Research on the Safety of Ship and Offshore Structure - on Low Cycle Resonance of a Ship in Severe Following Waves -

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Abstract

For the mechanism of ship capsizing, we can generally consider that it's caused due to pure loss of stability, parametric oscillation(low cycle resonance) of ship in waves and the broaching phenomena. Among them, low cycle resonance occurs due to the dynamic change of righting arm with respect to the relative position of ship to waves. The dynamic change depends on the encounter period of a ship in following waves. This paper discusses the following items : (1) An analytical expression of GZ curve varying with respect to the relative position of ship to waves, (2) Non-linear equation of motion describing low cycle resonance, (3) The effects of righting arm, stability range and encounter period on low cycle resonance.

1 Introduction

When a ship is travelling in following seas, the rolling motion will be developed to large amplitude such as lead up to capsizing in a very short time if the encounter wave period is nearly equal to a half of the natural rolling period. Such a rolling motion is so called low cycle or parametric resonance which is a dangerous situation for ships of poor stability even satisfying A167 and A562 of IMO resolution at the critical requirement. Although ships are usually designed with consideration of safety margin with respect to capsizing throughout various conditions, the combination of severe waves , loading condition , ship speed and heading angle may lead them to poor stability.

Analytical approaches to this problem were originally made by Grim[1], Kerwin[2] and Paulling[3]. They pointed out that low cycle resonance occurs due to the dynamic change of righting arm with respect to the relative position of ship to waves and the unstable regions of low cycle resonance are specified as the solution of linear differential equation of the so called Mathieu's type. It is impossible to simulate the rolling motion of large

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of the so called Mathieu's type. It is impossible to simulate the rolling motion of large amplitude leading up to capsizing by solving the linear equation of motion mentioned above, because the righting arm is actually described in a nonlinear function. Hence it is difficult to understand the rolling motion of large amplitude in detail.

This paper focuses on making an analytical approach to this problem by taking into account a non-linear righting arm GZ instead of linear one. Since this equation of motion is described in an analytical expression, it will be possible to investigate the features of low cycle resonance in detail.

2 Equation of Motion

In this section, an equation of motion is derived to perform an analytic approach to the low cycle resonance. The single degree of rolling motion can be described in the following form

$$(I_x + J_x)\ddot{\phi} + K_{\dot{\phi}}\dot{\phi} + WGZ(\xi_G, \phi) = 0 \tag{1}$$

where $I_x$is the mass moment of inertia of a ship, J_x added mass moment of inertia, $K_{\dot{\phi}}$ damping coefficient, W the weight of a ship and $GZ(\xi_G, \phi)$ righting arm at heeling angle ϕ at the relative position ξ_G of ship to waves.

Since $GZ(\xi_G, \phi)$ in Eq. (1) depends on not only ξ_G and ϕ but the hull form metacentric height GM, wave length λ and wave height H, it is almost impossible to describe the complete expression of $GZ(\xi_G, \phi)$ with an analytical function. We have to make a reasonable approximation for $GZ(\xi_G, \phi)$. For a simplified expression of GZ , it seems reasonable to use the sinusoidal function described in the following form

$$GZ = \frac{\phi_r}{\pi} GM sin\pi \frac{\phi}{\phi_r} \tag{2}$$

where $\phi_r$is the angle of vanishing stability and the initial metacentric height GM is given by

$$GM = \left\{\frac{dGZ}{d\phi}\right\}_{\phi=0} = 0 \tag{3}$$

It is necessary to take into account the variation ofGM which increases at the wave trough amidships and decreases at the wave crest amidships. Concerning this problem Kerwin gave a simple expression of $GM(wave)$ as

$$GM(wave) = GM + \Delta GM cosk(\xi_G - ct) \tag{4}$$

which corresponds to the wave profile ζ_w

$$\zeta_w = acosk(\xi - ct) \tag{5}$$

where $GM(wave)$ is the metacentric height in a wave, $\Delta GM$the small change from GM in still water, k wave number equal to $2\pi/\lambda$, c wave celerity and t time. Now let us consider the relationship between the variation of $GM(wave)$ and the relative position of ship to wave.

Fig.1 shows the wave profile and the variation of$GM(wave)$ which is increased at the wave trough $\xi_G/\lambda = 0$, equal to GM at the up slope $\xi_G/\lambda = 0.25$, decreased at the wave crest $\xi_G/\lambda = 0.5$, equal to GM at the down slope $\xi_G/\lambda = 0.75$ and again increased at the wave trough $\xi_G/\lambda = 1$. The remaining problem is how to determine the angle ϕ_r of vanishing stability which is larger at the wave trough amidships and smaller at the wave crest amidships than that of still water. Here a simplified assumption is made to determine ϕ_r by in same way as the variation of GM, that is

$$\phi_r = \phi_R[1 + \frac{\Delta\phi}{\phi_R} cosk(\xi_G - ct)] \tag{6}$$

where ϕ_R is the angle of vanishing stability in still water and $\Delta\phi$ the small change from ϕ_R. From the discussion mentioned above, $GZ(\xi_G, \phi)$ is given by

$$GZ(\xi_G, \phi) = \frac{\phi_r}{\pi} GM[1 + \frac{\Delta GM}{GM} cosk(\xi_G - ct)] sin\pi \frac{\phi}{\phi_r} \tag{7}$$

Finally, a non-linear equation of motion in Eq.(1) can be described in an analytical form as

$$\ddot{\phi} + 2\alpha_e\dot{\phi} + \frac{\phi_r}{\pi}\omega_\phi{}^2[1 + \frac{\Delta GM}{GM} cos(k\xi_0 - \omega_e t)] sin\pi \frac{\phi}{\phi_r} = 0 \tag{8}$$

where

$$\phi_r = \phi_R[1 + \frac{\Delta\phi}{\phi_R} cos(k\xi_0 - \omega_e t)] \tag{9}$$

$$2\alpha_e = \frac{K_{\dot{\phi}}}{I_x + J_x} \tag{10}$$

$$\omega_\phi{}^2 = \frac{W \cdot GM}{I_x + J_x} \tag{11}$$

Assuming that $\xi_G$is equal to ξ_0 at the time $t = 0$ and the ship speed U, $\xi_G$is equal to $\xi_0 + Ut$ at any time and encounter wave frequency $\omega_e = k(c - U)$. In addition, if the rolling angle is small in Eq.(8),

$$sin\pi \frac{\phi}{\phi_r} \simeq \pi \frac{\phi}{\phi_r} \tag{12}$$

So that Eq.(8) is linearized as

$$\ddot{\phi} + 2\alpha_e\dot{\phi} + \omega_\phi{}^2[1 + \frac{\Delta GM}{GM} cos(k\xi_0 - \omega_e t)] = 0 \tag{13}$$

This is the traditional equation of motion for low cycle resonance introduced by Grim in 1952.

3 Solution of Equation

Let us first consider the solution of linear equation to understand the outline of important parameter. In this case, the lowest and widest unstable region occurs at an exciting frequency of twice of the natural frequency. Thus, unstable roll will be excited if the ship encounter head or following seas with a frequency of encounter equal to twice of the natural frequency of roll. (For usual ship-wave proportion in which capsizing is likely, this occurs only in following seas.)

The problem considered here is investigation of the feature of rolling motion based on the solutions of linear and non-linear equation. That is why, according to the results of free running model test carried out before, the unstable rolling motion is developed to a large amplitude leading up to capsize in a very short time. It is of interest to know what to develop the rolling amplitude in a very short time.

3.1 Solution of Linear Equation

Since Eq.(13) is not a complete form of Mathieu's equation, in finding out the important parameters which are influential, Eq.(13) can be transformed into the complete form as follows.

$$\frac{d^2\Phi}{d\tau^2} + \frac{T_e}{T}^2[1 - (\frac{a_e}{\pi})^2 + \frac{\Delta GM}{GM} cos\tau]\Phi = 0 \tag{14}$$

by using the following relations

$$\phi(t) = \Phi(t)e^{-\alpha_e t}, \tau = \omega_e t \tag{15}$$

where a_e is the effective extinction coefficient given by $\alpha_e = 2a_e/T$ and T the natural period of roll, and T_e is encountering period.

In Eq.(14), a_e is too small in comparison with π . Thus a_e can not be a major factor on the rolling motion. The outline of rolling motion will be mainly determined by two parameters $\Delta GM/GM$ and T/T_e. Eq.(14) is known as Mathieu's equation. Solutions can be expressed in the form of special functions. This equation is a linear differential equation which has the presence of a time-dependent coefficient of the rolling motion variable Φ. The solutions to Mathieu's equation have a property of considerable importance in ship rolling problems in that the solution is unstable for certain values of the period. This implies that if the rolling motion described by Eq.(13) is taken place in an unstable region, the amplitude will grow up.

Let us now try to solve Eq.(13) by using a numerical approximation method to obtain the property above mentioned in Eq.(14). A numerical procedure is used to integrate the equation by adopting a step-by-step approximation. Fig.2 is a graph of $\Delta GM/GM$ versus T/T_e for the Mathieu's equation in which regions correspond to stable and unstable solutions. It is seen that unstable rolling occurs in the wide range T/T_e when $\Delta GM/GM$ has large values. Fig.3 is time histories of

rolling motion corresponding to the stable, critical and unstable regions respectively. The unstable rolling motion grow up gradually, that is to say, it takes a long time to have a large amplitude which leads up to capsizing. This appearance is caused by a linear GZ curve.

3.2 Solution of Non-Linear Equation

Let's give a try to solve Eq.(8) described with a non-linear GZ curve as shown in Fig.4 and 5. In the same way as before, Fig.6 shows the region for stability of solutions by $\Delta GM/GM$ versus T/T_e . There are three regions of stable, unstable and critical in which the rolling motion grows up until a physical constraint included in this equation is operated. Fig.7 indicates the time histories of rolling motion for stable, critical and unstable regions. The rolling motion in unstable region shows quite different behavior from that of a linear equation. This is due to a non-linear GZ curve and seems to be similar to an actual motion obtained from model experiments. Figs.8 and 9 are the time histories of rolling motion for a ship having different stability range at wave crest and trough amidship.

As the result of the analysis mentioned above, we know that the time histories obtained from non-linear equation in the unstable region do not make much difference to the experimental results5) shown in Fig.10 and 11. It implies that the non-linear righting arm should be taken into account to express the unstable roll of a large amplitude leading up to capsize in a very short time.

Furthermore, we can explain this easily. Because a ship can encounter the chance to have the unstable rolling motion even if in a irregular wave. When the ship has larger variation of GZ, the unstable rolling motion is easy to occur in the wider range of wave encounter period as shown in Fig.6. Therefore, in order to find out the range of low cycle resonance, it is necessary to evaluate exactly the variation of GZ in waves and to consider the combination of the variation of GZ, wave encounter period T_e , and natural rolling period T which depend on the ship hull configuration, speed and wave profile. From the view point of safety at sea, this is a important problem to the designers of ship hull configuration and the operators of a ship in rough seas because the domestic and international stability standards do not guarantee the critical limit of this problem.

4 Conclusions

An analytical study is attempted to investigate the fundamental nature of low cycle resonance. The following conclusions are drawn from the present research.

(1) A new equation describing a single rolling motion is presented to obtain the stable and unstable regions of roll. The regions are also compared with the results obtained from the traditional equation of motion.

(2) The time histories of rolling motion in the stable, unstable and critical rolling region are computed by a numerical approximation method. The unstable rolling motion obtained from a non-linear equation seems to be reasonable in comparison with experimental results.

(3) This approach will be available to consider the experimental results of free running model test because a single equation of motion is more understandable than the equations of motion of six degree freedom to know the outline of low cycle resonance.

Acknowledgement

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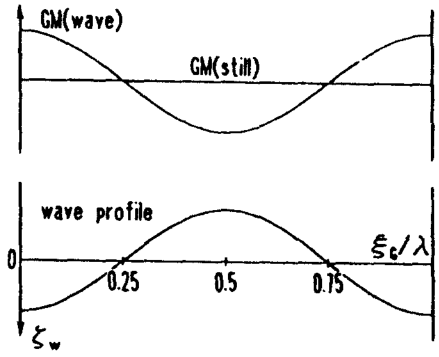

Figure 1: Variation of metacentric height with respect to the relative position of ship to waves

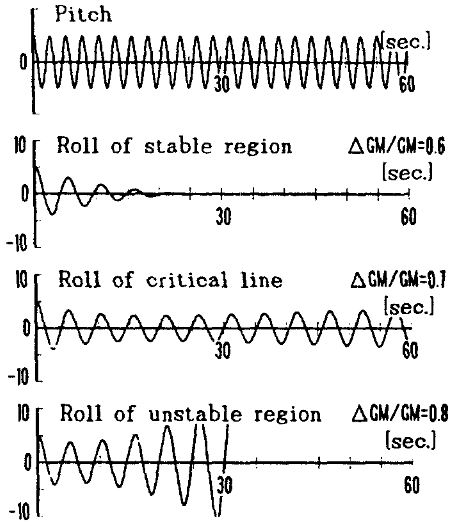

Figure 3: Time histories of stable and unstable regions for linear equation $(T/T_e = 2.292, a_e = 0.2)$

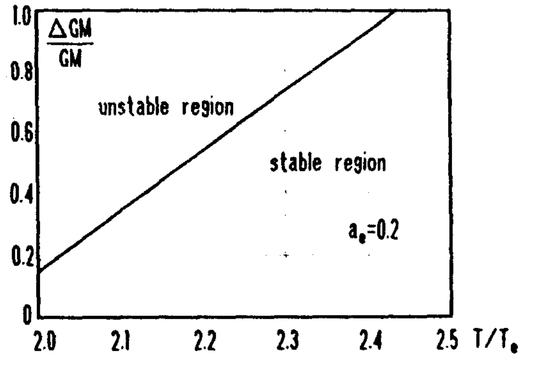

Figure 2: Stable and unstable regions for linear equation

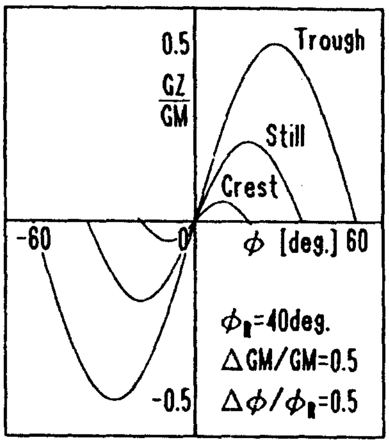

Figure 4: Righting arm for computing time histories in stable and unstable regions $(\Delta\phi/\phi = 0.5)$

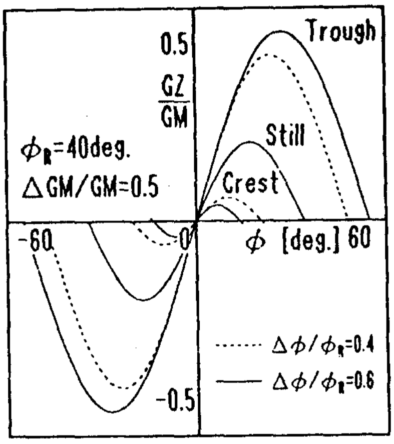

Figure 5: Righting arm for computing time histories $(\Delta\phi/\phi = 0.4, \Delta\phi/\phi = 0.6)$

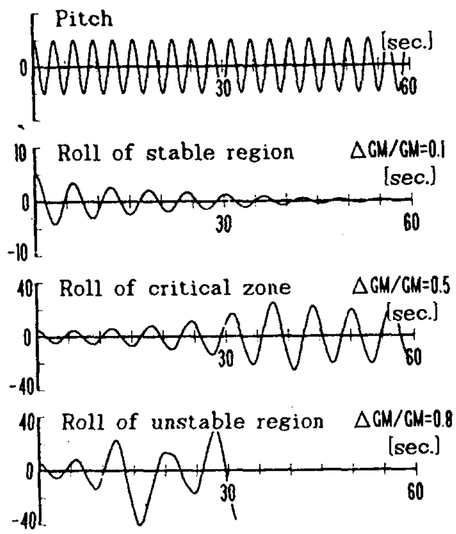

Figure 7: Time histories of stable and unstable regions for non-linear equation $(T/T_e = 1.911, a_e = 0.2, \phi_R = 40\ ^\circ \Delta\phi/\phi_R = 0.5)$

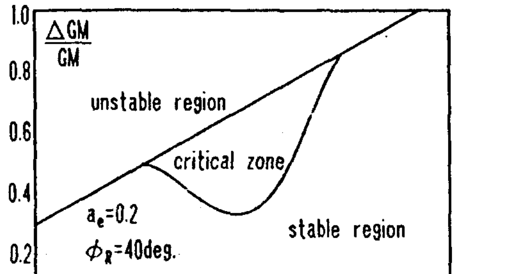

Figure 6: Stable and unstable regions for non-linear equation

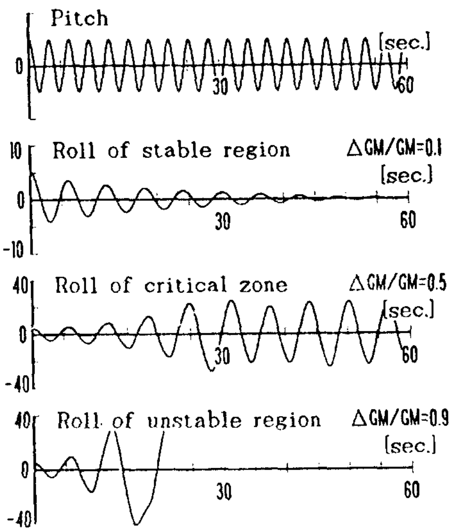

Figure 8: Time histories of stable and unstable regions for non-linear equation $(T/T_e = 1.911, a_e = 0.2, \phi_R = 40\ ^\circ \Delta\phi/\phi_R = 0.4)$

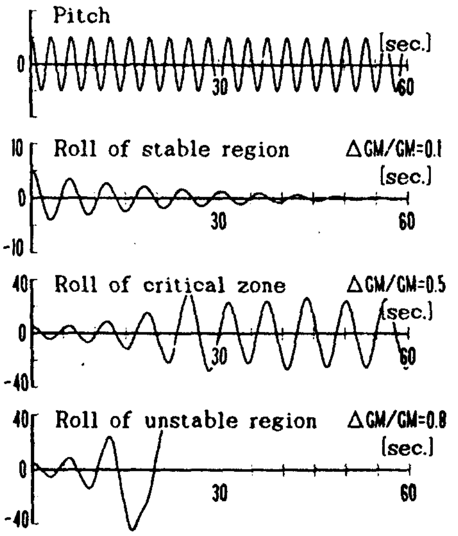

Figure 9: Time histories of stable and unstable regions for non-linear equation ($T/T_e = 1.911, a_e = 0.2, \phi_R = 40\ ^\circ \Delta\phi/\phi_R = 0.6$)

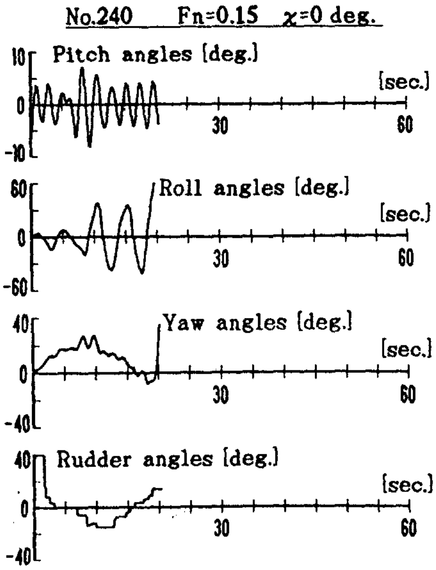

Figure 10: The results of free running model test for container ship in the unstable region

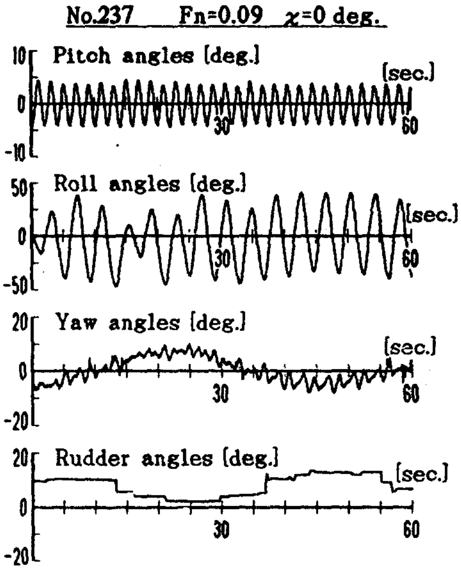

Figure 11: The results of free running model test for container ship in the critical zone