1995年 7月 電子工學會論文誌 第 32 券 A編 第 7 號

論文95-32A-7-1

레일리 페이딩을 겪는 셀룰라 이동통신시스팀의 일반화된 outage probability 해석

(Generalized Outage Probability Analysis for a Cellular Mobile Radio Systems in Rayleigh Fading Environment)

金成敏*, 尹東源**, 韓榮烈*

(Seongmin Kim, Dongweon Yoon, and Youngyearl Han)

요 약

본 논문에서는 레일리 페이딩을 겪는 여러 개의 동일채널간섭성분과 백색 가우시안 잡음이 혼재하는 이동통신시스팀에서 outage probability를 유도하고 해석한다. 유도된 결과는 레일리 페이딩 채널에서 잡음의 유무에 상관없이 적용가능하다. 가우시안 잡음이 존재하지 않는 경우 본 논문에서 유도한 outage probability는 기존의 다른 논문에서 페이딩만을 고려하여 얻은 결과와 동일함을 알 수 있다.

Abstract

In this paper, we generalize the method to calculate the outage probability in the presence of multiple Rayleigh faded cochannel interferences and additive white Gaussian noise. Our result is a computational formula that can be applied with or without Gaussian noise in Rayleigh faded cochannel interferences. Without Gaussian noise, the situation degenerates to usual case of the cochannel interferences. The result can be appiled also in the presence of Gaussian noise with or without cochannel interferences.

I. Introduction

Recently, as a demand for a mobile communication system increase rapidly, a finite frequency resource become exhausted. Therefore, frequency reuse is adopted to a cellular mobile radio system to efficiently use a finite frequency resource. A frequency set is reused over relatively long distant base station.[1] Then outage probability provides a mean of assessing frequency reuse. Outage probability is defined as the probability of failing to achieve adequate signal reception.[2]-[6]

Receiver performance is degraded by the multipath fading, cochannel interferences and environmental noise in a cellular mobile radio system. Multipath fading due to the waves

---

* 正會員, 漢陽大學校 電子通信工學科

(Dept. of Elec. Comm. Eng., Hanyang Univ.)

**正會員, 東西工科大學校 情報通信工學科

(Dept. of Info. & Comm. Eng., Dongseo Univ.)

接受日字:1995年1月18日, 수정완료일:1995年6月28日

reflected from the surrounding buildings and other structures is well represented by a Rayleigh probability density function and this multipath Rayleigh fading can severely degrades the signal transmission performance. Cochannel interference which is generated when two or more radio channels are assigned the same frequency set is inevitable in frequency reuse system. The effect of this component cannot be removed by a bandpass filter or a demodulator. That can be made less by being distant between cells.

In this paper, a generalized outage probability is studied which includes the effects of both multiple Rayleigh faded cochannel interferences and Gaussian noise. The outage probability for single cochannel interference plus nonrandom noise has been represented in [2] and extensive studies on outage probability have been reported for multiple cochannel interferences[4][5]. This paper studies a generalized outage probability which takes account of both the multiple cochannel interferences and the noise by deriving pdf of the m Rayleigh faded cochannel interferences and the Gaussian noise.

Ⅱ. Outage Probability Analysis

In a cellular mobile radio system having m cochannel interference, system is modeled by Fig.1

I_i is cochannel interference power and D is desired instantaneous signal power. It is well known that desired signal and undesired interfering signal experience Rayleigh fading. It is also assumed that the received power from interferences are same, and sum of total noise is Gaussian process.

Under this assumption, we derive the outage probability in single interference and multiple interferences environment.

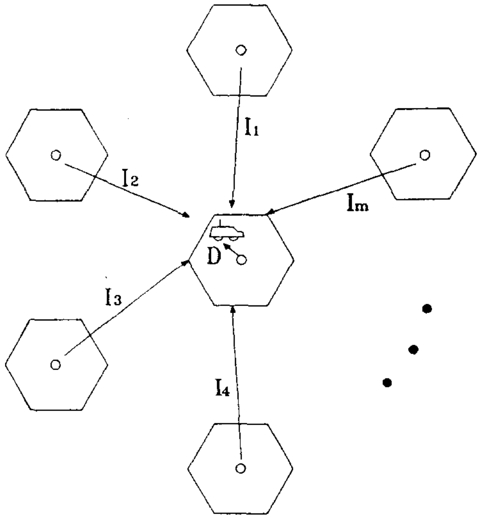


그림 1. m개의 동일채널간섭성분을 갖는 시스템 모델링

Fig. 1. System modeling with m cochannel interference.

Outage Probability is defined as the probability that the ratio of instantaneous power of the desired and the interfering signal is less than a parameter known as protection ratio α.

$$P_{out} = Prob\left(\frac{D}{I} < \alpha\right) \tag{1}$$

D and I represent random variables which stand for the desired and interfering signal power respectively. In a fully equipped hexagonal-shaped cellular system, there are six cochannel interfering cells in the first tier. Eqn.(1) assumed that noise power is small and can be neglected as compared with the interference power.

More general expression of outage probability can be expressed as follows including the effect of noise power

$$P_{out} = Prob\left(\frac{D}{I+N} < \alpha\right) = Prob\left(\frac{D}{U} < \alpha\right) \tag{2}$$

where N is a random variable of instantaneous power due to noise.

1. Outage probability in single interference environment

It is well known that in mobile systems both the desired radio signal and any cochannel interferences experience Rayleigh fading at the mobile receiver. A Rayleigh probability density function for the envelope of the desired signal results in an exponential probability density function for the instantaneous signal power

$$f_D(D)=\frac{1}{\sigma_d^2}e^{-\frac{D}{\sigma_d^2}}, \quad D\geq 0 \tag{3}$$

where $E[D]=\sigma_d^2$ is the local mean power of the desired signal. When the envelope of interfering signal due to one interference has a Rayleigh probability density function, the pdf of the instantaneous interference power becomes

$$f_I(I)=\frac{1}{\sigma_i^2}e^{-\frac{I}{\sigma_i^2}}, \quad I\geq 0 \tag{4}$$

where $E[I]=\sigma_i^2$ is the mean power of the undesired signal. Usually noise is Gaussian process with zero mean and variance σ_n^2, the pdf of the instantaneous noise power is given by

$$f_N(N)=\frac{1}{\sqrt{\pi N}\sigma_n}e^{-\frac{N}{\sigma_n^2}}, \quad N\geq 0 \tag{5}$$

Assuming that N and I are statistically independent, the pdf of the instantaneous power due to interference and noise can be obtained as follows

$$\begin{aligned} f_{I+N}(U) &= \int_{-\infty}^{\infty} f_{N,I}(N, U-N)\,dN \\ &= \int_0^U f_N(N)f_I(U-N)\,dN \\ &= \int_0^U \frac{1}{\sqrt{\pi N}\sigma_n}e^{-\frac{N}{\sigma_n^2}}\cdot\frac{1}{\sigma_i^2}e^{-\frac{U-N}{\sigma_i^2}}\,dN \end{aligned} \tag{6}$$

The pdf of U is given after several manipulations by

$$\begin{aligned} f(U) &= \frac{1}{\sigma_i^2\sigma_n\sqrt{k}}\,erf(\sqrt{k\cdot U})\,e^{-\frac{U}{\sigma_i^2}}, \quad U\geq 0 \\ k &= \frac{1}{\sigma_n^2}-\frac{1}{\sigma_i^2} > 0 \\ erf(x) &= \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \end{aligned} \tag{7}$$

As a check on this pdf, we try to integrate $f(U)$ from 0 to infinity to obtain 1.

The outage probability can be expressed as follows [6]

$$\begin{aligned} P_{out} &= Prob\left(\frac{D}{U}<\alpha\right) \\ &= \int_0^{\infty}\int_0^{\alpha\cdot U} f(D)\,dD\,f(U)\,dU \\ &= \int_0^{\infty}\left(1-e^{-\frac{\alpha\cdot U}{\sigma_d^2}}\right)\cdot\frac{1}{\sigma_i^2\sigma_n\sqrt{k}}\cdot erf(\sqrt{k\cdot U})\cdot e^{-\frac{U}{\sigma_i^2}}\,dU \end{aligned} \tag{8}$$

To integrate eqn. (8), we use following relation [8, pp.649]

$$\int_0^{\infty} erf(\sqrt{q\cdot x})\cdot e^{-p\cdot x}dx = \sqrt{\frac{q}{p+q}}\cdot\frac{1}{p} \tag{9}$$

sing eqn. (9), the outage probability simplifies to

$$\begin{aligned} P_{out} &= 1-\frac{\sigma_d^2/\sigma_i^2}{\sigma_d^2/\sigma_i^2+\alpha}\cdot\sqrt{\frac{\sigma_d^2/\sigma_n^2}{\sigma_d^2/\sigma_n^2+\alpha}} \\ &= 1-\frac{\gamma_i}{\gamma_i+\alpha}\cdot\sqrt{\frac{\gamma_n}{\gamma_n+\alpha}} \end{aligned} \tag{10}$$

where signal to noise and interference ratio γ is defined as

$$\begin{aligned} \gamma &= \frac{\sigma_d^2}{\sigma_n^2+\sigma_i^2} = \left(\left(\frac{\sigma_d^2}{\sigma_n^2}\right)^{-1}+\left(\frac{\sigma_d^2}{\sigma_i^2}\right)^{-1}\right)^{-1} \\ &= (\gamma_n^{-1}+\gamma_i^{-1})^{-1} \\ &\gamma_n=\frac{\sigma_d^2}{\sigma_n^2} \text{ is signal to noise ratio} \\ &\gamma_i=\frac{\sigma_d^2}{\sigma_i^2} \text{ is signal to interference ratio} \end{aligned} \tag{11}$$

In the event of Rayleigh fading only (i.e., $\sigma_n^2=0$), the outage probability becomes

$$P_{out}=P\left(\frac{D}{I}<\alpha\right)=1-\frac{\gamma_i}{\gamma_i+\alpha} \tag{12}$$

In the event of no cochannel interference being transmitted (i.e. $\sigma_i^2 = 0$), the outage probability can be expressed as

$$P_{out} = P\left(\frac{D}{N} < \alpha\right) = 1 - \sqrt{\frac{\gamma_n}{\gamma_n+\alpha}} \tag{13}$$

Fig.2. and Fig.3 show the numerical results of eqn.(10). Figures illustrate outage probability as a function of SIR (signal to interference ratio) for several value of SNR (signal to noise ratio) in single interference environment. From Fig.2. and Fig.3. we can notice that as the SNR increases, i.e. as the influence of noise decreases, outage probability decreases and as the protection ratio α increases, outage probability increases. It is also observed that for SIR ≤ 20dB the cochannel interference dominates as the major source of outage probability at low values of SNR.

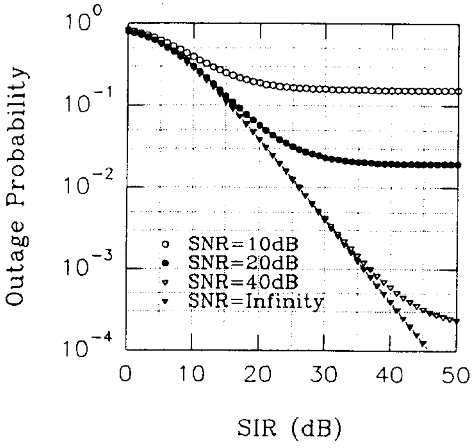


그림 2. 단일간섭에서의 outage probability
α = 6dB, 신호대 잡음비 = 10, 20, 40, 무한대 dB

Fig. 2. Outage probability for single interference case
α = 6dB, SNR = 10, 20, 40, Infinity dB

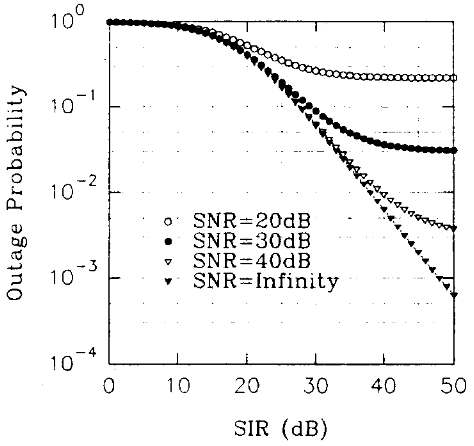


그림 3. 단일간섭환경에서의 outage probability
α = 18dB, 신호대 잡음비 = 20, 30, 40, 무한대 dB

Fig. 3. Outage probability for single interference case
α = 18dB, SNR = 20, 30, 40, Infinity dB

2. Outage probability in multiple interference environment

The outage probability in a cellular mobile radio system with m-cochannel interferences and background noise is calculated. As mentioned above, cochannel interferences experience Rayleigh fading and it is assumed that the each interference have the same mean power $E[I_i] = \sigma_i^2$. The probability density function of m exponential distributed interfering power is given by [5]

$$f(I) = \frac{I^{m-1}}{(m-1)!\,\sigma_i^{2m}} e^{-\frac{I}{\sigma_i^2}} \quad I \geq 0 \tag{14}$$

$$I = I_1 + I_2 + \cdots + I_m$$

Assuming that sum of the m-cochannel instantaneous interference power I and the noise power N are statistically independent, the pdf of the I and the N can be obtained by convolution. It can be derived as follows

$$f(U) = \frac{1}{\sigma_n \sigma_i^{2m} \Gamma\left(m+\frac{1}{2}\right)} e^{-\frac{U}{\sigma_i^2}} U^{m-\frac{1}{2}} {}_1F_1\left(\frac{1}{2}; m+\frac{1}{2}; -k\cdot U\right)$$

$$U = N + I_1 + I_2 + \cdots + I_m = N + I \geq 0$$

$$k = \frac{1}{\sigma_n^2} - \frac{1}{\sigma_i^2} \tag{15}$$

$${}_mF_n(\alpha_1, \cdots, \alpha_m;\ \beta_1, \cdots, \beta_n;\ x) = \sum_{j=0}^{\infty} \frac{(\alpha_1)_j \cdot (\alpha_2)_j \cdots (\alpha_m)_j}{(\beta_1)_j \cdot (\beta_2)_j \cdots (\beta_m)_j} \frac{x^j}{j!} \tag{16}$$

$$(\alpha)_j = \frac{\Gamma(\alpha + j)}{\Gamma(\alpha)}$$

where ${}_mF_n(\alpha_1 \cdots \alpha_m;\ \beta_1 \cdots \beta_n;\ x)$ is hypergeometric function [8, pp.1045] and $\Gamma(\cdot)$ is gamma function. To obtain outage probability, we use eqn. (3), eqn. (15) and eqn. (8).

$$P_{out} = \int_0^\infty \int_0^{\alpha \cdot U} f(D)\, dD f(U)\, dU = \int_0^\infty \left(1 - e^{-\frac{\alpha \cdot U}{\sigma_i^2}}\right) \cdot f(U)\, dU \tag{17}$$

To integrate eqn. (18), we use following relation. [8, pp.851]

$$\int_0^\infty e^{-\mu x} x^{\nu - 1} {}_1F_1(\alpha; \beta; \lambda x)\, dx = \Gamma(\nu) \mu^{-\nu} {}_2F_1\left(\alpha, \nu; \beta; \frac{\lambda}{\mu}\right), \quad \mu > 0,\ \nu > 0 \tag{18}$$

$${}_2F_1(\alpha,\ \beta;\ \beta;\ x) = (1 - x)^{-\alpha}, \quad \text{arbitrary } \beta \tag{19}$$

Using eqn. (18), eqn. (19) and eqn. (17), the outage probability is shown as follows

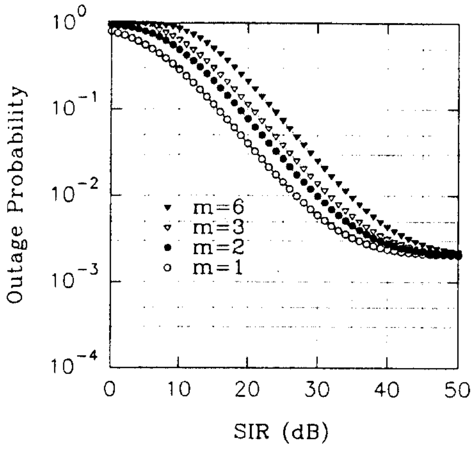
$$P_{out} = 1 - \left[\frac{\sigma_d^2/\sigma_i^2}{\sigma_d^2/\sigma_i^2 + \alpha}\right]^m \cdot \sqrt{\frac{\sigma_d^2/\sigma_n^2}{\sigma_d^2/\sigma_n^2 + \alpha}} \tag{20}$$

In the case of m is 1 in eqn. (20), the outage probability results in the eqn. (10) due to single interference. In the event of Rayleigh fading only (i.e. $\sigma_n^2 = 0$), the outage probability becomes

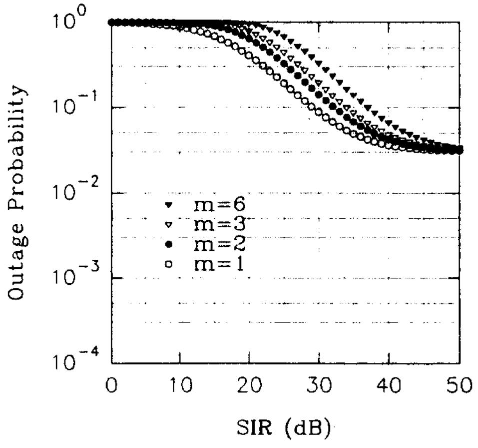
$$P_{out} = 1 - \left[\frac{\sigma_d^2/\sigma_i^2}{\sigma_d^2/\sigma_i^2 + \alpha}\right]^m \tag{21}$$

This result coincide with (1) of [9]. In the event of no cochannel interferences being transmitted (i.e. $\sigma_i^2 = 0$), the outage probability reduces to eqn. (3)

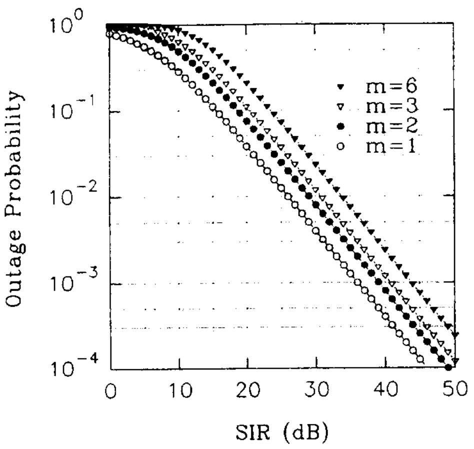
Fig.4, Fig.5 show the influence of multiple cochannel interferences on the outage probability as a function of SIR (signal to interference ratio) with fixed protection ratios and SNRs. Note that in Fig.4 SIR is defined as σ_d^2/σ_i^2. In each case, outage probability curves are presented for different value of α and SNR. Fig.5 shows the outage probability with respect to total interference power by defining SIR as $\sigma_d^2/m\sigma_i^2$.



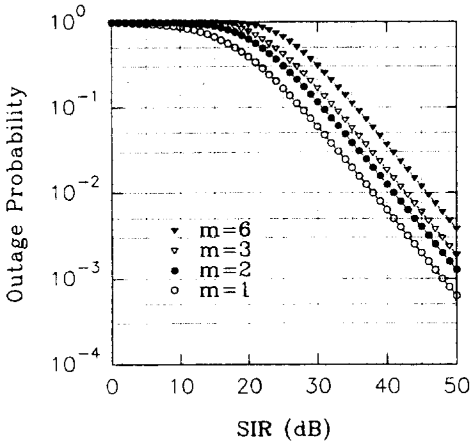
(a) SNR=30dB, α=6dB



(b) SNR=Infinity, α=6dB



(c) SNR=30dB, α=18dB



(d) SNR=Infinity, α=18dB

그림 4. 다중간섭환경에서의 outage probability
신호대 간섭비$=\sigma_d^2/\sigma_i^2$, m = 간섭성분의 수

Fig. 4. Outage probability for multiple interferences case
SIR $=\sigma_d^2/\sigma_i^2$, m = number of interferences.

Assuming that the total interference power is I, little or no difference in outage probabilities were observed for single ($I_1=I$)and multiple interferences ($I_1=I_2=\cdots I_m=I/m$) cases. This implies that the outage probability is influenced by the total interference power, not by the number of interferences.

Ⅲ. Conclusion

A generalized outage probability including the effects of both m cochannel Rayleigh faded interferences and additive white Gaussian noise has been investigated in the cellular mobile radio system. The derived result is a computational formula that can be applied with or without Gaussian noise in Rayleigh faded cochannel interferences. The obtained expression shows that single interference and multiple interferences are found to have approximately the same effect on outage probabilities provided that the mean total power of single interference and multiple interferences are same.

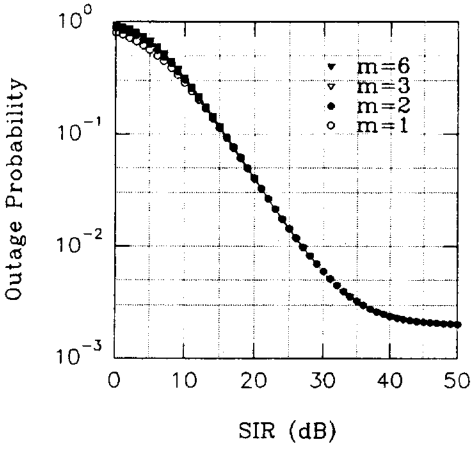


그림 5. 다중간섭환경에서의 outage probability
α = 6dB, 신호대잡음비 = 30dB, 신호대 간섭비$=\sigma_d^2/m\sigma_i^2$, m = 간섭성분의 수

Fig. 5. Outage probability for multiple interferences case
α = 6dB, SNR = 30dB, SIR = $\sigma_d^2/m\sigma_i^2$, m = number of interferences.

REFERENCE

[1] W. C. Y. Lee, "Mobile Communications Design Fundamentals", Wiley Inter Science.

[2] H. D. M. Gurr, "Generalised Outage Probability and BER Estimation Using Dual Protection Margins", 1994 Vehicular Technology Conference, Stockholm Sweden, June 8-10, Vol.3 pp.1440 - 1444.

[3] Y. D. Yao, "Investigations into Cochannel Interference in Microcellular Mobile Radio System", IEEE Trans.,1992, VT Vol 41, pp.114 - 123.

[4] K. W. Sowerby, "Outage Probability Calculations for a Mobile Radio System having Multiple Rayleigh Interferers", Electronic Letters 21st May 1987 vol.23 No.11 pp.600 - 601.

[5] Y. D. Yao, "Outage Probability Analysis for Microcell Mobile Radio Systems with Cochannel Interferences in Rician/Rayleigh Fading Environment", Electronic Letters 21st June 1990 vol.26 No.13 pp.864 - 866.

[6] R. Prasad, "Near-Far Effects on Performance of DS/SS CDMA Systems for Personal Communication Networks", 1993 Vehicular Technology Conference, New Jersey, May. pp.710 - 713.

[7] A. L. Mood, "Introduction to the Theory of Statistics", McGraw-Hill, Inc., 1982.

[8] I.S.Gradshteyn and I.M. Ryzhik, "Table of Integrals, Series, and Products", Academic Press, Inc., 1980.

[9] K. W. Sowerby, "Selection Diversity in Multiple Interferer Mobile Radio Systems", Electronic Letters 24th Nov. 1988 Vol.24 No.24.

저 자 소 개

金 成 敏(正會員)

1972년 2월 27일생. 1994년 2월 한양대학교 전자통신공학과 공학사. 1994년 3월 - 현재 한양대학교 전자통신공학과 석사과정

韓 榮 烈(正會員)

1938년 6월 10일생. 1960년 서울대학교 전자공학과 학사. 1976년 미주리 대학교 대학원 공학석사. 1979년 미주리 대학교 대학원 공학박사. 1980년 - 현재 한양대학교 전자통신공학과 교수. 1980년 - 1994년 통신공학회 이사, 상임이사, 부회장. 1995년 - 현재 통신공학회 수석 부회장

尹 東 源(正會員)

1966년 1월 22일생. 1989년 2월 한양대학교 전자통신공학과 공학사. 1992년 2월 한양대학교 전자통신공학과 공학석사. 1995년 8월 한양대학교 전자통신공학과 공학박사. 1995년 3월 현재 동서공과대학교 정보통신공학과 전임강사