Fuzzy Servo Design for Electromechanical Systems

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Abstract

In this paper, a fuzzy logic is applied to a model-following control(MFC) to form a fuzzy model following control(FMFC). The feedback gain the MFC is adjusted continuously through the fuzzy logic rule. The proposed fuzzy-MFC is applied is applied to synthesize controllers for linear time inverlant(LTI) systems with parameter uncertainties, and the robustness results of the proposed designs are compared.

Ⅰ. Introduction

Electromechanical systems require true servo designs, allowing a desired trajectory to be accurately followed. A controller that has good performance robustness is of importance for electromechanical system applications, where such a structure may provide enough tolerance of model uncertainties and disturbances. In this paper, a fuzzy logic is applied to the model-following controllers(MFC) of [7] to form fuzzy model following controllers(FMFC). As shown in [1, 7], the model-following, servo and tracking problems bear strong resemblance to each other. Therefore, it should be possible to formulate a genetic algorithm for all three cases.

The organization of this paper is as follows: In section 2, we discuss a H_∞ model-following control (MFC), where the system transfer function has a sub-optimal H_∞-norm tracking error of less than a given scalar. In section 3, we present a fuzzy-MFC which the feedback gain is adjusted continuously through the fuzzy logic rule. In section 4, the proposed fuzzy-MFC is applied to synthesize controllers for linear time inverlant (LTI) systems with parameter uncertainties. The robustness results of the proposed designs when plant parameter uncertainties exist are compared. Section 5 concludes the paper.

Ⅱ. Formulations of Servo Problem

We first consider a model-following control (MFC) problem. Let a plant, a model to be followed and a weighting function be given by the state-vector differential equation, respectively

$$\begin{aligned} &\text{plant: } \dot{x}_p = A_p x_p + B_p u, \quad y_p = C_p x_p \\ &\text{model: } \dot{x}_m = A_m x_m + B_m r, \quad y_m = C_m x_m \\ &\text{w.f.: } \dot{x}_f = A_f x_f + B_f u_f, \quad y_f = C_f x_f \end{aligned} \tag{1}$$

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The realization of a generalized plant G can be described by the state equation (see Francis [3] and Maciejowski [5])

$$\begin{aligned} \dot{x} &= Ax(t) + B_1 w(t) + B_2 u(t) \\ z(t) &= C_1 x(t) + D_{12} u(t) \\ y(t) &= C_2 x(t) + D_{21} w(t) \end{aligned} \tag{2}$$

where the signal $w(t) \in \mathbf{R}^{m1}$ is the external input vector, which contains all external inputs, including disturbances, sensor noise and commands; $x(t) \in \mathbf{R}^{m2}$ is the state vector; $z(t) \in \mathbf{R}^{p1}$ is the error vector; $y(t) \in \mathbf{R}^{p2}$ is the measured variable vector; and $u(t) \in \mathbf{R}^{n}$ is the control input vector;

The transfer function $G(s)$ will be denoted as

$$G(s) = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} := \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right] \tag{3}$$

Then the (suboptimal) H_∞ control problem is this: find the all admissible controllers K such that the H_∞ norm of the transfer matrix from w to z is smaller than a prescribed positive number (called the error bound) γ i.e.

$$\| G_{11} + G_{12} K (I - G_{22} K)^{-1} G_{21} \|_\infty \langle \gamma \tag{4}$$

Both G and K are real-rational proper transfer matrices. K is designed to minimize the H_2-norm or H_∞ -norm of the transfer matrix w to z. In both cases K is constrained to provide internal stability.

Let x be the state vector

$$x = [x_p^T \; x_m^T \; x_f^T]^T \tag{5}$$

and w be the input vector which consists of the reference input r.

Then, the model following problem will be described by the state equations

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ &= \begin{bmatrix} A_p & 0 & 0 \\ 0 & A_m & 0 \\ -B_f C_p & B_f C_m & A_f \end{bmatrix} x + \begin{bmatrix} 0 \\ B_m \\ 0 \end{bmatrix} w + \begin{bmatrix} B_p \\ 0 \\ 0 \end{bmatrix} u \\ z &= C_1 x + D_{12} u \\ &= \begin{bmatrix} 0 & 0 & C_f \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \beta I \end{bmatrix} u \\ y &= C_2 x + D_{21} w = x \end{aligned} \tag{6}$$

Note that z is a frequency weighted tracking error and $D_{12} = [0 \; \beta]^T$ is needed to constrain the control effort.

From the results of the H_∞ sub-optimal problem (see [5] and [6]), we have the following lemma [7];
Lemmal 1 *If*

$$K = -B_2^T P/\beta^2 \tag{7}$$

is a stabilizing controller then K solves(4) *where* $P = P^T \geq 0$ *satisfies Riccati equation,*

$$AP + PA - P(B_2 B_2^T/\beta^2 - B_1 B_1^T/\gamma^2)P + Q = 0 \tag{8}$$

with B_1 *and* B_2 *defined as* (6) *and* Q *is a nonnegative definite symmetric matrix*

□□□

We partition P, the solution of Riccati equation (8), as

$$P = P^T = \begin{bmatrix} P_0 & P_1 & P_2 \\ P_1^T & P_{11} & P_{12} \\ P_2^T & P_{12} & P_{22} \end{bmatrix} \rangle 0$$

Then, the optimal control law is obtained as

$$u = K_p x_p + K_m x_m + K_f x_f + K_r r \tag{9}$$

where

$$\begin{aligned} K_p &= -(1/\beta^2) B_p' P_0, \\ K_m &= -(1/\beta^2) B_p' P_{11}, \\ K_f &= -(1/\beta^2) B_p' P_{12}, \\ K_r &= -(1/\beta^2) B^T \\ &= [A^T - P(B_2 B_1^T/\beta^2 - B_1 B_1^T/\gamma^2]P B_1 \end{aligned} \tag{10}$$

with A, B_1, B_2 are defined as (6).

The servo problem results when $r = 0$, which $x_m(t)$ of(1) is an arbitrary member of a class of command inputs. Then the problem formulation follows the rest of model-following case (see [2, 4, 1] for further details). The tracking problem arises where y_m of (1) is a prescribed time-function on $[0, t_1]$.

Ⅲ. Fuzzy Model Following Controllers(FMFC)

Let $e(t)$ is the error $e(t) = x_m(t) - x_p(t)$, and $\hat{K}_m$, $\hat{K}_f$, $\hat{K}_r$ are the required controller parameters respectively. Then, from the model-following structure given in previous section, we have error differential equations

$$\dot{e}(t) = K_p e(t) + \phi_m x_m(t) + \phi_f x_f(t) + \phi_r r(t)$$

with $\phi_m = \hat{K}_m - K_m$, $\hat{K}_f - K_f$, $\hat{K}_r - K_r$. The objective is to adjust K_m, K_f, K_r in such a manner that

$$\lim_{t\to\infty} K_m = \hat{K}_m,\ \lim_{t\to\infty} K_f = \hat{K}_f,\ \lim_{t\to\infty} K_r = \hat{K}_r$$

From the eadptive law used in [6], we can propse

$$\begin{aligned} \dot{K}_m &= -K_m * K_Y * e(t), \\ \dot{K}_f &= -K_f * K_Y * e(t), \\ \dot{K}_r &= -K_r * K_Y * r(t) \end{aligned} \tag{11}$$

where K_Y is obtained by fuzzy logic rule.

The fuzzy MFC input signals are e and change in the error $\dot{e}$. The FSMC output signal is K_Y. Then change in the feedback gain can be obtained from Eq.(11).

Three membership functions associated with each input variable and five membership functions for the output variable. The membership functions associated with e, $\dot{e}$, and K_Y are given in Figure 1. The relationship between the two input fuzzy variables is expressed using nine(9) IF...THEN...fuzzy rules as summerized in Table 1. Block diagrams of the proposed fuzzy logic controller is shown in Figure 2.

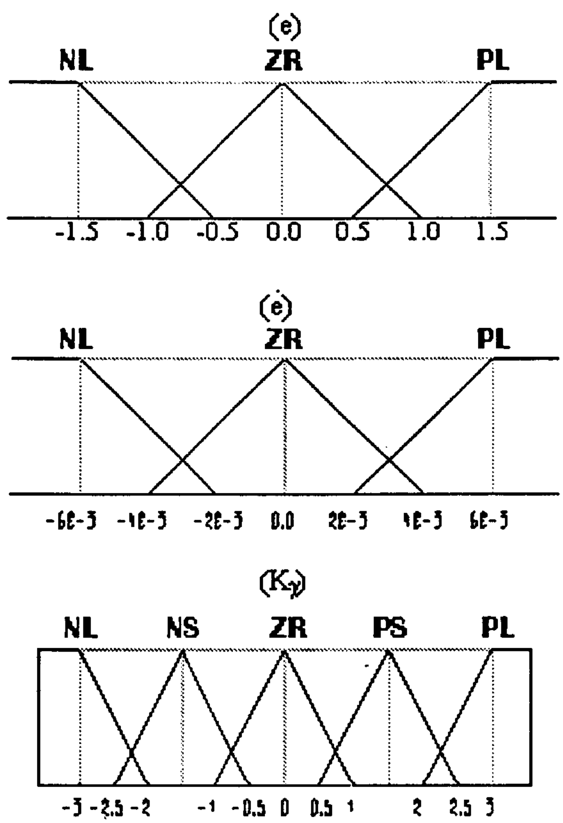

Figure 1. Membership functions of fuzzy-MFC

Table 1. Fuzzy rules used in the Fuzzy-MFC.
NL : negative large, PS : positive small, ZR : zero, etc.

Rule 1 If e NL and $\dot{e}$ is NF, make K_γ = NS
Rule 1 If e NL and $\dot{e}$ is ZR, make K_γ = NL
Rule 1 If e NL and $\dot{e}$ is PL, make K_γ = NL
Rule 1 If e ZR and $\dot{e}$ is NF, make K_γ = PS
Rule 1 If e ZR and $\dot{e}$ is ZR, make K_γ = ZR
Rule 1 If e ZR and $\dot{e}$ is PL, make K_γ = NS
Rule 1 If e PL and $\dot{e}$ is NF, make K_γ = PL
Rule 1 If e PL and $\dot{e}$ is ZR, make K_γ = PS
Rule 1 If e PL and $\dot{e}$ is PL, make K_γ = ZR

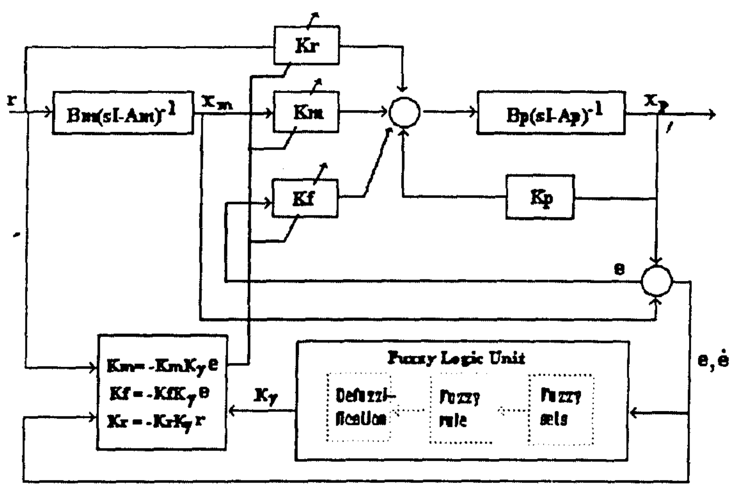

Figure 2. Fuzzy-MFC block diagram

Ⅳ. An Application Example

In order to demonstrate the performance robustness of the proposed designs, we considered a plant (A_p, B_m, C_m) taken from [7]:

$$A_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -2.37 & -0.01 \end{bmatrix},$$

$$B_p = \begin{bmatrix} 0 \\ 0.025 \\ 0 \\ 0.2 \end{bmatrix}, C_p = [1\ 0\ 1\ 0] \tag{12}$$

$$A_m = \begin{bmatrix} -0.260 & 1 & -0.260 & 0 \\ -0.025 & 0 & -0.025 & 0 \\ -4.500 & 0 & -4.500 & 1 \\ -0.200 & 0 & -2.570 & -0.01 \end{bmatrix},$$

$$B_m = B_p,\ C_m = C_p$$

For setpoint tracking, we select the frequency weighting function $\frac{1}{s}$. By incorporating (12) and the frequency weighting function into (6), A, B_1, B_2, are obtained. The Riccati equation (8) is solved to give the H_∞ model-following controllers.

Suppose that the plant has the following parameter uncertainty structure:

$$\Delta A_p = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.15 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The input value difference of e and $\dot{e}$, and output value fuzzy legions K_Y for this example are shown in Figure 1.

In Figure 3, the step responses of $H_\infty-MFC$ and $Fuzzy-MFC$ are compared, while the dotted line is the step response of the reference model. We can see that the $H_\infty-MFC$ design has bad performance specifications; while $Fuzzy-MFC$ has good performance robustness.

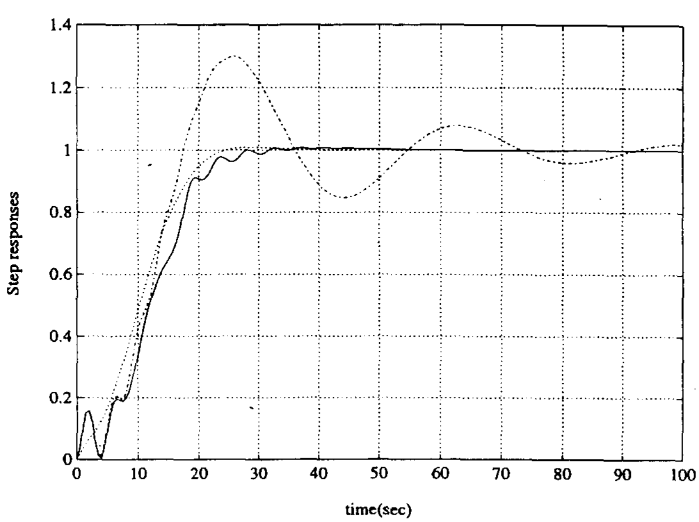

Figure 3. Step responses for fuzzy MFC
dash-dotted : MFC,
solid : fuzzy MFC,
dotted : model

V. Conclusion

In this paper, a fuzzy logic is applied to a model-following control(MFC) to form a fuzzy model following control(FMFC). The feedback gain the MFC is adjusted continuously through the fuzzy logic rule. The proposed fuzzy-MFC is applied to synthesize controllers for a linear time inveriant (LTI) system with parameter uncertainties. From the simulation result, we show that the $Fuzzy-MFC$ has good performance robustness.

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