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Dependent F Ratios Sharing a Common Denominator in ANOVA Table

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ABSTRACT

Let F_1 and F_2 be two F ratios with independent numerators and a common denominator. They are known to be positively dependent. The probabilities of simultaneous rejection and conditional rejection are numerically computed for both null and nonnull cases. The probabilities are presented in tables and graphics to show the influence of the seven parameters, the degrees of freedom of the numerators and the denominator, the non-centralities of the numerators, and the two levels of significance of the tests. The values of the correlation coefficient between F_1 and F_2 are also presented. Finally, a conjecture on the dependence order of the family of distributions of (F_1, F_2) is given.

1. INTRODUCTION

Several statisticians have studied dependent F ratios with independent numerators and a common denominator. Kimball (1951) evaluated the effect of dependency among the tests of significance, when each experiment is treated as a unit regardless of the number of hypotheses tested per experiment. He proved that the joint significance level of two dependent F tests is greater than that

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of independent F tests. Hurlburt and Spiegel (1976) illustrated this result by numerically evaluating the joint distribution of dependent F ratios for various degrees of freedom. By the numerical integration using the trapezoidal-rule, they evaluated the conditional probability under the null hypothesis that one F ratio is significant, given that another F ratio sharing the same denominator mean square is significant. They showed that the conditional Type I error is significantly larger than the unconditional Type I error.

Johnson and Herr (1984) discussed and computed the case where the two numerators are also dependent. They called such F's 'doubly dependent' in contrast with 'simply dependent' F's, whose numerators are independent. They used series, not numerical integration, for computing the power of the test. Many related papers are listed in their paper. Feingold and Korsog (1986) gave an explicit form of the correlation coefficient between two dependent F ratios for the nonnull case, and showed that the joint or conditional powers of dependent F tests are approximated by simple functions of the correlation coefficient.

In Section 2 of this paper, joint and conditional powers of 'simply dependent' F's, for both null and nonnull cases, are computed. The degrees of freedom of the numerators are restricted to 'even' integers. Hence the powers can be more easily and exactly computed by series or double series. Numerical results are presented in two tables and six graphics. Influence of the seven parameters is discussed.

The correlation coefficient between two F's has a simple closed form. In Section 3, the values of the correlation coefficient are also shown in table and graphics. In Section 4, the last section, the dependence order in the family of distributions of (F_1, F_2) is discussed, and a conjecture is given.

2. THE CONDITIONAL PROBABILITIES

Consider the following two way analysis of variance (ANOVA) table:

Source	Sum of squares	degree of freedom	F ratio
Row	SS_R	ν_1	$F_1 = (SS_R/\nu_1)/(SS_E/\nu)$
Column	SS_C	ν_2	$F_2 = (SS_C/\nu_2)/(SS_E/\nu)$
Error	SS_E	ν	

(1)

The null hypotheses H_1 and H_2 assume no row and column effects, respectively. Under the normality assumptions, the critical region where H_i is rejected is $F_i > a_i$, where a_i is the upper α-quantile of the F distribution with a pair of degrees of freedom (ν_i, ν), $i = 1, 2$.

Let $G_A(r)$ denote the gamma distribution with the probability density function $f(x;r) = x^{r-1}\, e^{-x} \,/\, \Gamma(r)$, $x > 0$ and $r > 0$. Let $N_C G_A(r,\delta)$ denote the non-central gamma distribution with the probability density function $f(x;r,\delta) = \sum_{k=0}^{\infty} (e^{\delta}\, \delta^k / k!)\, f(x; r+k)$, $x > 0, r > 0$ and $\delta \geq 0$. Recall that if r is a positive integer the distribution function $F(x) = F(x;r)$ of $G_A(r)$ is written as

$$\overline{F}(x) = 1 - F(x;r) = \int_x^{\infty} f(t;r)\, dt = \sum_{0 \leq k < r} e^{-x}\, x^k / k!. \tag{2}$$

The F statistics in the ANOVA table can be represented as $F_i = \nu X_i / \nu_i X$, $i = 1, 2$, where X is a $G_A(\nu/2)$ variable, and such that X_i is a $G_A(\nu_i/2)$ variable under the null hypothesis H_i , or a $N_C G_A(\nu = 12, \lambda = 12)$ variable under the alternative H_i^*, respectively, $i = 1$ and 2. λ_i is the noncentral parameter of the noncentral F distribution of F_i under H_i^*. All the variables X, X_1 and X_2 are independent. For simplicity , we write $m = \nu/2$, and $m_i = \nu_i/2$, $i = 1, 2$. It is following that we obtain exact formulas by using probability integral transformation of Peason, K.

[1] The joint power (the probability of simultaneous rejection of H_1 and H_2) in the null case is

$$\begin{aligned}
&Pr[(H_1^* \cap H_2^*) | H_1, H_2] \\
&= Pr[F_1 > a_1 \ \ and \ \ F_2 > a_2] \\
&= Pr[X_1 > \nu_1 X a_1/\nu \ \ and \ \ X_2 > \nu_2 X a_2/\nu] \\
&= \int_0^{\infty} \overline{F}(\theta_1 x\,; m_1)\, \overline{F}(\theta_2 x\,; m_2)\, f(x;m)\, dx \\
&= \{A^m \Gamma(m)\}^{-1} \sum_{i=0}^{m_1-1} \sum_{j=0}^{m_2-1} \frac{z_1^i\, z_2^j\, \Gamma(m+i+j)}{i!\, j!} \\
&= \sum_{0 \leq i < m_1} \sum_{0 \leq j < m_2} \binom{m+i+j-1}{i} \binom{m+j-1}{j} z_1^i z_2^j (1 - z_1 - z_2)^m, \quad (3)
\end{aligned}$$

where $\theta_i = m_i a_i / m = \nu_i a_i / \nu$, $z_i = \theta_i / A$, $A = 1 + \theta_1 + \theta_2$; $m, m_1, m_2 = 1, 2, \cdots$; and $0 < z_1,\ z_2 < 1$.

The power of F_2 in the null case is

$$
\begin{aligned}
Pr[(H_2^*)|H_2] &= Pr[F_2 > a_2] \\
&= \int_0^\infty \int_{\theta_2 x}^\infty f(x_2, x)\, dx_2\, dx \\
&= \{(1+\theta_2)^m \Gamma(m)\}^{-1} \sum_{j=0}^{m_2-1} \frac{z^j (m+j-1)!}{j!} \\
&= \sum_{0 \le j < m_2} \binom{m+j-1}{j} z^j (1-z)^m, \qquad (4)
\end{aligned}
$$

where $z = \theta_2/(1+\theta_2)$.

[2]The joint power in the nonnull case with noncentrality parameters λ_i, $i = 1, 2$.

$$
\begin{aligned}
&Pr[(H_1^* \cap H_2^*)|H_1^*, H_2^*] \\
&= \sum_{k,l} A_{kl} \sum_{0 \le i < m_1+k} \sum_{0 \le j < m_2+l} \binom{m+i+j-1}{i} \binom{m+j-1}{j} z_1^i z_2^j (1 - z_1 - z_2)^m, \qquad (5)
\end{aligned}
$$

and the power of F_2 in the nonnull case is

$$
Pr[(H_2^*)|H_2^*] = \sum_l A_l^{(2)} \sum_{0 \le j < m_2+l} \binom{m+j-1}{j} z^j (1-z)^m, \qquad (6)
$$

where $A_{kl} = A_k^{(1)} A_l^{(2)}$, $A_l^{(2)} = \delta_2^l\, e^{-\delta_2} / \, l\,!$; $k, l = 0, 1, 2, \cdots$.
The conditional probability that $F_1 > a_1$ given that $F_2 > a_2$, for positive constants a_1 and a_2, can be evaluated by

$$
Pr[(H_1^*|H_2^*)|H_1, H_2] = Pr[(H_1^* \cap H_2^*)|H_1, H_2] \;/\; Pr[(H_2^*)|H_2], \qquad (7)
$$

in the null case and

$$
Pr[(H_1^*|H_2^*)|H_1^*, H_2^*] = Pr[(H_1^* \cap H_2^*)|H_1^*, H_2^*] \;/\; Pr[(H_2^*)|H_2^*], \qquad (8)
$$

in the nonnull case. Define a double sequence in the nonnull case,

$$
\begin{aligned}
b_{kl} &= \sum_{0 \le i < m_1+k} \sum_{0 \le j < m_2+l} \binom{m+i+j-1}{i} \binom{m+j-1}{j} z_1^i z_2^j \\
&= \sum_{0 \le j < m_2+l} \left[\sum_{0 \le i < m_1+k} \binom{m+i+j-1}{i} z_1^i \right] \cdot \binom{m+j-1}{j} z_2^j, \qquad (9)
\end{aligned}
$$

for $k, l = 0, 1, 2, \cdots$. Since

$$\sum_{i \geq 0} \binom{m+i+j-1}{i} z_1^i = \frac{1}{(1-z_1)^{m+j}}, \tag{10}$$

it follows

$$\begin{aligned} b_{kl} &\rightarrow \sum_{j \geq 0} \frac{1}{(1-z_1)^{m+j}} \cdot \binom{m+j-1}{j} z_2^j \\ &= \left(\frac{1}{1-z_1-z_2}\right)^m, \quad k, l \rightarrow \infty. \end{aligned} \tag{11}$$

Further, the bivariative Poisson distribution is convergent, and we rewrite the joint power of non-central F's ratio as

$$Pr[(H_1^* \cap H_2^*)|H_1^*, H_2^*] = C \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} A_{kl} \times b_{kl}, \tag{12}$$

where $C = (1 - z_1 - z_2)^m = (1 + \theta_1 + \theta_2)^{-m}$.

For example, consider two way experimental layout with five levels of row factor and seven levels of column fator. Suppose that the F tests yield $F_{4,24} = 3.5$ and $F_{6,24} = 3.1$, which are significant at $0.01 < \alpha < 0.05$. For the row factor, using (4) with $\nu_1 = 4$, $\nu = 24$, $m_1 = 2$, $m = 12$, $a_1 = 3.5$, $\theta_1 = 0.583$, the p-value of the test is $Pr[(H_1^*)|H_1] = 0.02183813$. For the column factor, using (4) with $\nu_2 = 6$, $\nu = 24$, $m_2 = 3$, $m = 12$, $a_2 = 3.1$, $\theta_2 = 0.775$, the p-value of the test is $Pr[(H_2^*)|H_2] = 0.02158189$. The joint p-value of H_1^* and H_2^* is

$$\begin{aligned} Pr[(H_1^* \cap H_2^*)|H_1, H_2] &= \{2.358^{12} \cdot \Gamma(12)\}^{-1} \sum_{i=0}^{1} \sum_{j=0}^{2} (0.247)^i (0.329)^j \, \Gamma(12+i+j)/i\,!j\,! \\ &= 0.001965837. \end{aligned} \tag{13}$$

From these the conditional p-value of event H_1^* given event H_2^* under the null hypotheses H_1 and H_2 is $Pr[(H_1^*|H_2^*)|H_1, H_2] = 0.09108735$. We calculate the joint and conditional significance level, namely $Pr[(H_1^* \cap H_2^*)|H_1, H_2]$ and $Pr[(H_1^*|H_2^*)|H_1, H_2]$, in table [1]. The nominal significance levels of the tests are 0.05 for both H_1 and H_2 in the left column of table [1], and 0.05 for H_1 and 0.01 for H_2 in the right column. The conditional probabilities in the left column are always larger than those in the right column.

From (5) and (6), we obtain the joint and conditional p-values of noncentral

F ratios. In the case $a_1 = 3.5, a_2 = 3$, $\nu = 12$, $\nu_1 = 6, \nu_2 = 10$, and $\lambda_1 = \lambda_2 = 6$, $Pr[(H_1^* \cap H_2^*)|H_1^*, H_2^*] = 0.06976792$, and $Pr[(H_1^*|H_2^*)|H_1^*, H_2^*] = 0.4766672$. We find that the probability of the occurrence of not making a second Type II error, given that a first Type II error was not made, is larger than the unconditional probability. We calculate the joint and conditional power of the test when the nominal significance level is $\alpha = 0.05$, table [2].
From table [1], figures [1] and [2], we find that the joint probability $Pr[(H_1^* \cap H_2^*)|H_1, H_2]$ of rejections of dependent F's is decreasing in the denominator degree of freedom ν, and increasing in the numerator degrees of freedom ν_1 and ν_2 . When the noncentrality parameter λ_i increases , the joint and conditional power increase in table [2] and figure [3]. It seems that two F ratios are more dependent, if the noncentrality parameter λ_i has a larger value. Formulas (3) and (5) suggest that the analysis of two way experimental designs can be extended to the problem of dependent of tests having the same error term in the analysis of variance of multiway experimental designs.

Let the events $H_i^* : F_i > a_i$ $(i = 1, 2)$ be represented in a 2×2 table. The complement events are H_1 and H_2. Let the joint probability be denoted by $\pi = Pr[(H_1^* \cap H_2^*)|H_1^*, H_2^*]$, and let the marginal probabilities be denoted by $p_1 = Pr[(H_1^*)|H_1^*]$ and $p_2 = Pr[(H_2^*)|H_2^*]$.

	H_2^*	H_2	Total
H_1^*	π	$(p_1 - \pi)$	p_1
H_1	$(p_2 - \pi)$	$(1 - p_1 - p_2 + \pi)$	$1 - p_1$
Total	p_2	$1 - p_2$	1

(14)

The dependence between two events is best measured by the odds-ratio

$$\gamma = \frac{\pi(1 - p_1 - p_2 + \pi)}{(p_1 - \pi)(p_2 - \pi)} . \tag{15}$$

In table [2], figures [4], [5] and [6], we compute and illustrate the odds-ratios. Figures [4] and [6] suggest that the odds-ratio is an increasing function of ν_1 and ν_2, and a decreasing function of ν. Figure [5] suggests that the odds-ratio as a function of the noncentrality parameters λ_1 and λ_2 is not monotone. All the values are for $\alpha = 0.05$. The results in the cases $\alpha = 0.1$ and $\alpha = 0.2$, not published in this paper, show similar patterns.

3. THE CORRELATION COEFFICIENT

Feingold and Korsog (1986) showed that the dependence between two non-central F statistics in the ANOVA table is well represented by the correlation between the statistics. The correlation between non-central F_1 and F_2 is

$$\rho = \prod_{i=1}^{2} \left\{ 1 + \frac{(\nu-2)(\nu_i + 2\lambda_i)}{(\nu_i+\lambda_i)^2} \right\}^{-\frac{1}{2}}, \quad for\ \nu > 4. \tag{16}$$

They gave approximations of $Pr[H_1^*|H_2^*]$, $Pr[H_1^* \cup H_2^*]$ and $Pr[H_1^*|H_2]$ by linear or quadratic functions of ρ.

Lemma 1. The correlation between non-central F_1 and F_2 is increasing in ν_i and λ_i , and decreasing in ν .

Proof. ρ is clearly decreasing in ν. Set

$$f(\nu_i, \lambda_i) = (\nu_i + 2\lambda_i)/(\nu_i+\lambda_i)^2 \ . \tag{17}$$

Partial differentiation of $\log f(\nu_i, \lambda_i)$ with respect to ν_i and λ_i are

$$\frac{\partial \log f(\nu_i,\lambda_i)}{\partial \nu_i} = \frac{-(\nu_i + 3\lambda_i)}{(\nu_i+2\lambda_i)(\nu_i+\lambda_i)} < 0 \ , \tag{18}$$

$$\frac{\partial \log f(\nu_i,\lambda_i)}{\partial \lambda_i} = \frac{-2\lambda_i}{(\nu_i+2\lambda_i)(\nu_i+\lambda_i)} < 0 \ . \tag{19}$$

Hence $f(\nu_i, \lambda_i)$ is a decreasing function of ν_i and λ_i. Therefore the correlation between F_1 and F_2 is an increasing function of ν_i and λ_i. Q.E.D.

To visualize the function, set $s_i = \nu_i/(\nu-2)$, and $t_i = \lambda_i/(\nu-2)$, $(i = 1,2)$. Then the correlation between non-central F_1, F_2 is

$$\rho = \prod_{i=1}^{2} \left\{ 1 + \frac{s_i + 2t_i}{(s_i+t_i)^2} \right\}^{-\frac{1}{2}} = \rho_1 \cdot \rho_2 \ , \tag{20}$$

where

$$\rho_i = \left\{ 1 + \frac{s_i + 2t_i}{(s_i+t_i)^2} \right\}^{-\frac{1}{2}} . \tag{21}$$

Figure [7] shows the contour of ρ_i, in the correlation coefficient between two non-central F ratios, as an increasing function of s_i and t_i .
Some numerical values are shown in table [3]. For example, the correlation between F_1^* and F_2^* statistics is $\rho = 0.3820$ by $\rho_1 \cdot \rho_2$, if F_1^*, F_2^* have a singly non-central F distribution with $\nu_1 = 10,\ \nu_2 = 2,\ \nu = 10$ and noncentrality parameter $\lambda_1 = 5,\ \lambda_2 = 2$, respectively.

4. REMARKS

This article has established a direct method for calculating the exact value of joint and conditional probability of F ratios with the noncentrality parameters, the same denominator degree of freedom and even value of the numerators. The results can be more exactly computed joint and conditional power of noncentral F ratios anywhere critical region of dependent F test and are simpler mathematically than the results from Hurlburt and Spiegel (1976), Johnson and Herr (1984).

The influence of the parameters ν, ν_1, ν_2, λ_1, and λ_2 is best shown by the correlation coefficient ρ in Section 3. However ρ is just a weak measure of dependence. In fact Figure 5 shows that the influence of λ_1 and λ_2 is not simple.

It is conjectured that the distributions of (F_1, F_2) in our family is more dependent if ν_1 and ν_2 increase and ν decreases. This result is consistent with Kimball's (1951) result. A nice measure of dependence is the odds-ratio.

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Table 1. $Pr[(H_1^* \cap H_2^*)|H_1, H_2]$ and $Pr[(H_1^*|H_2^*)|H_1, H_2]$,null case.

ν_1	ν_2	ν	$\alpha = 0.05$ for H_1 and H_2		$\alpha = 0.05$ for H_1 and $\alpha = 0.01$ for H_2	
			$Pr(H_1^* \cap H_2^*)$	$Pr(H_1^*\|H_2^*)$	$Pr(H_1^* \cap H_2^*)$	$Pr(H_1^*\|H_2^*)$
2	2	2	.026	.513	.008	.840
		4	.016	.317	.006	.551
		10	.008	.156	.002	.243
		30	.004	.082	.001	.104
		120	.003	.058	.001	.062
	10	10	.011	.225	.003	.345
		30	.005	.110	.001	.144
		120	.003	.064	.001	.071
	30	30	.007	.133	.002	.179
		120	.004	.072	.001	.082
	120	120	.004	.085	.001	.099
4	4	4	.022	.437	.007	.739
		5	.019	.376	.006	.644
		10	.011	.226	.004	.370
		16	.008	.159	.003	.243
		30	.005	.106	.001	.144
		120	.003	.063	.001	.070
	10	10	.014	.281	.005	.452
		15	.011	.210	.003	.324
		30	.007	.131	.002	.181
		40	.005	.110	.001	.145
		120	.003	.069	.001	.078
	30	30	.008	.164	.002	.232
		120	.004	.080	.001	.094
	120	120	.005	.098	.001	.119
10	10	10	.018	.362	.006	.606
		15	.014	.275	.004	.449
		30	.008	.168	.002	.250
		100	.004	.084	.001	.102
		120	.004	.078	.001	.093
	30	30	.011	.222	.003	.336
		120	.005	.095	.001	.117
	120	120	.006	.123	.002	.160
30	30	30	.015	.309	.005	.500
		90	.007	.148	.002	.207
		120	.006	.123	.002	.165
	120	120	.009	.175	.003	.247
120	120	120	.014	.275	.004	.427

Table 2. Power $Pr[(H_2^*)|H_2^*]$, simultaneous power $Pr[(H_1^* \cap H_2^*)|H_1^*, H_2^*]$, conditional power $Pr[(H_1^*|H_2^*)|H_1^*, H_2^*]$, and odds-ratio of H_1^*, H_2^* (O.R.) for $\alpha = 0.05$, nonnull case.

ν_1	ν_2	ν	$\lambda_i = 0.5\nu_i$				$\lambda_i = \nu_i$			
			$Pr(H_2^*)$	$Pr(H_1^* \cap H_2^*)$	$Pr(H_1^*\|H_2^*)$	O.R.	$Pr(H_2^*)$	$Pr(H_1^* \cap H_2^*)$	$Pr(H_1^*\|H_2^*)$	O.R.
2	2	2	.073	.039	.536	30.161	.096	.055	.568	27.267
		4	.090	.033	.364	8.486	.133	.055	.416	7.288
		5	.096	.031	.321	6.105	.145	.055	.380	5.237
		10	.111	.025	.229	2.809	.178	.054	.304	2.476
		30	.124	.021	.165	1.471	.208	.052	.252	1.384
		120	.131	.018	.141	1.107	.221	.051	.232	1.087
	10	10	.145	.040	.279	4.335	.268	.090	.335	3.699
		30	.205	.038	.183	1.835	.413	.107	.259	1.684
		120	.249	.036	.145	1.185	.505	.118	.233	1.154
	30	30	.287	.055	.191	2.176	.602	.152	.252	2.059
		120	.430	.063	.146	1.274	.818	.187	.228	1.275
	120	120	.722	.103	.142	1.486	.989	.219	.222	1.797
4	4	4	.094	.045	.484	16.649	.142	.076	.537	14.012
		5	.101	.044	.436	11.340	.159	.079	.498	9.449
		10	.125	.040	.319	4.338	.215	.088	.412	3.660
		16	.139	.037	.268	2.733	.246	.093	.377	2.377
		30	.152	.034	.224	1.793	.277	.097	.350	1.631
		120	.166	.031	.185	1.173	.308	.101	.328	1.138
	10	10	.145	.052	.357	5.897	.268	.116	.432	4.884
		15	.169	.051	.301	3.724	.326	.128	.392	3.165
		30	.205	.049	.239	2.115	.413	.146	.353	1.898
		40	.218	.049	.223	1.793	.440	.151	.344	1.642
		120	.249	.047	.189	1.237	.505	.165	.327	1.194
	30	30	.287	.072	.250	2.627	.602	.207	.343	2.438
		120	.430	.082	.190	1.355	.818	.261	.320	1.352
	120	120	.722	.133	.184	1.648	.989	.306	.310	2.071
10	10	10	.145	.068	.469	8.938	.268	.153	.572	7.190
		15	.169	.070	.414	5.227	.326	.179	.548	4.315
		30	.205	.072	.348	2.633	.413	.219	.531	2.314
		100	.245	.072	.294	1.399	.498	.266	.534	1.334
		120	.249	.072	.290	1.327	.505	.270	.535	1.275
	30	30	.287	.105	.367	3.536	.602	.312	.519	3.201
		120	.430	.126	.292	1.500	.818	.428	.523	1.504
	120	120	.722	.203	.280	1.957	.989	.502	.507	2.637
30	30	30	.287	.157	.546	5.375	.602	.452	.750	4.914
		90	.407	.209	.512	2.091	.791	.651	.822	2.229
		120	.430	.219	.511	1.787	.818	.685	.838	1.929
	120	120	.722	.354	.491	2.598	.989	.812	.821	4.295
120	120	120	.722	.586	.812	4.491	.989	.980	.991	16.254

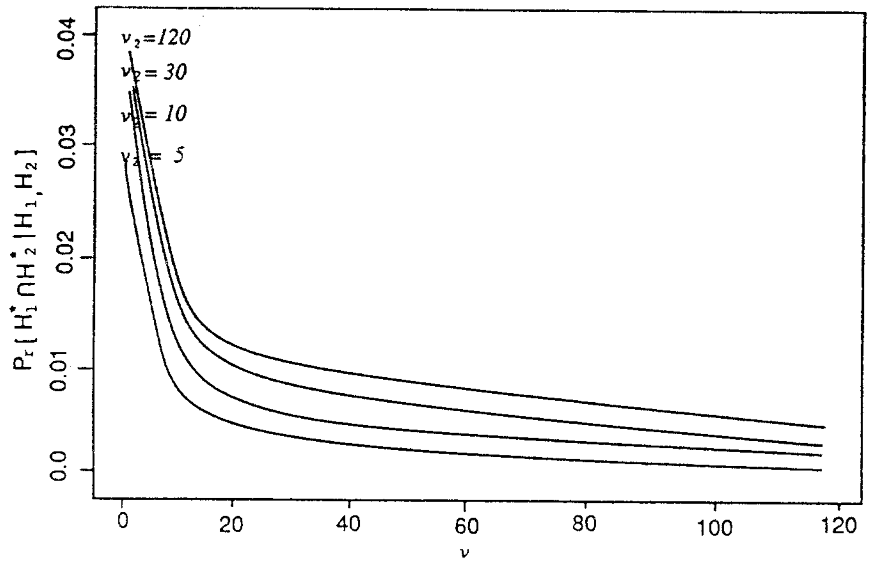

Figure 1. Probability of simultaneous rejection of F's as a function of ν, null case.

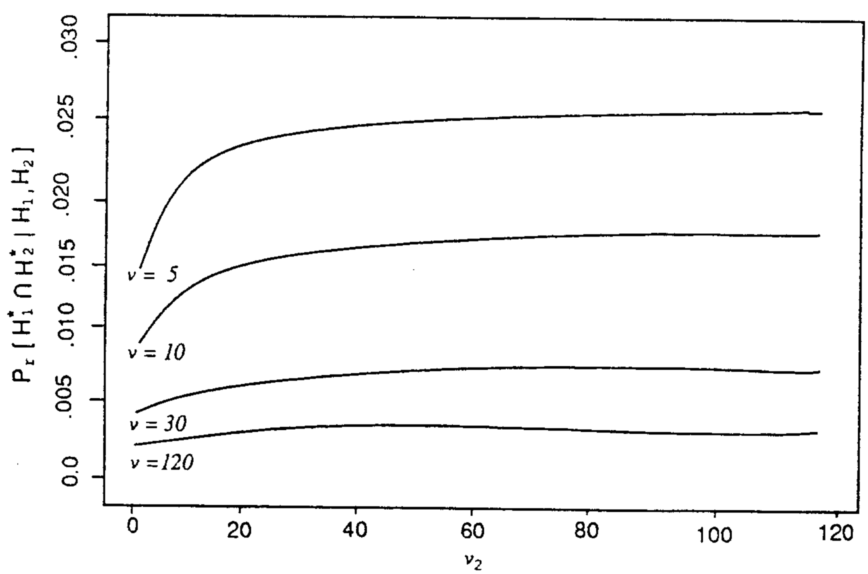

Figure 2. Probability of simultaneous rejection of F's as a function of ν_2 $(\nu_1 = 4)$.

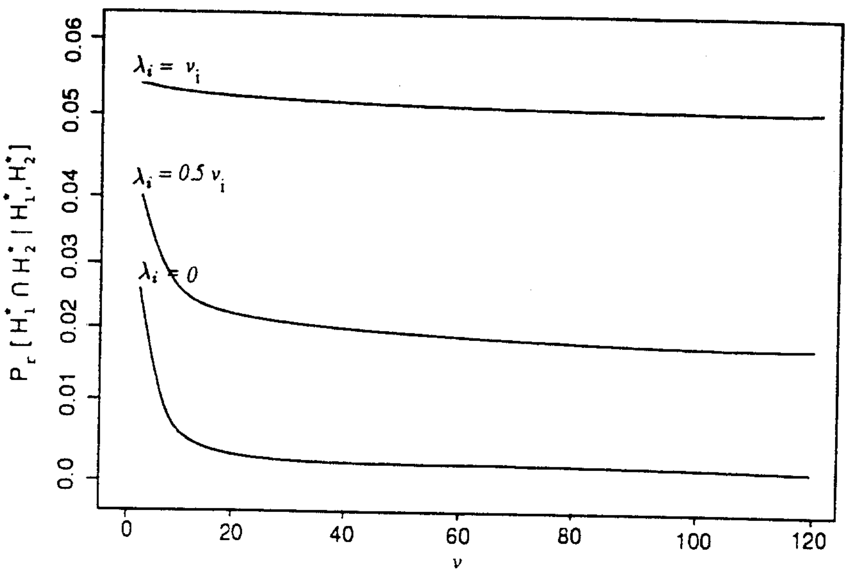

Figure 3. Probability of simultaneous rejection of F's as a function of ν, nonnull case ($\nu_1 = \nu_2 = 4$).

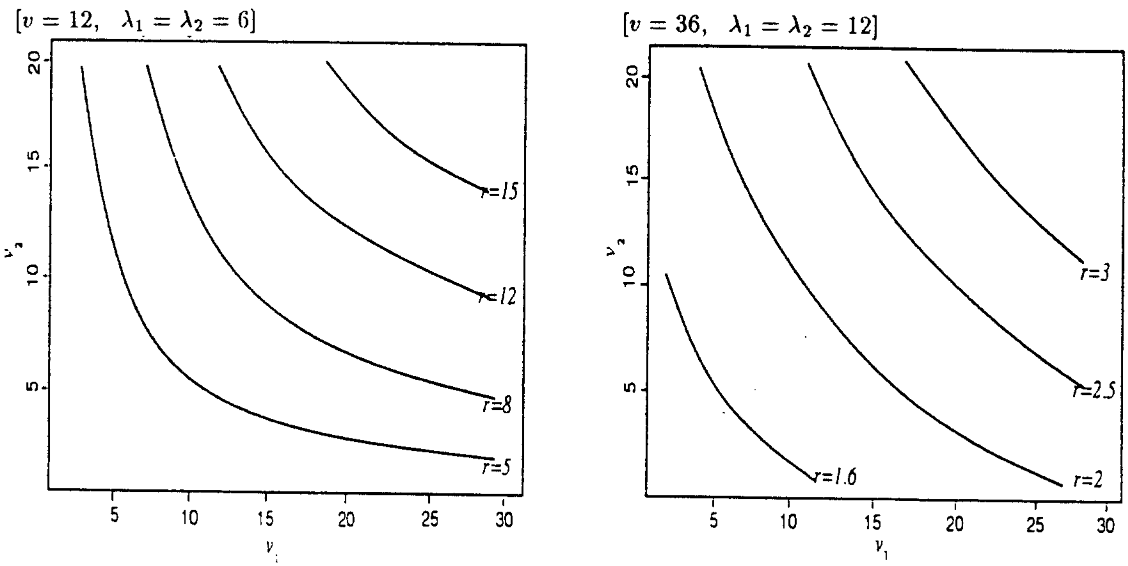

Figure 4. Contour of odds-ratio γ of powers as a function of ν_1 and ν_2.

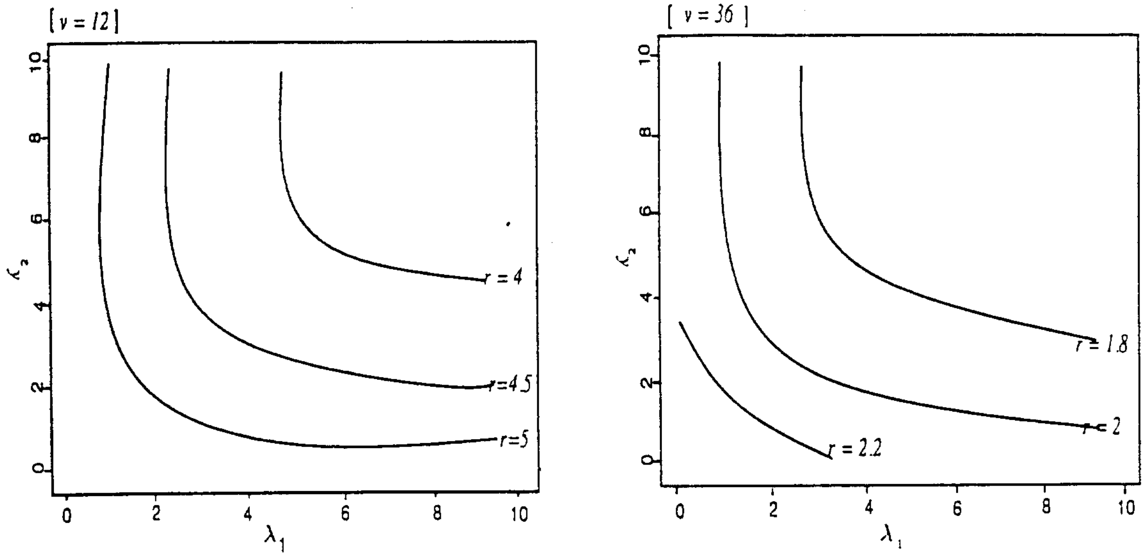

Figure 5. Contour of odds-ratio γ of powers as a function of λ_1 and λ_2.

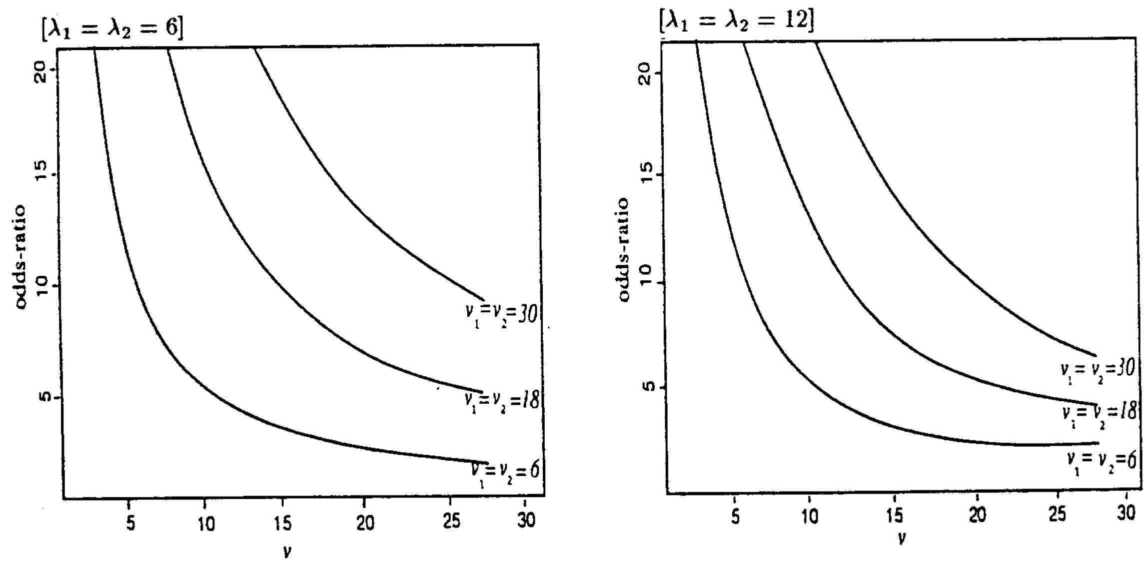

Figure 6. Odds-ratio powers as a function of ν with a parameter $\nu_1 = \nu_2$.

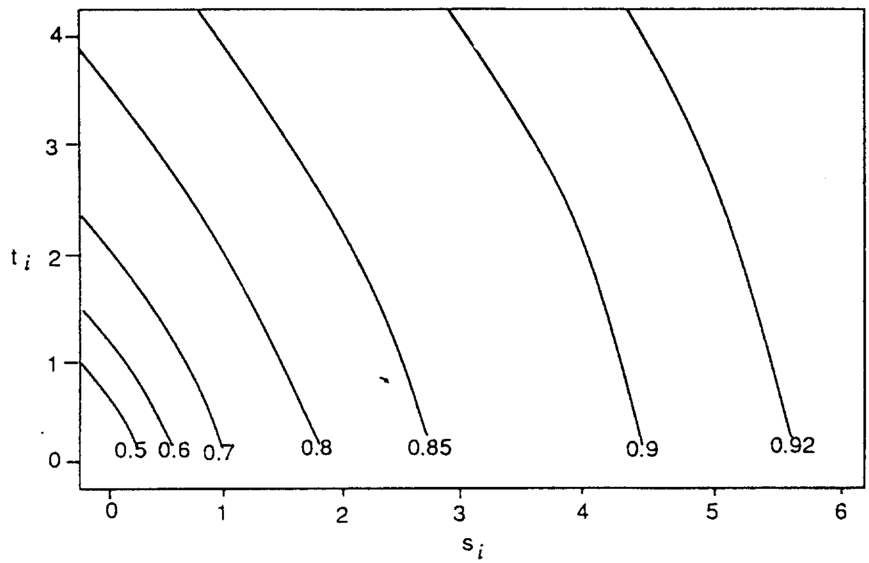

Figure 7. ρ_i in the correlation between two non-central F ratios with $s_i = \nu_i/(\nu - 2)$, $t_i = \lambda_i/(\nu - 2)$.

Table 3. ρ_i for known ν_i, ν the degrees of freedom of numerator and denominator and noncentrality parameter λ_i.

$\nu_i \backslash \nu$	5			10			30			120		
	$\lambda_i = 0$	$\lambda_i = .5\nu_i$	$\lambda_i = \nu_i$	$\lambda_i = 0$	$\lambda_i = .5\nu_i$	$\lambda_i = \nu_i$	$\lambda_i = 0$	$\lambda_i = .5\nu_i$	$\lambda_i = \nu_i$	$\lambda_i = 0$	$\lambda_i = .5\nu_i$	$\lambda_i = \nu_i$
1	.500	.522	.555	.333	.351	.378	.186	.197	.213	.092	.097	.106
2	.632	.655	.686	.447	.469	.500	.258	.273	.295	.129	.137	.149
5	.791	.808	.830	.620	.643	.674	.389	.409	.439	.202	.213	.231
10	.877	.889	.904	.745	.764	.791	.513	.535	.568	.280	.295	.319
30	.953	.958	.964	.889	.899	.913	.719	.739	.767	.450	.472	.503
120	.988	.989	.991	.968	.972	.976	.900	.910	.923	.710	.730	.759