철근콘크리트 휨부재의 신뢰성

Reliability of RC Beams Designed for Flexure

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요　약

ACI규준에 의하여 설계된 철근콘크리트 보의 휨보강의 신뢰성을 분석하였다. 분석된 결과를 기존의 연구와 비교하였다. 인장철근비가 변함에 따라 일관된 신뢰성이 보장되지 못하고 부적합하게 변하는 것으로 분석되었다. 규준의 최대철근비 조항에 적합하게 설계된 보라도 취성파괴가 매우 높은 것으로 분석되었다. 특히 국내의 철근콘크리트 구조물에서는 이러한 현상이 두드러지는 것으로 분석되었다. 이와 같은 현상의 원인을 규명하였고 그 대책을 제시하였다.

Abstract

Reliability of RC beams designed for flexure under the provisions of ACI Building Code is analyzed. The results are compared with those obtained previously. It is shown that in some cases the reliability is inadequate and changes substantially with reinforcement ratio. The probability of brittle failure appears to be rather high. The reasons for these phenomena are revealed and some measures to remedy the situation are recommended. Much attention is given to the conditions as they stand at present in Korea.

Keywords : ACI Building Code, RC beams, flexural strength, reliability, brittle failure, concrete strength control.

1. Introduction

Reliability of RC beams designed for flexure under the provisions of ACI Building Code [1] was investigated in several papers(e.g., [2-8]). For example, the implication of the analysis in Ref [8] is that ACI Code 318-89 gives a uniform reliability. All component reliability indices vary from 3.2 to 4.2.

The aim of this paper is to show that in some cases probability of brittle failure is rather high, reliability is inadequate and reliability indices go well below 3.2. The reasons for these phenomena are revealed and some measures to remedy the situation are recommended. Much attention is given to the conditions as they stand at present in Korea.

2. Initial data

In beams designed for flexure three modes of bending failure are possible depending on whether the beam is lightly, moderately or over-reinforced. In Ref [8] to distinguish be-

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• 본 논문에 대한 토의를 1995년 8월 31일까지 학회로 보내주시면 1995년 10월호에 토의회답을 게재하겠습니다.

tween the modes the following limit-state functions are used:

$$g_1 = A_s - \frac{200}{f_y} b_w d \tag{1}$$

$$g_2 = A_s - \frac{0.85\beta_1 f_c'}{f_y} \frac{87000}{87000+f_y} b_w d \tag{2}$$

Here

A_s =area of tension reinforcement (in^2);

b_w, d, h =width, effective depth and height of beam cross section (in);

f_c', f_y =compressive strength of concrete, yield stress of steel (ksi);

β_1 =the ratio of the depth of stressed block in the compression zone to the distance between the outside compression surface and the neutral axis (here, according to Ref [1], β_1 =0.85).

It is assumed in Ref [8] that conditions $g_1 < 0$ and $g_2 > 0$ hold for light- and over-reinforcement, respectively. Otherwise (i.e., if $g_1 \geq 0$ and $g_2 0 \leq$) the beam is moderately reinforced.

In Ref [8] the limit state functions for lightly, moderately and over-reinforced beams are, respectively, as follows:

$$g_3 = B_f(1.25 b_w h^2 \sqrt{f_c'}) - M \tag{3}$$

$$g_4 = B_f A_s f_y \left(d - \frac{A_s f_y}{1.7 f_c' b_w}\right) - M \tag{4}$$

$$g_5 = B_f\left(\frac{1}{3} b_w d^2 f_c'\right) - M \tag{5}$$

Here

B_f=factor characterizing flexural model uncertainty;

M=external bending moment.

For generality let us divide both sides of eqn (2) by $b_w d$ and both sides of eqns. (3), (4), (5) by $b_w d^2$. Then

$$g_2^r = \rho - \frac{0.85\beta_1 f_c'}{f_y} \frac{87000}{87000+f_y} \tag{6}$$

$$g_3^r = B_f(1.65\sqrt{f_c'}) - M^r \tag{7}$$

$$g_4^r = B_f \rho f_y \left(1 - \frac{\rho f_y}{1.7 f_c'}\right) - M^r \tag{8}$$

$$g_5^r = B_f\left(\frac{1}{3} f_c'\right) - M^r \tag{9}$$

Here $\rho = A_s/(b_w d)$ is reinforcement ratio and superscript r stands for relative values of limit-state functions and external moments. For the sake of definiteness it is assumed in eqns (3), (7) that h=1.15d.

As was mentioned above, in Ref [8] conditions $g_1 < 0$ and $g_2 > 0$ are used to distinguish between the cases of light- and over-reinforcement, respectively. Below we shall employ a similar condition $g_2^r > 0$ for over-reinforcement. To distinguish the light reinforcement another criterion instead of the condition $g_1 < 0$ will be used. The beam is lightly reinforced if its uncracked strength is greater than cracked strength. Therefore the condition $g_3^r > g_4^r$, or

$$1.65\sqrt{f_c'} > \rho f_y \left(1 - \frac{\rho f_y}{1.7 f_c'}\right) \tag{10}$$

holds for lightly reinforced beams.

Six random variables were considered in the course of reliability analysis: yield strength of steel f_y^*, compressive strength of concrete f_c^*, measure of concrete splitting strength $\sqrt{f_c^*}$, flexural model uncertainty B_f^*, external moments M_1^{r*} and M_2^{r*}, produced by dead and live loads, respectively(sing* stands for random values).

In all, 8 cases were considered. For these cases characteristics of random values f_y^* and f_c^* are presented in Table 1.

For all cases flexural model uncertainty B_f^* is assumed normally distributed with mean value

Table 1 Description of random variables f_y^* and f_c^*

Case No	Random values and their characteristics							
	f_y^*					f_c^*		
	Distribution	Mean value, ksi	c.o.v.	Lower bound, ksi	Upper bound, ksi	Distribution	Mean value, ksi	c.o.v.
1	beta	43.3	0.107	33	62	normal	3.125	0.18
2	beta	71.0	0.093	54	102	normal	3.125	0.18
3	normal	42.3	0.12	–	–	lognormal	1.95	0.19
4	normal	42.3	0.12	–	–	lognormal	2.19	0.19
5	normal	42.3	0.12	–	–	lognormal	2.40	0.19
6	normal	61.9	0.10	–	–	lognormal	1.95	0.19
7	normal	61.9	0.10	–	–	lognormal	2.19	0.19
8	normal	61.9	0.10	–	–	lognormal	2.40	0.19

1.1, c.o.v.=0.12 [8]. The description of random values $\sqrt{f_c^*}$, M_1^{r*} and M_2^{r*} is given below.

Cases 1,2 are typical for American and Canadian practice: the initial data are taken from Ref [8]. Cases 3 to 8 are typical for Korean practice: the initial data are taken from Ref [9]. Concrete in cases 3 and 6; 4 and 7; 5 and 8 is defined as poor, medium and good, respectively. Mean values of concrete strengths are 0.65 f_c', 0.73 f_c' and 0.8 f_c' for poor, medium and good concretes, respectively. Grade 40 and 60 reinforcing bars are nominally used in cases 1, 3, 4, 5 and 2, 6, 7, 8, respectively. Nominal strength of concrete is 3.0 ksi for all 8 cases.

3. Probability of brittle failure

In Ref [8] only moderately reinforced beams with limit-state functions (4) are considered: it is correctly reasoned that probabilities of light- and over-reinforcement are very small. However, the beam *initially* (deterministically) designed as moderately reinforced can be *actually* lightly or over-reinforced. Let us estimate the probabilities of these events for above 8 cases.

The beam is moderately reinforced if its reinforcement ratio satisfies the following conditions:

$$\rho_{min} \le \rho \le \rho_{max} \tag{11}$$

where $\rho_{min}=0.005$; $\rho_{max}=\frac{3}{4}\rho_b=0.0278$ for $f_y=40$ ksi; $f_c'=3$ ksi and $\rho_{min}=0.0033$; $\rho_{max}=\frac{3}{4}\rho_b=0.016$ for $f_y=60$ ksi; $f_c'=3$ ksi. The balanced reinforcement ratio ρ_b is determined by the following formula:

$$\rho_b=\frac{0.85\beta_1 f_c'}{f_y}\frac{87000}{87000+f_y} \tag{12}$$

If material strengths are random values then the balanced reinforcement ratio ρ_b is a random value too. Let us denote it by ρ_b^*. Using eqn (6) probabilities $P(\rho>\rho_b^*)=P(g_2^{r*}>0)$ and $P(\rho>\frac{3}{4}\rho_b^*=\rho_{max}^*)$ have been determined.

Probability $P(\rho>\rho_b^*)$ is the probability that the beam initially designed as moderately reinforced with reinforcement ratio satisfying conditions (11) is actually over-reinforced. In much the same way probability $P(\rho>\frac{3}{4}\rho_b^*)$ is the probability that the provisions of ACI Building Code for moderately reinforced beams are violated. Probabilities $P(\rho>\rho_b^*)$ and $P(\rho>\frac{3}{4}\rho_b^*)$ are determined for eleven ξ values: $\xi=0$, 1, 0.2, ⋯, 1. The ξ values are associated with the reinforcement ratio ρ in the following way:

$$\xi=\frac{\rho-\rho_{min}}{\rho_{max}-\rho_{min}} \tag{13}$$

Calculations were performed using three approaches:

1) Monte Carlo simulation with subsequent approximation of the results by Pearson's curves and numerical integration [10,11]; sample

size was 5,000 (the description of this approach is given in more detail in section 4).

2) Crude Monte Carlo simulation with sample size 5,000.

3) Crude Monte Carlo simulation with sample size 15,000 (for some ρ values).

All results were in close agreement. They are presented in Tables 2, 3.

As can be seen from Tables 2, 3, probabilities $P(\rho > \rho_b^*)$ and $P(\rho > \frac{3}{4}\rho_b^*)$ are very high for high ρ values and gradually go down as the ρ values decrease. To take an example, in case 3 for $\xi=1$ probabilities $P(\rho > \rho_b^*)$ and $P(\rho > \frac{3}{4}\rho_b^*)$ are 0.8191 and 0.9756, respectively. The probability of brittle failure is fairly high even for relatively low ρ values: for example, $P(\rho > \rho_b^*)$ ranges from 0.0131(case 8) to 0.1124(case 3) for $\xi=0.5$, $\rho=\rho_{min}+0.5(\rho_{max}-\rho_{min})$.

The reason for this phenomenon is quite apparent. Even for cases 1, 2 where the actual strength of concrete is the highest the mean value of concrete strength 3.125 ksi is rather low: it is only slightly above $f_c=3.0$ ksi; at the same time c.o.v.=0.180 is rather high. As a result the probability that concrete strength will fall below f_c is high (it equals 0.413). For cases 3 and 6 the situation is much worse: the mean value of concrete strength is the lowest and c.o.v.=0.19 is the highest. The probability that concrete strength will fall below f_c equals 0.99. From physical considerations as well as from eqn (6) one can see that the probability of over-reinforcement increases as concrete strength decreases. In view of high probability of low concrete strength values the probability of over-reinforcement is high. For low ξ values ($\xi \leq 0.5$) probabilities $P(\rho > \rho_b^*$ and $P(\rho > \frac{3}{4}\rho_b^*)$ are lower in cases 5 and 8 than in cases 1 and 2, respectively.

This is associated with the distinctions in distribution laws of concrete strength: due to positive skewness of lognormal distribution random realizations of concrete strength are generally higher in cases 5 and 8 than in cases 1 and 2, respectively, even with larger mean values of concrete strength in cases 1 and 2.

Comparing cases 1 and 2; 3 and 6; 4 and 7; 5 and 8 (different f_y^* but the same f_c^*) one can see that more often than not the probability of over-reinforcement for fixed ξ values decreases with steel strength. At first glance this result seems erroneous: the probability of over-reinforcement should increase with steel strength. However, the result becomes evident if reinforcement ratio ρ is taken into account. As can be seen from formula (13), $\rho=\rho_{min}+\xi(\rho_{max}-\rho_{min})$. For a fixed ξ value $\rho_{60}<\rho_{40}$, where ρ_{60} and ρ_{40} are ρ values for reinforcing bars of grades 60 and 40, respectively. It is clear that probability of over-reinforcement decreases as the reinforcement ratio decreases. From the two tendencies-increase in steel strength and decrease in reinforcement ratio-the latter exerts more influence. Therefore the probability of over-reinforcement decreases with steel strength.

Table 2 Probability $P(\rho > \rho_b^*)$

Case No	ξ 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	0.0000	0.0000	0.0002	0.0014	0.0064	0.0178	0.0523	0.1098	0.1966	0.3026	0.4178
2	0.0000	0.0000	0.0002	0.0016	0.0064	0.0186	0.0462	0.0946	0.1646	0.2568	0.3630
3	0.0000	0.0000	0.0008	0.0065	0.0316	0.1124	0.2446	0.4157	0.5785	0.7119	0.8191
4	0.0000	0.0000	0.0002	0.0016	0.0104	0.0464	0.1234	0.2432	0.3963	0.5506	0.6721
5	0.0000	0.0000	0.0000	0.0008	0.0035	0.0173	0.0618	0.1444	0.2571	0.3986	0.5410
6	0.0000	0.0000	0.0008	0.0050	0.0266	0.0969	0.2197	0.3798	0.5494	0.6862	0.7978
7	0.0000	0.0000	0.0002	0.0012	0.0078	0.0344	0.1011	0.2129	0.3530	0.4998	0.6356
8	0.0000	0.0000	0.0000	0.0006	0.0022	0.0131	0.0495	0.1172	0.2211	0.3506	0.4836

Table 3 Probability $P(\rho > \frac{3}{4}\rho_b^*)$

Case	ξ										
No	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	0.0000	0.0002	0.0003	0.0136	0.0562	0.1396	0.2685	0.4243	0.5664	0.6935	0.7922
2	0.0000	0.0002	0.0036	0.0152	0.0512	0.1302	0.2434	0.3859	0.5291	0.6635	0.7791
3	0.0000	0.0008	0.0116	0.0865	0.2541	0.4844	0.6811	0.8226	0.9078	0.9529	0.9756
4	0.0000	0.0020	0.0032	0.0296	0.1306	0.3002	0.5078	0.6793	0.8196	0.8902	0.9428
5	0.0000	0.0000	0.0001	0.0121	0.0663	0.1888	0.3614	0.5553	0.6948	0.8137	0.8858
6	0.0000	0.0008	0.0126	0.0855	0.2456	0.4696	0.6772	0.8156	0.9033	0.9511	0.9734
7	0.0000	0.0002	0.0035	0.0284	0.1203	0.2832	0.4800	0.6616	0.7958	0.8836	0.9386
8	0.0000	0.0000	0.0012	0.0118	0.0591	0.1677	0.3346	0.5137	0.6734	0.7942	0.8768

On occasion, the increase in steel strength exerts more influence than the decrease in reinforcement ratio. To take an example, for low ξ values ($\xi \leq 0.4$) probability $P(\rho > \frac{3}{4}\rho_b^*)$ increases with steel strength (see Table 3).

To estimate the probability of light reinforcement a similar methodology was used. In the course of Monte Carlo simulation condition (10) was checked for $\rho = \rho_{min}$ in all eight cases (see Table 1). Three random variables f_c^*, $\sqrt{f_c^*}$, f_y^* were taken into account. The distributions for f_c^* and f_y^* are given in Table 1. The random variable $\sqrt{f_c^*}$ was assumed to be fully correlated with f_c^*. It turned out that the probability of light reinforcement was very close to zero (less than 10^{-4}). Therefore in what follows the probability of failure due to light reinforcement will be neglected.

4. Reliability of beams

Since the probabilities of brittle failure due to over-reinforcement in some cases are rather high, two failure modes described by limit-state functions (8), (9) should be taken into account in the course of beam reliability analysis. So far, in most, for reliability analysis of RC beams FORM/SORM methods have been used (e.g., see Ref [8]). The methods can take into account only one limit-state function. Therefore it is difficult, if not impossible, to use them in the case of two limit-state functions. Below is given another approach. For all 8 cases live-to-dead load ratio is assumed to be 3.0.

The initial data for calculations were prepared in the following way.

1. Specify reinforcement ratio ρ satisfying conditions (11).

2. Determine the nominal relative moment capacity of the beam M_n^r. Assume that the factorized relative external moment M_f^r equals ϕM_n^r (ϕ is a strength reduction factor, $\phi = 0.9$ [1]).

3. Take $M_1^r = M_f^r / 6.5$; $M_2^r = 3M_1^r$, where M_1^r and M_2^r are unfactored moments produced by dead and live loads, respectively. The coefficient 6.5 is easily obtained from two following equations:

$$1.4M_1^r + 1.7M_2^r = M_f^r;\ M_2^r = 3M_1^r \qquad (14)$$

Here 1.4 and 1.7 are load factors for dead and live loads, respectively.

4. According to Ref [8,9], assume that M_1^r, M_2^r(for cases 1, 2) and 1.05 M_1^r, 1.038 M_2^r(for cases 3 to 8) are mean values of the random moments M_1^{r*} and M_2^{r*} produced by dead and live loads, respectively. Assume that M_1^{r*} is normally distributed with c.o.v.=0.10 and M_2^{r*} fits a type 1 extreme value distribution with c.o.v.=0.25 (cases 1,2) and with c.o.v.=0.24 (cases 3 to 8).

5. In eqns. (8), (9) assume that $M^r = M_1^{r*} + M_2^{r*}$, $f_y = f_y^*$, $f_c = f_c^*$, $B_f = B_f^*$, $\sqrt{f_c} = \sqrt{f_c^*}$, where f_y^*, f_c^*, B_f^*, $\sqrt{f_c^*}$ are random values.

All random variables exclusive of $\sqrt{f_c^*}$ are assumed to be mutually statistically independent. In Ref [8] $\sqrt{f_c^*}$ is introduced as an additional statistically independent normally distributed basic variable; its mean value equals the square root of the nominal compressive strength, and c.o.v.=0.18. This definition was used for cases 1, 2 (see Table 1). For cases 3 to 8 the definition seems to be inappropriate because of rather low mean values of concrete strength. To be on the safe side in cases 3 to 8 the random value $\sqrt{f_c^*}$ was assumed to be fully correlated with f_c^*.

Perform calculations in the following order:

1. Using Monte Carlo simulation obtain a set of realizations of random variables f_y^*, f_c^*, B_f^*, $\sqrt{f_c^*}$, M_1^*, M_2^*.
2. Check condition (6) to determine whether the beam is moderately or over-reinforced.
3. Choose the corresponding limit-state function among (8), (9) and calculate its value g^r.
4. Perform steps 1 to 3 m times. As a result obtain m values $g_1^r, \cdots, g_m^r$.
5. Fit an appropriate Pearson's curve y(z) to describe probability density functions of g^r values.
6. Calculate the reliability of the beam R by numerical integration:

$$R = \int_0^{+\infty} y(z)dz \tag{15}$$

All calculations were performed with sample size m=5,000 and for some cases were checked by crude Monte Carlo simulation with sample size 15,000. Reliabilities obtained by the two methods were in close agreement. In addition for each case calculations were also performed using one failure mode, corresponding to the moderately reinforced beam and described by failure function (8).

Calculation results are presented in Figs. 1 to 4. Here the reliability indices β are plotted vs. ξ.

Case 1 (the solid and dashed lines in Fig. 1) will be our initial concern. The obtained results are in complete agreement with the values of probabilities $P(\rho > \rho_{max}^*)$ discussed above. As can be seen from the Figure, if two failure modes are taken into account (solid line), the reliability index β=3.20 remains unchanged for $0 \leq \xi \leq 0.7$ because in this case for the most part limit-state function (8) is used and it gives the highest g^r values. As ξ increases from 0.7 to 1, the probability $P(\rho > \rho_{max}^*)$ increases as well (see Table 2). As a result in the course of Monte Carlo simulation in increasing number of cases limit-state function (9) is used and this function gives lower g^r values in comparison with limit-state function (8). For ξ=1 ($\rho = \rho_{max}$) reliability index β drops to 2.7.

Thus, reliability of beams is low if reinforcement ratio is high (ρ is close to ρ_{max}). Under this condition the moment capacity of the beam is governed predominantly by concrete; reinforcement does not contribute to the beam moment capacity (see eqn (9)). Therefore the

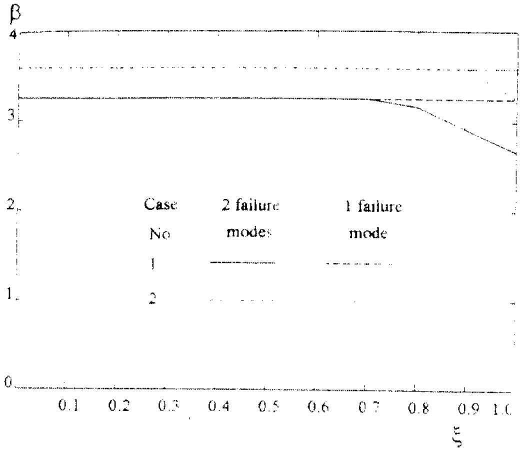

Fig. 1 Flexural reliability index β vs. relative relative reinforcement ratio ξ, cases 1, 2

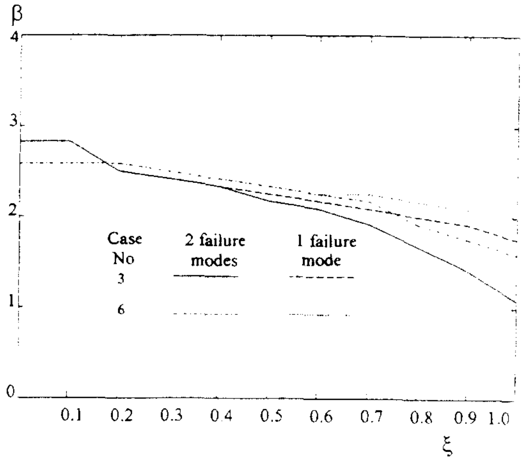

Fig. 2 Flexural reliability index β vs. relative relative reinforcement ratio ξ, cases 3, 6

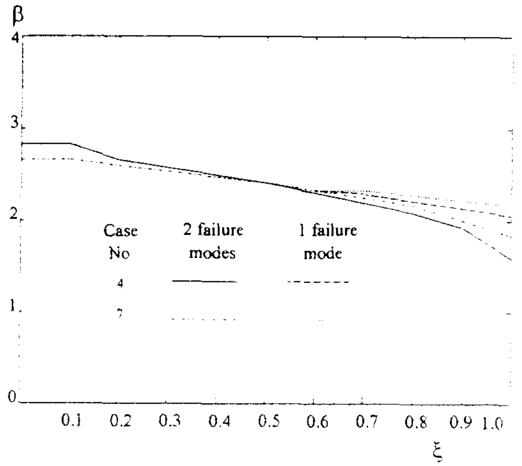

Fig. 3 Flexural reliability index β vs. relative relative reinforcement ratio ξ, cases 4, 7

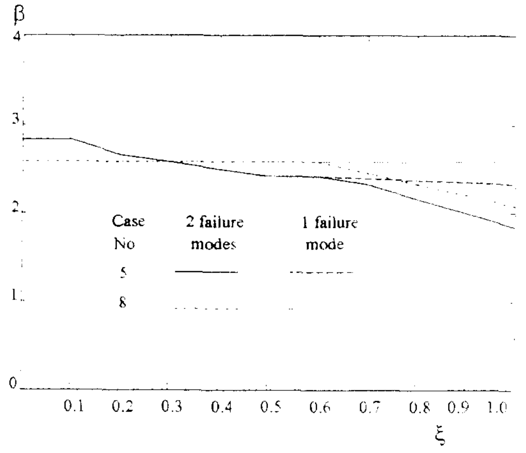

Fig. 4 Flexural reliability index β vs. relative relative reinforcement ratio ξ, cases 5, 8

reliability is low.

In case of low and average reinforcement ratio ($0 \leq \xi \leq 0.7$, see Fig. 1) the beam is actually moderately reinforced, i.e., the probability of over-reinforcement is small. Flexural strength of such beam is governed by both materials - concrete and steel (see eqn (8)). The materials behave as though they support each other.

Assume, for example, that concrete strength is low. Then if strength of reinforcement is sufficiently high, the depth of concrete compression zone increases and the beam can support the external moment with a reduced value of the arm of the internal couple. Similarly, if strength of reinforcement is low, but strength of concrete is sufficiently high, the depth of concrete compression zone decreases and the beam can support the external moment with a larger value of the arm of the internal couple.

Maximum reliability index $\beta=3.20$ is the same as reliability index obtained in [8] for the same conditions. It also coincides with the reliability index calculated taking into account one failure mode (the dashed line in Fig. 1). However, if only one failure mode is considered, the reliability index $\beta=3.20$ remains constant on the whole interval $0 \leq \xi \leq 1$. From this it follows again that the failure mode related to over-reinforcement must not be neglected.

The shape of the graph in case 2 is similar to that in case 1, but reliability in case 2 is higher.

The graphs in cases 3 to 8 (Figs. 2, 3, 4) differ substantially in shape from those in cases 1, 2 (Fig. 1) and the reliability is lower. To take an example, for $=1$ if two failure modes are taken into account the β values range from 1.1 (case3) to 2.1 (case8). The distinctive feature of the graphs in cases 3 to 8 is that the reliability decreases with ξ (in all cases) and increases with concrete strength (compare cases 3, 4, 5 as well as 6, 7, 8). This feature holds irrespective of whether one or two failure modes are taken into account. Let us discuss this subject in more detail.

To clarify the heart of the matter let us compare a decline in moment capacity of the beams as concrete strength decreases for different values of reinforcement ratio. Take a set of random realizations f_y, f_c', B_f, M_1^r, M_2^r of basic variables. Calculate the relative moment capacity of the beam M_{c1}^r, M_{c2}^r for two reinforcement ratios ρ_1, $\rho_2 (\rho_1 < \rho_2)$, respectively. Consider a reduced value of concrete strength $k_c f_c' (0 < k_c < 1)$. Calculate the relative moment capacity of the beam $\bar{M}_{c1}^r$, $\bar{M}_{c2}^r$, taking the same values of all parameters as for M_{c1}^r, M_{c2}^r, excluding concrete strength that equals $k_c f_c'$. We shall investigate the sign of the difference

$$D_c = \frac{\bar{M}_{c1}^r}{M_{c1}^r} - \frac{\bar{M}_{c2}^r}{M_{c2}^r} (\rho_1 < \rho_2, 0 < k_c < 1) \quad (16)$$

We shall assume that only one failure mode corresponding to moderate reinforcement is taken into account. Substitute the values of M_{c1}^r, M_{c2}^r, $\bar{M}_{c1}^r$, $\bar{M}_{c2}^r$ from eqn (8) into (16):

$$D_c = \frac{B_f \rho_1 f_y (1 - \frac{\rho_1 f_y}{1.7 k_c f_c'})}{B_f \rho_1 f_y (1 - \frac{\rho_1 f_y}{1.7 f_c'})} - \frac{B_f \rho_2 f_y (1 - \frac{\rho_2 f_y}{1.7 k_c f_c'})}{B_f \rho_2 f_y (1 - \frac{\rho_2 f_y}{1.7 f_c'})} \quad (17)$$

Denote

$$A = \frac{1.7 f_c'}{f_y} \quad (18)$$

Then

$$D_c = \frac{A(\rho_2 - \rho_1)(1 - k_c)}{k_c (A - \rho_1)(A - \rho_2)} \quad (19)$$

Since $A > 0$, $A - \rho_1 > 0$, $A - \rho_2 > 0$, $\rho_1 < \rho_2$ and $0 < k_c < 1$, then

$$D_c > 0 \quad (20)$$

Thus, the relative decline in moment capacity increases with reinforcement ratio ρ depending on concrete strength: from eqn (19) one can see that the lower concrete strength drops (i.e., the lower k_c value is taken) the more the above increase is.

Random realizations of the external moments M_1^r, M_2^r do not depend on concrete strength. From eqn (8) one can conclude that limit-state function g_4^r decreases as concrete strength reduces, i.e., the reliability decreases as well.

In cases 1, 2 concrete strength is higher than in cases 3 to 8. Therefore if one failure mode is taken into account the reliability of the beam remains constant irrespective of reinforcement ratio. In cases 3 to 8 concrete strength is markedly less. Therefore reliability decreases substantially with reinforcement ratio.

In much the same way as in case 1, if two failure modes are taken into account in cases 3 to 8, then for low and average reinforcement ratio (ξ is between 0 and 0.5-0.6) the reliability is the same as for one failure mode (the beam is actually moderately reinforced). As the reinforcement ratio increases the probability of over-reinforcement increases as well. The moment capacity of over-reinforced beams is governed only by concrete. Since concrete strength is low, the moment capacity is low too. As a result, the reliability decreases in comparison with that obtained for one failure mode.

A decline in moment capacity of the beams as steel strength decreases can be compared in much the same fashion. Equation similar to eqn (17) takes the form

$$D_s = \frac{B_f\rho_1 k_s f_y (1 - \frac{\rho_1 k_s f_y}{1.7f_c'})}{B_f\rho_1 f_y (1 - \frac{\rho_1 f_y}{1.7f_c'})} - \frac{B_f\rho_2 k_s f_y (1 - \frac{\rho_2 k_s f_y}{1.7k_c f_c'})}{B_f\rho_2 f_y (1 - \frac{\rho_2 f_y}{1.7f_c'})} \qquad (21)$$

where $0<k_c<1$. After some algebra:

$$D_s = \frac{Ak_s(\rho_1 - \rho_2)(1-k_s)}{(A-\rho_1(A-\rho_2)} < 0 \qquad (22)$$

Thus, the relative decline in moment capacity decreases with reinforcement ratio ρ depending on steel strength: the lower the steel strength drops, the more the above decrease is.

From this it follows that two tendencies exist: as reinforcement ratio increases the decrease in concrete or steel strength results in lower or higher reliability, respectively. Mutual arrangement of the graphs in Figs. 1 to 4 is attributable to this fact: the decrease in steel strength (cases 1, 3, 4, 5 in comparison with cases 2, 6, 7, 8, respectively) can result in increase or decrease in reliability depending on which tendency prevails for different reinforcement ratios ρ.

Figs. 1 to 4 indicate that reliability of the beams designed under the provisions of ACI Building Code is non-uniform and inadequate for ρ values close to ρ_{max}. The reliability is particularly low in cases typical for Korean practice because of low concrete strength [9]. The problem arises how to remedy the situation.

5. Recommendations

In authors' opinion, matters can be straightened out by the following measures.

As was mentioned above, in case of over-reinforcement the inadequate reliability arises from the low strength of concrete, which in its turn is directly related to the evaluation and acceptance rules. Under the provisions of ACI Building Code the concrete is considered acceptable if two criteria are met:

1) No single test strength shall be more than 500 psi below the specified compressive strength f_c'.

2) The average of any three consecutive test results must equal or exceed the specified compressive strength, f_c'.

The second criterion implies that the minimum required average compressive strength of concrete is equal to the specified compressive strength, f_c'. Consider the concrete in a batch. Assume that concrete strength is normally distributed with mean value f_c'. According to the second criterion, the concrete is accepted. In this batch the probability $P(f_c^* > f_c')$ that actual concrete strength f_c^* exceeds f_c' is very low and equals 0.5. As was discussed above, in practice this exceedance probability is even lower; it is particularly low in cases typical for Korean practice [9].

Several years ago similar drawbacks in the Russian Code for RC structures design came to light [10]. To remedy the situation it was decided to change the definitions of characteristic and design strengths of concrete. Previously characteristic strength B_n was specified with exceedance probability 0.95. Design strength R_d was defined as the ratio $R_d = B_n / \gamma_c$, where γ_c is a partial safety factor ($\gamma_c > 1$). To get rid of the cases with inadequate reliability additional requirements on characteristic B_n and design R_d strengths of concrete were imposed: probability that concrete strength exceeds B_n and R_d should be not less than

0.95 and 0.9986, respectively, with partial safety factor γ_c being unchanged. Then the control procedures in the State Standard GOST 18105-86 "Concrete. Rules for Acceptance Control" were changed to meet these requirements. This procedure is described in detail in Ref [12,13].

It is appropriate to consider a possibility to apply a similar approach to ACI Code 318-89. Specified compressive strength of concrete f_c' can be defined with a certain exceedance probability. Then the rules for acceptance control of concrete can be changed in such a way as to satisfy this definition. In this case the minimum average required compressive strength of concrete will, of course, exceed f_c'.

By these means the cases with inadequate reliability are eliminated. However, excessive reliability can appear in some cases (e.g., for $0 \leq \xi \leq 0.7$, see Fig. 1, case 1). In such an event a material combination factor can be introduced to regulate reliability [10].

The material combination factor is an additional partial safety factor. It is similar to load factors. It takes into account low probabilities of simultaneously low values of strength of several materials, when the materials behave as if they support each other. By comparison, load factors take into account low probabilities of simultaneously high values of several loads. With the material combination factor a uniform reliability can be achieved.

This approach can be applied not only to beams, but to other structures as well.

6. Conclusion

From the above discussion the following conclusions related to flexural strength of RC beams can be drawn.

1. Reliability of RC beams designed under the provisions of the ACI Building Code is non-uniform and changes substantially with reinforcement ratio ρ. The lowest values of reliability occur for beams with ρ values close to $\frac{3}{4}\rho_b$.
2. For the above case the probability of brittle failure is rather high (up to 0.42).
3. In cases typical for Korean practice [9] the reliability is particularly low (reliability index β can go down to 1.1) and the probability of brittle failure is particularly high (up to 0.82).
4. In the course of reliability analyses of RC beams two failure modes, corresponding to moderate and over-reinforcement should be taken into account.
5. In investigated cases the probability of brittle failure due to light reinforcement is very low and can be neglected.
6. To decrease the probability of brittle failure due to over-reinforcement the rules for acceptance control of concrete can be changed.
7. To achieve a uniform reliability a material combination factor can be used.

References

1. ACI, Building Code Requirements for Reinforced Concrete, American Concrete Institute Standard Committee 318, Detroit, 1989.
2. B.R.Ellingwood and A.H-S.Ang, Risk-based evaluation of design criteria, J. Struct. Div., ASCE, 100(9) (1974) 1771-1788.
3. M.K.Ravindra, N.C.Lind and W.Siu, Illustration of reliability-based design, J. Struct. Div., ASCE, 100(9) (1974) 1789-1811.
4. B.R.Ellingwood, Probabilistic codified design, presented at Structural Reliability: Theory and Applications, University of California, Berkeley, 1983.
5. J.G.MacGregor, S.A.Mirza and B.R.Ellin-

gwood, Statistical Analysis of resistance of reinforced and prestressed concrete members, ACI J., 80(3) (1983) 167-176.

6. M.Israel, B.Ellingwood and R.Corotis, Reliability-based code formulation for reinforced concrete buildings, J. Struct. Div., ASCE, 113(10) (1987) 2235-2251.
7. I.Y.Chung and S.K.Park, A study on the probability of brittle failure and its effect on the flexural safety of the under-reinforced beams, J. The Architectural Institute of Korea, 4(4) (1988) 205-214.
8. R.Lu, Y.Luo and J.P.Conte, Reliability evaluation of reinforced concrete beams, Structural Safety, 14 (1994) 277-298.
9. S.H.Kim and H.K.Cho, Reliability evaluation of structures. A case of reinforced concrete buildings under dead, live and wind loads. Final Report. KICT, Korea (1989), 95-106.
10. M.B.Krakovski, Improvement of Russian Code for RC structures design on the basis of reliability theory, Proc. of the 5th IFIP WG 7.5 Conf., Takamatsu, Japan (1993), 141-148.
11. M.B.Krakovski, Structural reliability estimation using Monte Carlo simulation and Pearson's curves, Struct. Engrg. and Mech., An Intern. J.,3(3) (1995) 201-213.
12. M.B.Krakovski, A probabilistic model of acceptance control of concrete used in the Russian Standard GOST 18105-86, Structural Safety & Reliability, Proc. of ICOSSAR'93, 1387-1390.
13. M.B.Krakovski, Monte Carlo simulation of the acceptance control of concrete, tructural Safety 17(1) (1995) 43-56.

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