j} \times (n-1) \right] \times M_{pt}$$

(c) Mode C_i [max. $i = (n-1) / 2$]

$$P_c = \frac{(n+1)}{A} \times \left[\frac{4}{i} + \frac{2}{i} \times (m-1) \right] \times M_{pt} \\ + \frac{(m+1)}{B} \times \left[\frac{4}{(m+1)} + \frac{4}{(m+1)} \times (i-1) \right] \times M_{pt}$$

1.2 (m x n) = (odd x even) grids

(a) Mode B_j [max. $j = (m-1) / 2$]

$$P_c = \frac{(n+1)}{A} \times j \times \left[\frac{2}{n} + \frac{2}{(n+2)} \right] \times M_{pt} \\ + \frac{(m+1)}{B} \times \frac{4}{j} \times \left[1 + \frac{(n-1)}{2} \right] \times M_{pt}$$

- (b) Mode C_{i-j} [max. $i = n/2$; max. $j = n/2 + 1$]
 [when $i = \max.$, then half value in { }]
 [when $j = \max.$, then half value in |]]

$$P_c = \frac{(n+1)}{A} \times (m+1) \times \left[\frac{\{1\}}{i} + \frac{|1|}{j} \right] \times M_{pl} \\ + \frac{4}{B} \times \left[1 + \frac{\{i-1\}}{2} + \frac{|j-1|}{2} \right] \times M_{pt}$$

1.3 (m x n) = (even x odd) grids

- (a) Mode B_{j-k} [max. $j = m/2$; max. $k = m/2 + 1$]
 [when $j = \max.$, then half value in { }]
 [when $k = \max.$, then half value in |]]

$$P_c = \frac{4}{A} \times \left[1 + \frac{\{j-1\}}{2} + \frac{|k-1|}{2} \right] \times M_{pl} \\ + \frac{(m+1)}{B} \times (n+1) \times \left[1 + \frac{\{1\}}{j} + \frac{|1|}{k} \right] \times M_{pt}$$

- (b) Mode C_i [max. $i = (n-1) / 2$]

$$P_c = \frac{(n+1)}{A} \times \frac{4}{i} \times \left[1 + \frac{(m-1)}{2} \right] \times M_{pl} \\ + \frac{(m+1)}{B} \times i \times \left[\frac{2}{m} + \frac{2}{(m+2)} \right] \times M_{pt}$$

1.4 (m x n) = (even x even) grids

- (a) Mode B_{j-k} [max. $j = m/2$; max. $k = m/2 + 1$]
 [when $j = \max.$, then half value in { }]
 [when $k = \max.$, then half value in |]]

$$P_c = \frac{(n+1)}{A} \times \left[\left(\frac{2}{n} + \frac{2}{(n+2)} \right) \times \left(1 + \frac{(j-1)}{2} + \frac{(k-1)}{2} \right) \right] \times M_{pl} \\ + \frac{(m+1)}{B} \times \left[\left(\frac{\{2\}}{j} + \frac{|2|}{k} \right) \times \left(1 + \frac{(n-1)}{2} \right) \right] \times M_{pt}$$

- (b) Mode C_{i-j} [max. $i = n/2$; max. $j = n/2 + 1$]
 [when $i = \max.$, then half value in { }]
 [when $j = \max.$, then half value in |]]

$$P_c = \frac{(n+1)}{A} \times \left[\left(\frac{\{2\}}{i} + \frac{|2|}{j} \right) \times \left(1 + \frac{(m-1)}{2} \right) \right] \times M_{pl} \\ + \frac{(m+1)}{B} \times \left[\left(\frac{2}{m} + \frac{2}{(m+2)} \right) \times \left(1 + \frac{(i-1)}{2} + \frac{(j-1)}{2} \right) \right] \times M_{pt}$$

2. Fixed grids under a point load

2.1 (m x n) = (odd x odd) grids

- (a) Mode A

$$P_c = \frac{2(n+1)}{A} \times \left[\frac{4}{(n+1)} + \frac{2}{(n+1)} \times (m-1) \right] \times M_{pl} \\ + \frac{2(m+1)}{B} \times \left[\frac{4}{(m+1)} + \frac{2}{(m+1)} \times (n-1) \right] \times M_{pt}$$

(b) Mode B_j [max. $j = (m - 1) / 2$]

$$P_c = \frac{2(n+1)}{A} \times \left[\frac{4}{(n+1)} + \frac{4}{(n+1)} \times (j-1) \right] \times M_{pt} \\ + \frac{(m+1)}{B} \times \left[\frac{4}{j} + \frac{2}{j} \times (n-1) \right] \times M_{pt}$$

(c) Mode C_i [max. $i = (n - 1) / 2$]

$$P_c = \frac{(n+1)}{A} \times \left[\frac{4}{i} + \frac{2}{i} \times (m-1) \right] \times M_{pt} \\ + \frac{2(m+1)}{B} \times \left[\frac{4}{(m+1)} + \frac{4}{(m+1)} \times (i-1) \right] \times M_{pt}$$

2.2 (m x n) = (odd x even) grids

(a) Mode B_j [max. $j = (m - 1) / 2$]

$$P_c = \frac{2(n+1)}{A} \times j \times \left[\frac{2}{n} + \frac{2}{(n+2)} \right] \times M_{pt} \\ + \frac{(m+1)}{B} \times \frac{4}{j} \times \left[1 + \frac{(n-1)}{2} \right] \times M_{pt}$$

(b) Mode C_{i-j} [max. $i = n/2$; max. $j = n/2 + 1$]

$$P_c = \frac{(n+1)}{A} \times (m+1) \times \left[\frac{1}{i} + \frac{1}{j} \right] \times M_{pt} \\ + \frac{8}{B} \times \left[1 + \frac{(i-1)}{2} + \frac{(j-1)}{2} \right] \times M_{pt}$$

2.3 (m x n) = (even x odd) grids

(a) Mode B_{j-k} [max. $j = m/2$; max. $k = m/2 + 1$]

$$P_c = \frac{8}{A} \times \left[1 + \frac{(j-1)}{2} + \frac{(k-1)}{2} \right] \times M_{pt} \\ + \frac{(m+1)}{B} \times (n+1) \times \left[1 + \frac{1}{j} + \frac{1}{k} \right] \times M_{pt}$$

(b) Mode C_i [max. $i = (n - 1) / 2$]

$$P_c = \frac{(n+1)}{A} \times \frac{4}{i} \times \left[1 + \frac{(m-1)}{2} \right] \times M_{pt} \\ + \frac{2(m+1)}{B} \times i \times \left[\frac{2}{m} + \frac{2}{(m+2)} \right] \times M_{pt}$$

2.4 (m x n) = (even x even) grids

(a) Mode B_{j-k} [max. $j = m/2$; max. $k = m/2 + 1$]

$$P_c = \frac{2(n+1)}{A} \times \left[\left(\frac{2}{n} + \frac{2}{(n+2)} \right) \times \left(1 + \frac{(j-1)}{2} + \frac{(k-1)}{2} \right) \right] \times M_{pt} \\ + \frac{(m+1)}{B} \times \left[\left(\frac{2}{j} + \frac{2}{k} \right) \times \left(1 + \frac{(n-1)}{2} \right) \right] \times M_{pt}$$

(b) Mode C_{i-j} [max. $i = n/2$; max. $j = n/2 + 1$]

$$P_c = \frac{(n+1)}{A} \times \left[\left(\frac{2}{i} + \frac{2}{j} \right) \times \left(1 + \frac{(m-1)}{2} \right) \right] \times M_{pt} \\ + \frac{2(m+1)}{B} \times \left[\left(\frac{2}{m} + \frac{2}{(m+2)} \right) \times \left(1 + \frac{(i-1)}{2} + \frac{(j-1)}{2} \right) \right] \times M_{pt}$$

Table 2: Weights and dimensions of Pontoon deck (Simply-supported Beams)

m x n	TP(mm)	Twg(mm)	Tws(mm)	Wp	Wb	Wt	Wp/Wt
1 x 3	22.11	12.63	6.91	9.32	1.60	10.92	0.85
4	20.39	12.63	7.01	8.60	1.95	10.55	0.81
5	19.10	12.04	7.08	8.05	2.25	10.30	0.78
6	18.08	12.46	7.12	7.62	2.65	10.27	0.74
7	17.24	11.35	7.16	7.27	2.90	10.17	0.72
8	16.54	12.19	7.19	6.97	3.34	10.31	0.68
9	15.94	10.57	7.22	6.72	3.56	10.28	0.65
10	15.41	11.89	7.24	6.50	4.04	10.54	0.62
11	14.95	9.65	7.26	6.30	4.21	10.51	0.60
12	14.54	11.55	7.27	6.13	4.74	10.86	0.56
2 x 3	22.11	12.09	5.60	9.32	1.82	11.14	0.84
4	20.39	11.46	5.75	8.60	1.95	10.55	0.82
5	19.10	11.62	5.84	8.05	2.23	10.29	0.78
6	18.08	10.60	5.90	7.62	2.30	9.93	0.77
7	17.24	11.23	5.95	7.27	2.67	9.94	0.73
8	16.54	10.26	5.99	6.97	2.76	9.74	0.72
9	15.94	10.80	6.03	6.72	3.12	9.84	0.68
10	15.41	9.86	6.07	6.50	3.23	9.73	0.67
11	14.95	10.32	6.09	6.30	3.57	9.87	0.64
12	14.54	9.39	6.11	6.13	3.68	9.81	0.62
3 x 3	22.11	10.03	5.80	9.32	1.91	11.22	0.83
4	20.39	10.13	5.26	8.60	2.00	10.59	0.81
5	19.10	10.12	5.04	8.05	2.10	10.15	0.79
6	18.08	10.23	5.08	7.62	2.31	9.93	0.77
7	17.24	9.96	5.14	7.27	2.44	9.71	0.75
8	16.54	10.19	5.19	6.97	2.70	9.68	0.72
9	15.94	9.78	5.25	6.72	2.81	9.53	0.70
10	15.41	10.14	5.29	6.50	3.11	9.61	0.68
11	14.95	9.57	5.32	6.30	3.18	9.48	0.66
12	14.54	10.04	5.34	6.13	3.50	9.63	0.64
4 x 3	22.11	10.64	5.33	9.32	2.42	11.74	0.79
4	20.39	9.33	5.41	8.60	2.20	10.80	0.80
5	19.10	10.72	5.00	8.05	2.59	10.65	0.76
6	18.08	9.47	5.00	7.62	2.39	10.01	0.76
7	17.24	10.66	5.00	7.27	2.80	10.08	0.72
8	16.54	9.42	5.00	6.97	2.57	9.54	0.73
9	15.94	10.52	5.00	6.72	3.10	9.82	0.68
10	15.41	9.14	5.00	6.50	2.83	9.33	0.70
11	14.95	10.35	5.00	6.30	3.39	9.69	0.65
12	14.54	8.99	5.00	6.13	3.13	9.26	0.66
5 x 3	22.11	8.56	5.80	9.32	2.17	11.49	0.81
4	20.39	8.69	5.16	8.60	2.25	10.85	0.79
5	19.10	8.68	5.04	8.05	2.38	10.44	0.77
6	18.08	8.87	5.00	7.62	2.44	10.06	0.76
7	17.24	8.73	5.00	7.27	2.52	9.79	0.74
8	16.54	9.05	5.00	6.97	2.56	9.53	0.73
9	15.94	8.88	5.00	6.72	2.64	9.36	0.72
10	15.41	8.95	5.00	6.50	2.82	9.32	0.70
11	14.95	8.71	5.00	6.30	2.88	9.18	0.69
12	14.54	8.93	5.00	6.13	3.10	9.23	0.66

m = Number of girders, n = Number of stiffeners
 Tp = Thick. of Plate(mm), Wp = Plate weight(tonnes)
 Twg = Thick. of Girder(mm), Wb=Beam weight(tonnes)
 Tws = Thick. of Stiffener(mm), Wt=Total weight(Wp+Wb)

Table 3: Weights and dimensions of Pontoon deck
(Fixed ends Beams)

m x n	Tp(mm)	Twg(mm)	Tws(mm)	Wp	Wb	Wt	Wp/Wt
1 x 3	22.11	8.61	6.91	9.32	1.25	10.57	0.88
4	20.39	7.95	7.01	8.60	1.56	10.16	0.85
5	19.10	7.45	7.08	8.05	1.88	9.94	0.81
6	18.08	7.05	7.12	7.62	2.22	9.84	0.77
7	17.24	6.72	7.16	7.27	2.55	9.82	0.74
8	16.54	6.45	7.19	6.97	2.90	9.87	0.71
9	15.94	6.23	7.22	6.72	3.26	9.98	0.67
10	15.41	6.03	7.24	6.50	3.61	10.11	0.64
11	14.95	5.82	7.26	6.30	3.97	10.27	0.61
12	14.54	5.65	7.27	6.13	4.32	10.45	0.59
2 x 3	22.11	8.74	5.60	9.32	1.25	10.57	0.88
4	20.39	8.08	5.75	8.60	1.41	10.01	0.86
5	19.10	7.58	5.84	8.05	1.60	9.65	0.83
6	18.08	6.92	5.90	7.62	1.78	9.40	0.81
7	17.24	6.60	5.95	7.27	2.00	9.27	0.78
8	16.54	6.34	5.99	6.97	2.23	9.20	0.76
9	15.94	6.12	6.03	6.72	2.47	9.19	0.73
10	15.41	5.92	6.07	6.50	2.72	9.22	0.70
11	14.95	5.71	6.09	6.30	2.96	9.27	0.68
12	14.54	5.55	6.11	6.13	3.21	9.34	0.66
3 x 3	22.11	6.86	6.23	9.32	1.35	10.67	0.87
4	20.39	6.84	5.76	8.60	1.45	10.05	0.86
5	19.10	7.03	5.40	8.05	1.57	9.63	0.84
6	18.08	7.02	5.12	7.62	1.64	9.27	0.82
7	17.24	6.93	5.14	7.27	1.81	9.08	0.80
8	16.54	6.72	5.19	6.97	1.98	8.96	0.78
9	15.94	6.56	5.25	6.72	2.17	8.89	0.76
10	15.41	6.34	5.29	6.50	2.35	8.84	0.73
11	14.95	6.14	5.32	6.30	2.52	8.82	0.71
12	14.54	5.97	5.34	6.13	2.70	8.83	0.69
4 x 3	22.11	6.86	5.62	9.32	1.40	10.71	0.87
4	20.39	6.84	5.19	8.60	1.48	10.08	0.85
5	19.10	7.03	5.00	8.05	1.60	9.65	0.83
6	18.08	7.02	5.00	7.62	1.65	9.28	0.82
7	17.24	6.81	5.00	7.27	1.75	9.02	0.81
8	16.54	6.84	5.00	6.97	1.88	8.85	0.79
9	15.94	6.70	5.00	6.72	2.02	8.74	0.77
10	15.41	6.49	5.00	6.50	2.15	8.65	0.75
11	14.95	6.31	5.00	6.30	2.28	8.58	0.74
12	14.54	6.07	5.00	6.13	2.41	8.54	0.72
5 x 3	22.11	6.86	5.12	9.32	1.49	10.80	0.86
4	20.39	6.84	5.00	8.60	1.56	10.15	0.85
5	19.10	7.03	5.00	8.05	1.67	9.72	0.83
6	18.08	7.02	5.00	7.62	1.72	9.34	0.82
7	17.24	7.11	5.00	7.27	1.79	9.06	0.80
8	16.54	6.95	5.00	6.97	1.87	8.84	0.79
9	15.94	6.83	5.00	6.72	1.98	8.70	0.77
10	15.41	6.64	5.00	6.50	2.08	8.58	0.76
11	14.95	6.48	5.00	6.30	2.18	8.49	0.74
12	14.54	6.28	5.00	6.13	2.27	8.40	0.73

m = Number of girders,

n = Number of stiffeners

Tp = Thick. of Plate(mm),

Wp = Plate weight(tonnes)

Twg = Thick. of Girder(mm),

Wb=Beam weight(tonnes)

Tws = Thick. of Stiffener(mm), Wt=Total weight(Wp+Wb)

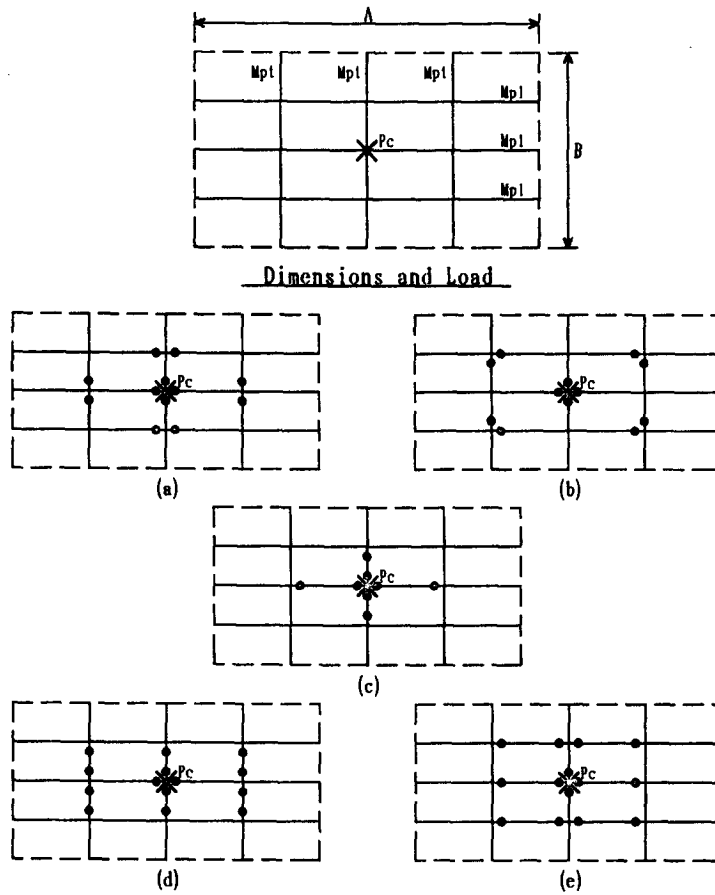


Figure 1: 3 x 3 Grid with simply-supported ends

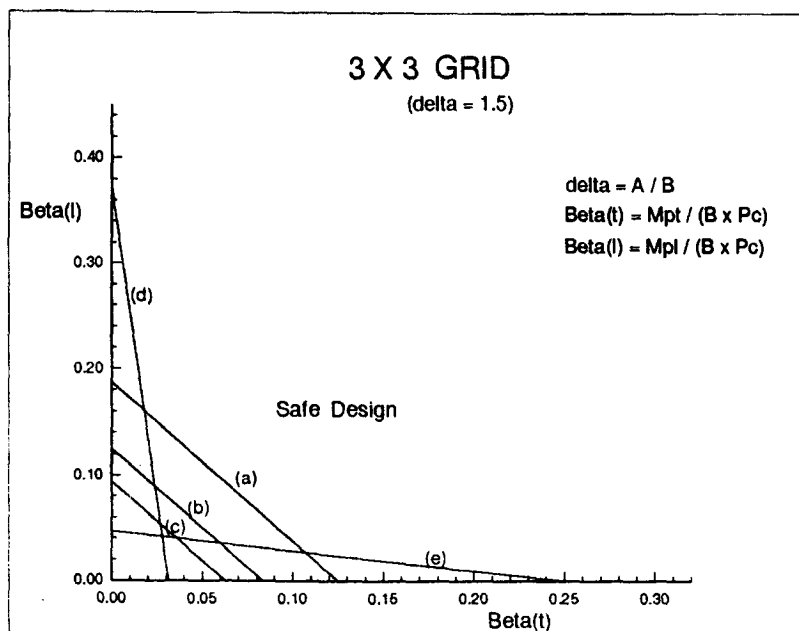


Figure 2: Design space for 3 x 3 grid ($\delta = 1.5$)

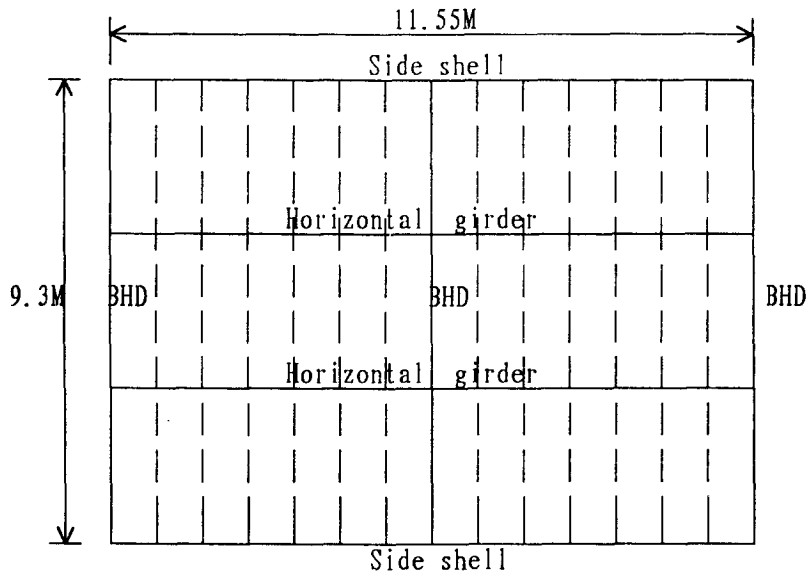


Figure 3: Pontoon deck for design study

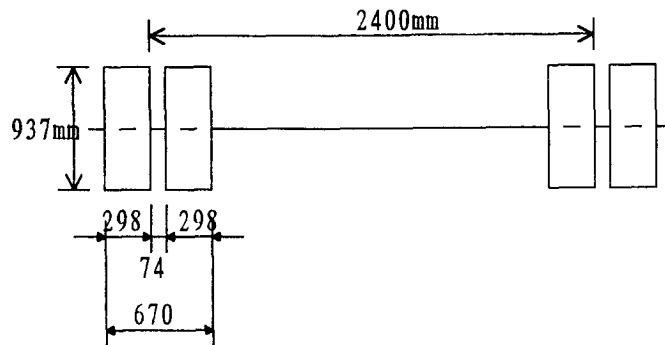


Figure 4: Tire prints of loading wheels

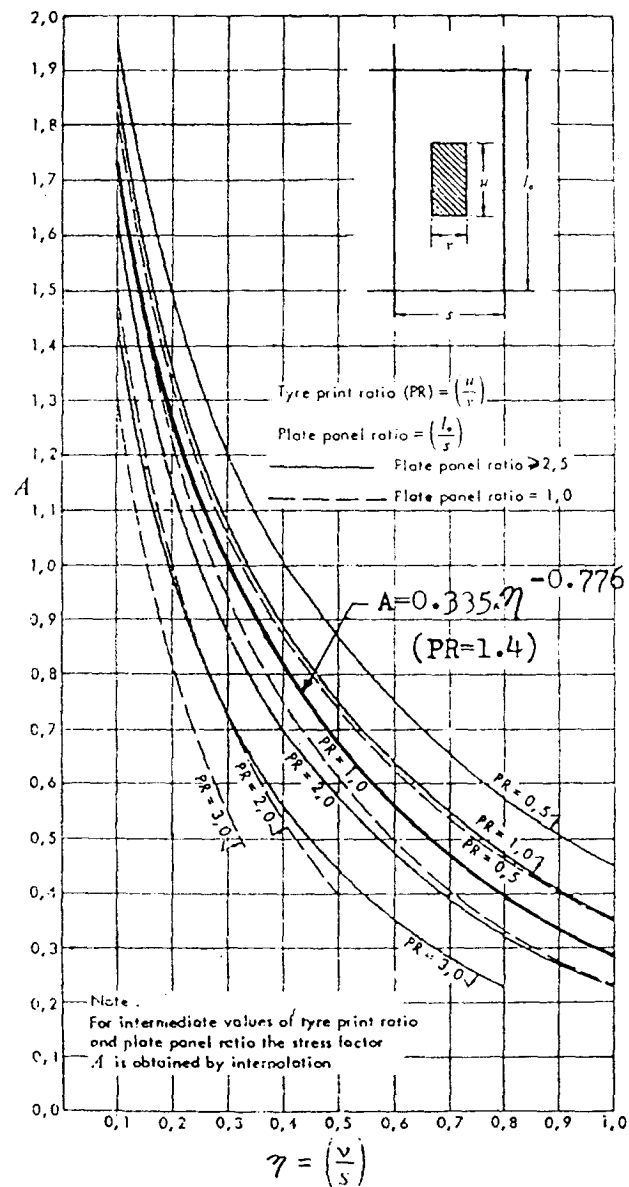
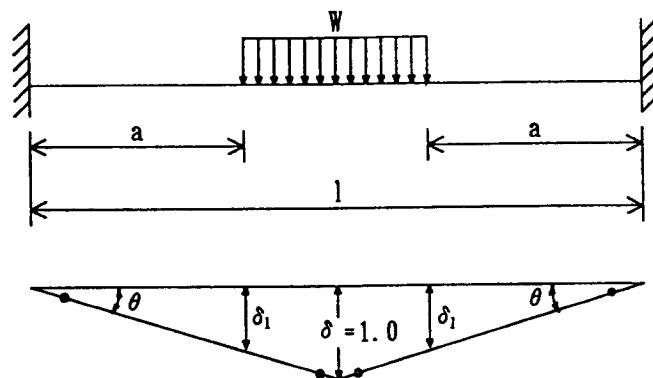


Figure 5: Tire print load stress factor(A)



$$4 \cdot \theta \cdot M_p = \frac{1}{2} W \cdot l - \frac{1}{2} W \cdot a \cdot \frac{2a}{l} \cdot 2$$

where, $\theta = \frac{2}{l}$

$$M_p = \frac{W}{16} (l - 2a)(l + 2a)$$

Figure 6: Local collapse of beams under wheel load

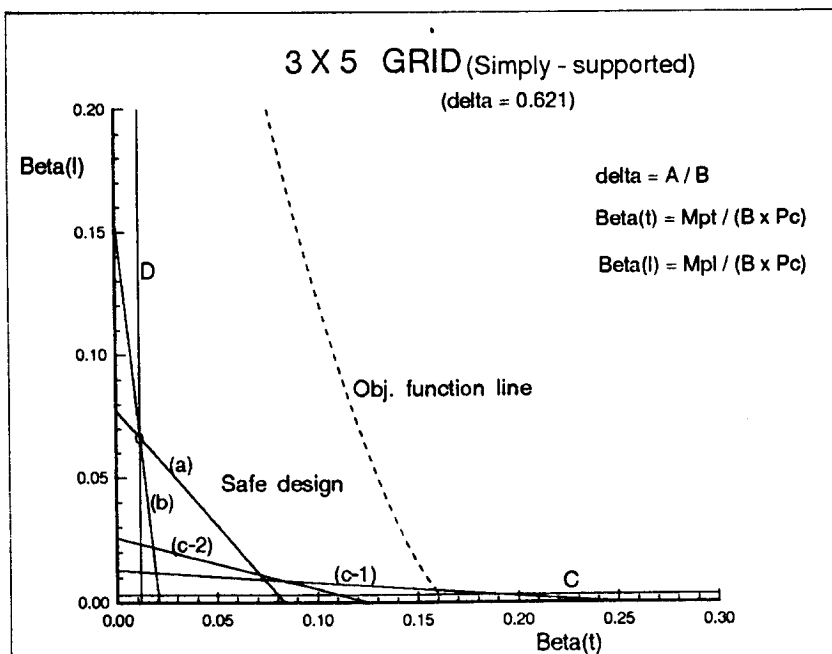


Figure 7: Optimum design point for 3 x 5 grid

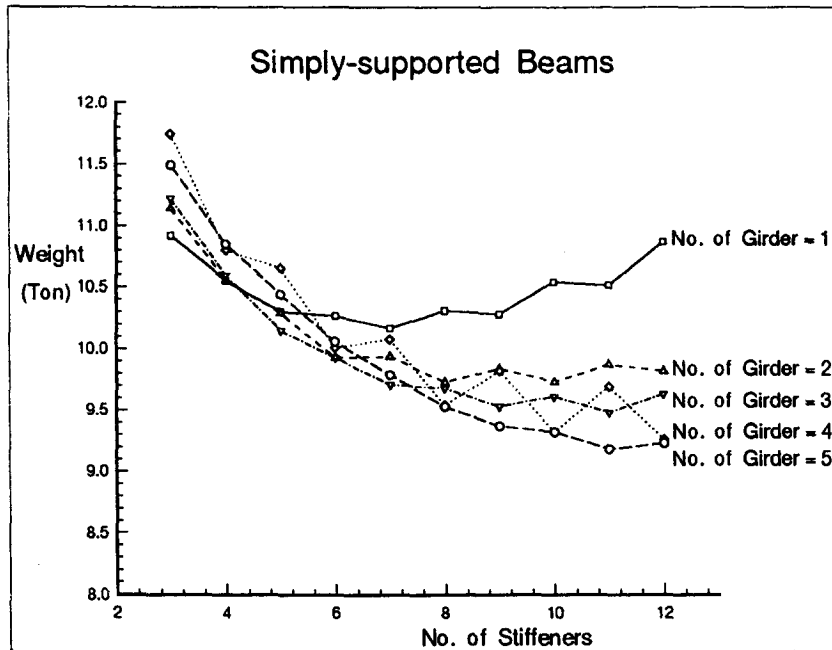


Figure 8: Weight curve for simply-supported beams

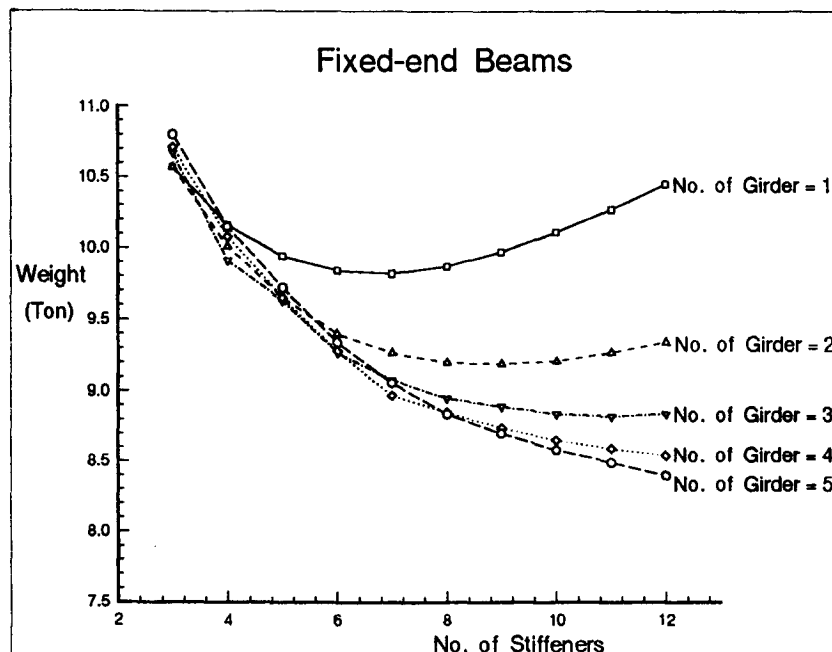


Figure 9: Weight curve for fixed-ends beams