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1995년 12월 pp. 358-365

# Operating Characteristic Properties of Two Types of Multi-Level Skip-Lot Sampling Plans[1)]

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## Abstract

This paper presents another system of multi-level skip-lot sampling plan, which can directly return to normal inspection from any skipping inspection level when a sudden shift for the worse in the process mean occurs. All the other properties of the proposed sampling plan are similar to those of the Choi's(1993) plan. The formula of the operating characteristic function for the proposed $n$-level skip-lot sampling plan is derived. Some operating characteristic properties for the proposed plan are graphically compared with those of the Choi's plan.

## 1. Introduction

Choi(1993) proposed a multi-level skip-lot sampling plan(MLSkSP1), which has merits that we can freely choose not only the number, $i$, of consecutive lots to be inspected and accepted but also the fraction, $f$, of lots to be inspected. It has seen that the plan MLSkSP1 are desirable in the aspect that it can reduce the cost of inspection when the level of submitted quality is high. The plan MLSkSP1, however, has a shortcoming that it may take long time for it to return to normal inspection from higher-level skipping inspections when the quality of submitted lots suddenly grows worse.

In this paper, to overcome that demerit, another system of multi-level skip-lot sampling plan(MLSkSP2) is developed. The plan MLSkSP2 immediately swich to the normal inspection when a lot is rejected on any skipping inspection level. That is the only difference between the plan MLSkSP1 and the plan MLSkSP2. All the notations and symbols defined in Choi, therefore, are used if there is no difference.

The procedure of the plan MLSkSP2 is the following.

(1) Start with normal inspection which inspects every lot, using the *reference sampling plan* that is a given lot-inspection plan by the method of attributes (single sampling, double sampling, etc.).

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1) This research was supported by NON-DIRECTED RESEARCH FUND, Korea Research Foundation, 1994.

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(2) When $i_1$ consecutive lots are accepted on normal inspection, swich to the first skipping inspection at rate $f_1$.

(3) During the $k^{th}$, $k$ = 1, 2, ..., $n-1$, skipping inspection:

When $i_{k+1}$ consecutively inspected lots are accepted, swich to the $(k+1)^{th}$ skipping inspection at rate $f_{k+1}$.

(4) During the $n^{th}$ skipping inspection at rate $f_n$:

Unless a lot is rejected, continue that skipping inspection.

(5) When a lot is rejected on any skipping inspection, swich to the normal inspection.

## 2. Formula of Operating Characteristic Function

The Markov chain approach taken in Choi is applied to obtain the probability of acceptance for the plan MLSkSP2. The state space of the Markov chain for the plan MLSkSP2 is exactly the same as that of the plan MLSkSP1, that is,

$$\{ N_R, N_1, N_2, \cdots, N_{i_1}, S_{1A1}, \cdots, S_{1Ai_2}, S_{1R}, S_{1N0}, S_{1N_1}, \cdots, S_{1Ni_{2-1}}, \cdots, \cdots, S_{(n-1)A1},$$
$$\cdots, S_{(n-1)Ai_n}, S_{(n-1)R}, S_{(n-1)N0}, S_{(n-1)N_1}, \cdots, S_{(n-1)N(i_{n-1})}, S_{nA}, S_{nR}, S_{nN}\}.$$

All the elements of the state space are defined in terms of Choi as follows:

$N_R$ = lot rejected on normal inspection.

$N_j$ = number of consecutively accepted lots during normal inspection is $j$, $j$ = 1, 2, $\cdots$, $i_1$.

$S_{kAj}$ = number of consecutively inspected and accepted lots during the $k^{th}$, $k$ = 1, 2, $\cdots$, $n-1$, skipping inspection at rate $f_k$ is $j$, $j$ = 1, 2, $\cdots$, $i_{k+1}$.

$S_{kNj}$ = lot skipped during the $k^{th}$, $k$ = 1, 2, $\cdots$, $n-1$, skipping inspection at rate $f_k$, and previous number of inspected and accepted lots on the $k^{th}$ skipping inspection at rate $f_k$ is $j$, $j$ = 1, 2, $\cdots$, $i_{k+1}-1$.

$S_{kR}$= lot rejected during the $k^{th}$, $k$ = 1, 2, $\cdots$, $n$, skipping inspection at rate $f_k$.

$S_{nA}$ = lot inspected and accepted during $n^{th}$ skipping inspection at rate $f_n$.

$S_{nN}$ = lot skipped during $n^{th}$ skipping inspection at rate $f_n$.

The one-step transition probability matrix $M$ for the plan MLSkSP2, which is different from that of the plan MLSkSP1, is given by

$$M=\begin{pmatrix} P_{00} & & & & \\ & P_{11} & & & 0 \\ & & P_{22} & & \\ P_1 & & & \ddots & \\ & & 0 & & P_{(n-1)(n-1)} \\ & & & & P_{nn} \end{pmatrix},$$

where the diagonal submatrices of $M$ are the same as those of the plan MLSkSP1, which have been defined by

$$P_{00} = \begin{array}{c} \\ N_R \\ N_1 \\ \vdots \\ N_{i_1-1} \end{array} \begin{array}{c} N_R \; N_1 \; N_2 \cdots \; N_{i_1} \\ \begin{pmatrix} Q & P & & & \\ Q & & P & & \\ & & & \ddots & \\ Q & & & & P \end{pmatrix} \end{array}$$

and for $k = 1, 2, \cdots, n-1$ ,

$$P_{kk} = \begin{array}{c} \\ S_{(k-1)Ai_k} \\ S_{kA1} \\ \vdots \\ S_{kA(i_{k+1}-1)} \\ S_{kR} \\ S_{kN0} \\ S_{kN1} \\ \vdots \\ S_{kN(i_{k+1}-1)} \end{array} \begin{array}{c} S_{kA1} \; S_{kA2} \cdots S_{kAi_{k+1}} \; S_{kR} \; S_{kN0} \; \cdots \; S_{kN(i_{k+1}-1)} \\ \begin{pmatrix} f_kP & & & & f_kQ & 1-f_k & & \\ & f_kP & & & f_kQ & & & \\ & & \ddots & & \vdots & & \ddots & \\ & & & f_kP & f_kQ & & & 1-f_k \\ & & & & & & & \\ f_kP & & & & f_kQ & 1-f_k & & \\ & f_kP & & & f_kQ & & & \\ & & \ddots & & \vdots & & \ddots & \\ & & & f_kP & f_kQ & & & 1-f_k \end{pmatrix} \end{array},$$

where $S_{(k-1)Ai_k} = N_{i_1}$ when $k = 1$.

$$P_{nn} = \begin{array}{c} \\ S_{(n-1)Ai_n} \\ S_{nA} \\ S_{nR} \\ S_{nN} \end{array} \begin{array}{c} S_{nA} \; S_{nR} \; S_{nN} \\ \begin{pmatrix} f_nP & 1-f_n & f_nQ \\ f_nP & 1-f_n & f_nQ \\ & & \\ f_nP & 1-f_n & f_nQ \end{pmatrix} \end{array}.$$

Note that all the values of elements in the submatrices $P_{00}$, $P_{kk}$'s and the following submatrix $P_1$ that have no entry equal to zeros. The submatrix $P_1$ is newly defined by

$$
P_1 = \begin{array}{c} \\ N_{i_1} \\ \vdots \\ S_{1R} \\ \vdots \\ S_{2R} \\ \vdots \\ S_{nR} \\ \vdots \\ S_{nN} \end{array}
\begin{array}{c} N_R \; N_1 \; N_2 \; \cdots \; N_{i_1} \\ \left( \begin{array}{ccccc} & & & & \\ & & & & \\ Q & P & & & \\ & & & & \\ Q & P & & & \\ & & & & \\ Q & P & & & \\ & & & & \\ & & & & \end{array} \right) \end{array} .
$$

Since the Markov chain of the plan MLSkSP2 has the same properties as those of the plan MLSkSP1, we can uniquely obtain the long-run or stationary probabilities, $\pi_i$'s, of all the given states by using the same mathod in Choi. The key probabilities of $\pi_{N_R}$, $\pi_{S_{1R}}$, $\pi_{S_{2R}}$, $\cdots$, $\pi_{S_{nR}}$ are derived from the system of equations after some tedious calculations, and the solutions are given by

$$\pi_{N_R} = \frac{Q}{B}\frac{1-P^{i_1}}{P^{i_1+i_2+\cdots+i_n}},$$

$$\pi_{S_{1R}} = \frac{Q}{B}\frac{1-P^{i_2}}{P^{i_2+i_3+\cdots+i_n}},$$

$$\cdots,$$

$$\pi_{S_{(n-1)R}} = \frac{Q}{B}\frac{1-P^{i_n}}{P^{i_n}},$$

$$\pi_{S_{nR}} = \frac{Q}{B},$$

where

$$B = \frac{1}{f_n} + \frac{1}{f_{n-1}}\frac{1-P^{i_n}}{P^{i_n}} + \cdots + \frac{1}{f_1}\frac{1-P^{i_2}}{P^{i_2+i_3+\cdots+i_n}} + \frac{1-P^{i_1}}{P^{i_1+i_2+\cdots+i_n}}.$$

Thus the general formula of the operating characteristic(OC) function for the plan MLSkSP2 is explicitly given by

$$P_a(f_1,\cdots,f_n;i_1,\cdots,i_n) = 1-(\pi_{N_R}+\pi_{S_{1R}}+\pi_{S_{2R}}+\cdots+\pi_{S_{nR}}),$$
$$= 1-\frac{Q}{B}A,$$

where

$$A = 1+\frac{1-P^{i_n}}{P^{i_n}}+\frac{1-P^{i_{n-1}}}{P^{i_{n-1}+i_n}}+\cdots+\frac{1-P^{i_1}}{P^{i_1+i_2+\cdots+i_n}}.$$

## 3. Comparisons and Conclusions

All the acceptance probabilities of the plan MLSkSP2 for $n = 1, 2, \cdots$ can be derived from the general formula of $P_a(f_1,\cdots,f_n;i_1,\cdots,i_n)$ by suitably adjusting $f_k$'s and $i_k$'s for $k=1, 2, \cdots, n$. In the same manner as the plan MLSkSP1, therefore, if we let $f_1 = f_2 = \cdots = f_{n-1} = 1$, $f_n = f$ and $i_1 = i_2 = \cdots = i_{n-1} = 0$, $i_n = i$, then $P_a(f_1,\cdots,f_n; i_1,\cdots,i_n)$ is reduced to

$$P_a(f,i)=\frac{fP+(1-f)P^i}{f+(1-f)P^i},$$

which is exactly Perry's(1973a) formula for the single-level skip-lot sampling plans SkSP-2. Also by letting $f_1 = f_2 = \cdots = f_{n-2}=1$, $f_{n-1} = f_1$, $f_n = f_2$ and $i_1 = i_2 = \cdots = i_{n-2} = 0$, $i_{n-1} = i_1$, $i_n = i_2$, we can obtain the probabilites of acceptance of the two-level skip-lot sampling plan as follows:

$$P_a(f_1,f_2;i_1,i_2) = \frac{f_2[P^{i_1}+f_1(P-P^{i_2})]+(f_1-f_2)P^{i_1+i_2}}{f_2[P^{i_1}+f_1(1-P^{i_1})]+(f_1-f_2)P^{i_1+i_2}},$$

which goes to Perry's(1973b) formula $P_a^{2L.2}(f_1,f_2\ ;\ i)$ for the plan *Plan 2L.2* when $i_1 = i_2$ .

In order to obtain the OC functions for more higher-level MLSkSP2 plans than the two-level, it is sufficient to similarly adjust the inspection parameters $f_k$'s and $i_k$'s to the cases of the single- and two-level plans.

Figure 1 and Figure 2 show the OC curves of 3-level MLSkSP2 plans of the reference plan for $n = 20$, and $c = 1$. Those of Figure 1 are $P_{a2}(1/2,1/5,1/10; 4,4,4)$, $P_{a2}(1/2,1/5,1/10; 4, 8, 12)$, $P_{a2}(1/2,1/5,1/10; 8, 8, 8)$ and $P_{a2}(1/2,1/5,1/10; 12, 12, 12)$ in turn from above, and those of Figure 2 are $P_{a2}(1/2,1/5,1/10; 4, 8, 12)$, $P_{a2}(1/2,1/5,1/10; 8, 8, 8)$ and

$P_{a2}(1/2, 1/5, 1/10;\ 12,\ 8,\ 4)$ in turn from above. Note that $P_{a2}(\cdot)$ represents the OC curve of the plan MLSkSP2.

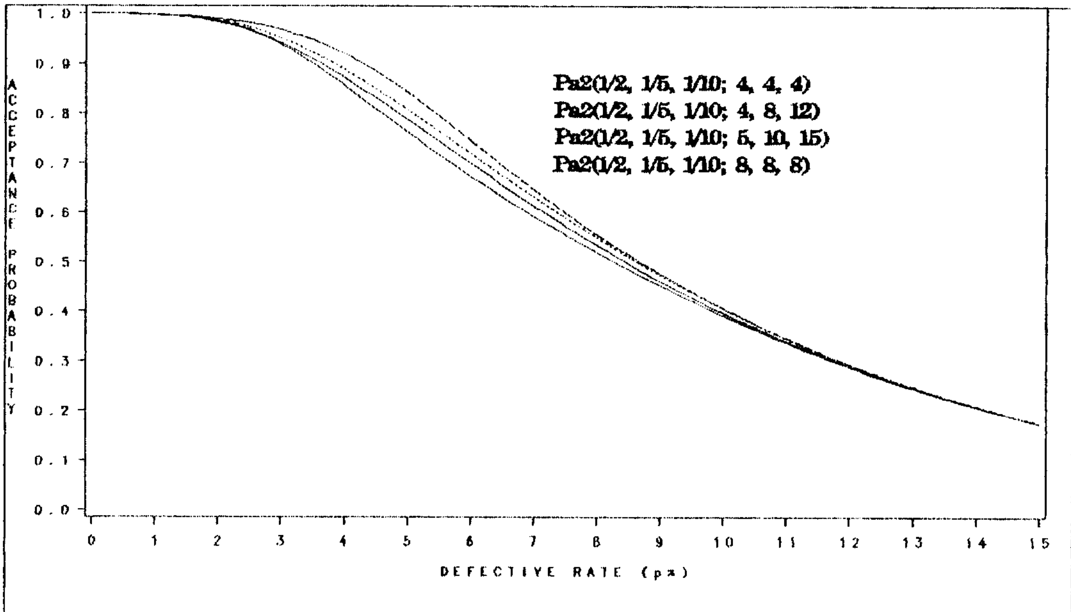


FIGURE 1. OC Curves for 3-Level MLSKSP2 Plans

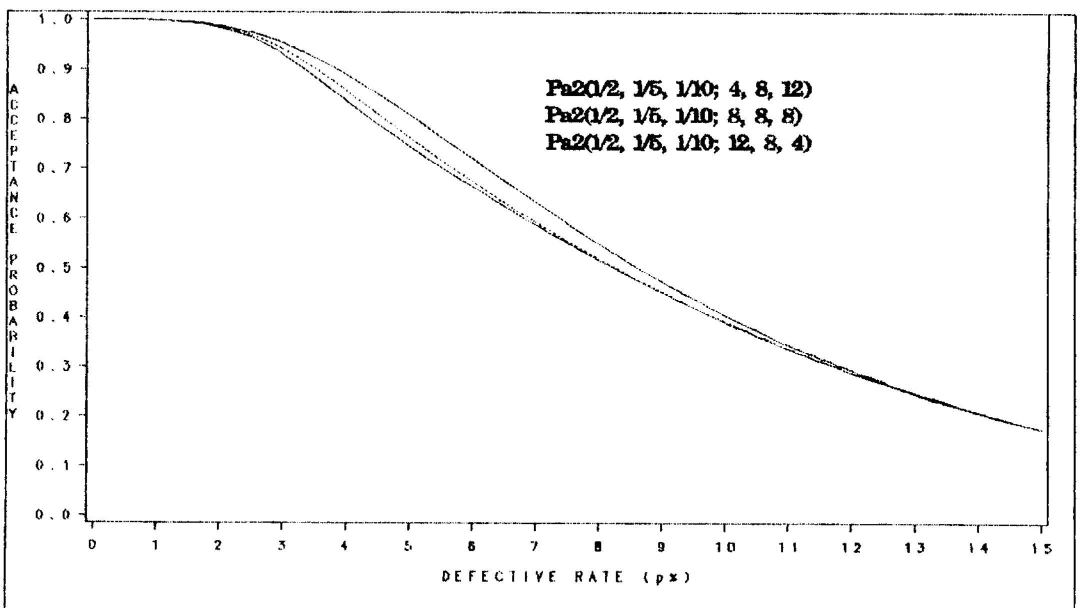


FIGURE 2. OC Curves fof 3-Level MLSKSP2 Plans

From Figure 1, we can see that the acceptance probabilities of the plan MLSKSP2 all but $P_{a2}(1/2,1/5,1/10;\ 8,\ 8,\ 8)$ decrease for the given defective rate $p$ as the sum of the parameters $i_k$'s increases. From Figure 2, we can see that the acceptance probability of the plan MLSKSP2 in the case $i_1 \geq i_2 \geq i_3$ is lowest among others considered when the sums of the parameters $i_k$'s are equal. From Figure 1 and Figure 2, therefore, the larger the parameter $i_1$ on the normal inspection is, the lower the acceptance probability of the plan MLSKSP2 is. That is, the parameter $i_1$ highly affects the acceptance probability of the plan MLSKSP2.

Figure 3 compares the OC curves between 3-level MLSkSP1 and MLSkSP2 plans of the reference plan for $n = 20$, and $c = 1$, they are $P_{a1}(1/2,1/5,1/10\ ;\ 4,4,4)$, $P_{a2}(1/2,1/5,1/10\ ;\ 4,4,4)$, $P_{a1}(1/2,1/5,1/10;\ 4,\ 8,\ 12)$ and $P_{a2}(1/2,1/5,1/10;\ 4,\ 8,\ 12)$ in turn from above. Also note that $P_{a1}(\cdot)$ represents the OC curve of the plan MLSkSP1.

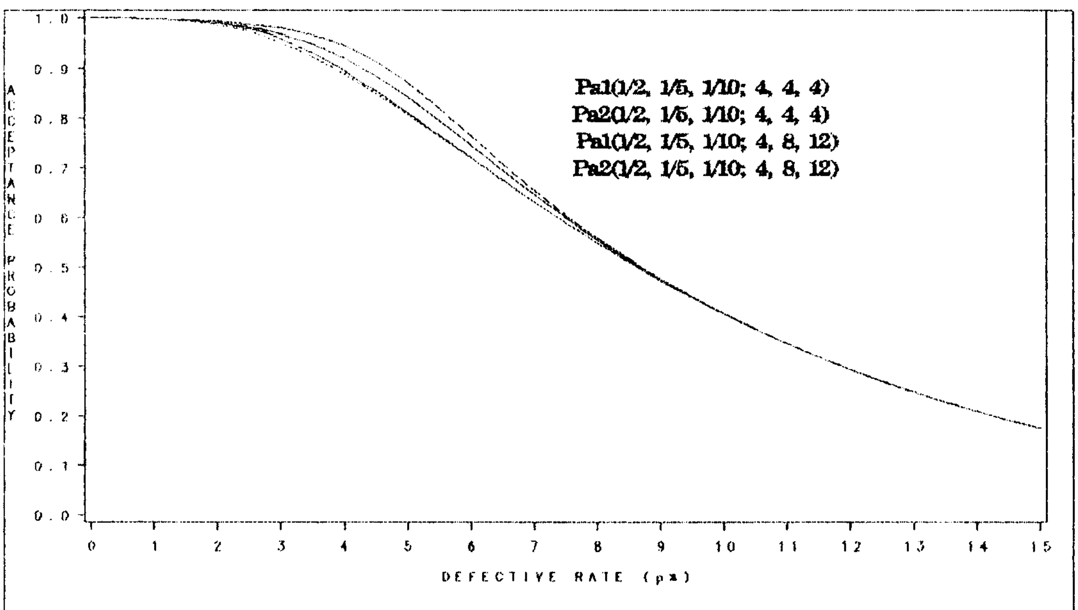


**FIGURE 3. Comparison of OC Curves for 3-Level MLSKSP1 and MLSKSP2 Plans**

From Figure 3, we can see that, in the case that the parameters $i_k$'s are equal, the acceptance probability of the plan MLSkSP1 is higher than that of the plan MLSkSP2 for all given $p$, which is true for the case that the parameters $i_k$'s are all different. In order to select higher quality lots as far as possible, therefore, the consummer might not like to choose the plan MLSkSP1, for the submitted lots are more easily accepted on the plan MLSkSP1 than the plan MLSkSP2. That comes from the fact that when a lot is rejected on any skipping inspection on the plan MLSkSP2, the inspection is swiched to the normal inspection, but on the plan MLSkSP1, the inspection is swiched to the only one level lower skipping inspection

Now we come to a conclusion that, in general, the acceptance probabilities of the proposed plans MLSkSP2 are lower than those of the plans MLSkSP1 under the given conditions as we expected. For the consummer, therefore, the plan MLSKSP2 is more desirable than the plan MLSKSP1 paticularly when the defective rate is not low and not high.

## References

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