地域硏究, 第11卷 第1號
1995年 6月 pp.31~60

Probabilistic Location Choice and Markovian Industrial Migration a Micro-Macro Composition Approach

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1. Introduction

A model of industrial location choice in a static world and the resulting industrial migration in a dynamic setting is developed for an abstract economy whose state is identified by two components ; the kinds of economic activity it involves and the location where that activity takes place. Economic agents in the economy are classified into three generations : the younger generation, the working generation, and the older generations : the younger generation, the working generation, and the older generation. Members in the younger generation are in the process of developing human capital which will determine the future earnings stream in the course of their economic life. Members in the working generation consist of both wage earners and proprietors in the labor force and provide services from their accumulated human capital for the activity which offers the highest expected earnings. Their earned income is either retained for the future or spent to increase the satisfaction level for themselves and for members in other generations, this consumption-investment-transfer mix depends on individual time preference and risk aversion in a competitive market economy. Finally, members of the older generation are retired from the labor force and consume their retained earnings from their job experience optimally chosen.

Even though the abstract economy has a simple structure identified only by the kind and place of activity it provides, the existence of other generations and the activity-specificity of accumulated human capital prohibit an economic agent in the working generation from making the industry-region choice decision solely on the basis of the observed earnings differential of the current activity he is providing relative to all alternative activities identified by the industry-region pair in the economy. Rather, the agent would make the choice based on his version of expected earnings. Thus, the agent's predicted earnings depends on two information sets regarding ability and environment. The former is related to the cost and benefit of acquiring new activity-specific job-qualifications and their competitiveness, and the latter to the location-specific regional economic profiles such as types of customers, size of the market, amenity, and congestion, along with the behavior of local governments regarding specific industries and the community.

To an economist, the resulting choices of individual agents are observed in the form of distributions of economic activities on a rectangular array of industry-region pairs representing the structure of an abstract economy. An inference problem on the choice behavior of economic agents to the economist is the problem of comparing the shape of this activity distribution with the shape of the actual(or expected) earnings and other relevant distributions on the array wich has identical structure. However, a direct comparison of the activity data matrix with the earnings data matrix may not lead to a proper inference due to differences in the unit of measurement and in the magnitude of differences. Our strategy adopted in this study is to normalize these matrices into biproportional matrices whose elements all add up to one. We call this resulting matrix the share distribution matrix and a probabilistic choice model is going to be constructed to characterize the shape of this probability-like distribution. An industrial migration model, in a difference equation form, will be presented to approximate the time evolution of this share distribution. The normalizing factors taken out from these matrices form a dynamic reduced form model that can predict the time evolution of these jointly endogenous aggregates and will complement the loss of information in the models based on these share distributions.

While the elements in the activity data matrix represent the total number of activities of a specific industry in a specific region identified by the industry-region coordinate, the elements in the activity share matrix represent the relative frequency of finding a person currently employed being located in that industry-region pair. The entry in each cell of the share distribution can be treated as a probability resulting from an individual's choice behavior only under the assumption that every job in the economy is equally likely to be taken by any economic agent. However, in reality, activity specificity of human capital and familiarity with regional economic profiles tend to keep the economic agent in the current cell with high probability and, moreover, these tendencies do not seem to be uniform across industry and/or region. Unless some significant changes in the expected profitability differential wipe out the relative benefit of staying in the same business in the current location, the economic agent would not consider the reshuffling of activity allocation. In the recent history of the U.S. economy, however, numerousexamples of changes in expected profitability differentials can be cited. Some examples are plant closings due to foreign competition, the development of new products, and the change in government industrial policy on regulations and business tax laws. These developments lead to active relocating of economic activity and present thick clouds of uncertainty in the industrial location choice decision.

In order to address the practical utility of investigation the problem of industrial migration, we present two surface diagrams of activity data matrices for the U.S. economy for the years 1969 and 1986, eighteen years later. A 10×17 activity matrix shows employment distribution of industry-region pairs. Rows show ten major non-farm industries ; agricultural services (AGR), mining(MIN), construction (CNS), manufacturing(MFG), transportation and utilities(TRU), wholesale trade (WTR), retail trade(RTR), finance(FIN), services(SER), and government sectors (GVT). Columns show seventeen northeast states ; six states(ME, NH, VT, MA, CT, and RI) in the New England region, six states(NY, PA, MD, DC, DE, and NJ) in the Mideast region, and five states(OH, MI, WI, IL, IN) in the Great Lakes region. Seventeen states are ordered by magnitude of state earnings in 1986, i.e., VT,

DE, RI, ME, NH, DC, CT, WI, MD, IN, MA, NJ, MI, OH, PA, IL, and NY. The vertical magnitude shows the total number of full-time and part-time jobs distributed over industry-region pairs. The data is based on the April 1988 release of state personal Income Estimates by the Bureau of Economic Analysis and a description of the data is given at the end of the text. From the inspection of two activity surfaces(Figures 1.a and 2.a), we realize that a significant decline in manufacturing industries in Ohio and Michigan(Figures 1.b and 2.b) and a dramatic increase in the service sector in New York State (Figures 1.c and 2.c) has resulted in a major change in the shape of the activity surface in the last two decades. Gradual industrial migration toward service-oriented industries from production oriented industries and symptoms of agglomeration toward financial centers such as New York, Chicago, and Los Angeles are observed from the snap-shot of two activity distributions in two remote periods. For an economist to provide an adequate explanation on this observed phenomenon and to suggest an acceptable prediction to the decision maker, what would be a necessary conceptualization in constructing an empirically verifiable migration model?

One would conjecture that even though the activity distribution does not seem to have regularity upon which an analytical paradigm can be constructed, the distribution of earnings per activity may be able to provide some useful insight on the problem, because an individual optimization scheme that secures a stable earning stream cannot be independent of rewards that a chosen action might generate. From the evidence of historical data, this conjecture gets some strong support in general, but not unanimous support. Two surfaces of yearly earnings per job for the years 1969 and 1986(Figures 3.a and 4.a) exhibit a smoother surface for most regions and most industries except the mining industry(Figure 3.b and 4.b). Quite unlike the activity distribution, which is proportional to the size of the entries in the industry-region pair, the distribution of realized earnings form provided activity exhibit a more uniform distribution over industries relative to that over regions(Figures 3.c and 4.c). The retail trade industry shows the least variation across region and the mining industry shows the opposite. If we decide to include all industries in our analysis, is there any analytic or deterministic way that we can handle this stable variable with irregular data points? Recent developments in complex dynamics may provide answers for some of these phenomena, but we are still waiting for a positive answer.

Another conjecture would be an assessment of an appropriate probability distribution whose finite moments can capture industry-specific and region-specific variation such that risk-minimizing predictions about the mean of these activities can be made. Then a predictive earnings-per-activity distribution is obtained as a function of finite number of parameters. Considering the randomness of individual choice decisions a stochastic treatment of this data matrix would be attractive. However, the usual independence and identicality assumption on the adopted distribution would be an untenable assumption for the observed data matrix whose mean and variances are shifting over time. One obvious choice is to take out the systematic component affecting the first finite moments. We transform the data matrix into a biproportional share matrix and this transformed matrix is then normalized by the add-up-to-one condition. The transformation preserves the same distributive shape as the original data matrix. Now the problem of examining the phenomena of industrial location choice and associated industrial migration is reduced to the problem of examining any source of change in these distributions.

Because all estimates representing our assessment of underlying structure are some linear or nonlinear transformation of our observations on the variables we are interested in, the first step in econometric analysis is to examine the empirical distribution that the unstructured data carries. Figures 5.a and 5.b exhibit the empirical distributions of activity(Number of jobs distributed over 10 industries and 17 States) and associated yearly earnings-per-activity for selected years 1969, 1978, and 1986. For the purpose of comparison of distributions, yearly data points are centered at yearly sample means, and, as might be expected, the most disperse distribution is of the latest period. An extreme form of skewe-dness to the right causes the median, which gives. 5 of the cdf value, to lie far below the zero point, As we might expect, the adoption of any symmetric distribution without an appropriate nonlinear transformation will result in serious misspeci-fication. In modeling probabilistic industrial location choice, we adopt the extreme value distribution, the generalization of which was developed by McFadden in a series of papers(1978, 1981, 1984) in the context of residential location choice. Figures 6.a and 6.b illustrates the cdf and pdf of the extreme value distribution in the standard form, which we hope would be consistent with the appropriately transformed data matrix. The transformation will also provide the chance to incorporate our structural knowledge about the underlying economic agent's optimal choice behavior in reference to another biproportional share matrix, the expected earnings matrix.

The purpose of this study is to develop a model of industrial location choice at a given time, and its time evolution in a dynamic setting. The model can then provide empirically relevant predictions and feasible policy recommendations by analyzing the change of the empirical distribution of economic activity in reference to the change of the empirical distribution of expected earnings per activity. The latter distribution has an explicit structure which considers the underlying changes of industry-specific, region-specific, ans/or both attributes. Now, the distribution of the predicted(expected) earnings differential is perceived by economic agents as a probability distribution of random profitability upon which the industrial migration decision is based. n a static setting, this profitability matrix follows the generalized extreme value distribution, a distribution utilized in the now well known random utility models.

The maximum likelihood estimator of the multivariate error components model is obtained to handle the nature of panel data, the pooling of the cross sectional industry-region pair and the time series. Under the Markovian assumption, the job evolution equation is obtained from the set of equations of motion for the probability-like share evolution. The transition probability matrix of this activity share evolution is obtained from the profitability matrix and its prediction is compared with the actual change in the activity configuration over time. The simulation results, in the absence of the local government transfer payment and benefit programs, will be compared with the model prediction.

2. The Model and the Method of Analysis :

At the beginning of each period, an economic agent makes a decision to provide one unit of economic activity to the i^{th} industry at the j^{th} location after comparing his expected earnings from that activity with all alternative expected(predicted) earnings when the same activity is provided to other industries and/or locations. At the end of each period, the economist observes that n_{ij} activities have been performed in industry i and region j in the economy. The structure of the economy is identified by a categorical, mutually exclusive and completely exhaustive, industry-region pair. The indices of industry and region is by (i, j), where $i=1, \cdots, a$ and $j=1, \cdots, l$, and where a is the total number of industrial groups and l the total number of regional groups. The total activities operated for a given time is the sum of these realized activities in each cell :

$$(2.1)\quad n_{..}=\sum_i \sum_j n_{ij}$$

To the economist, individual choice is observed through a rectangular array arrangement of the total number of activities performed during the given period. We call this rectangular arrangement the *activity matrix*, denoted by $N=[n_{ij}]$. Each column, $n_{aj}=[n_{ij}]$, $i=1, \cdots, a$ for given j, represents the operation of various industries for a given region and each row $n_{li}=[n_{ij}]'$, $j=1, \cdots, l$ for given i, represents the operation of a given industry in various regions. The column sum, $n_a=[n_{i.}]$, $n_{i.}=\sum_j n_{ij}$, $i=1, \cdots, a$, represents the operation of $n_{..}$ activities at varius locations for a given time. In matrix-vector notation, the economic activity of the whole economy during a given period is summarized in either the activity matrix, the industrial activity vectior, or the regional activity vectior.

$$N=[n_{ij}],\ 1_a' N1_l=n_{..}$$
$$(2.2)\quad N=[n_{a1},\cdots,n_{aj},\cdots,n_{al}], N1_l=n_a=[n_i].$$
$$N'=[n_{l1},\cdots,n_{li},\cdots,n_{la}], N'1_a=n_l=[n_{.j}].$$

For a statistician who does not concern himself with the underlying economic behavior, this rectangular arrangement would be perceived as the familiar contingency table or the cross-classified categorical data matrix.

Two-way classification ANOVA can be used to examine the significance of industrial effects, regional effects, and their interactions using the exact F-test under the null hypothesis of zero effects. Since insufficient partitioning of the data always leads to a loss of information, these multivariate statistical analysis would be useful in examining the significance of categorical classification. However, the validity of these parametric test statistics in finite samples is doubtful when an inappropriate row-column classification leads to unbalanced degrees of freedom or nonnormality due to outliers in the cell. The statistician may use nonparametric or information tests to explore the characteristics of the economy under this activity-location classification. However, there is a more serious defect in the statistician's assumption that is difficult for the economist to follow. The assumption is that the relative frequency of each cell represents the probability, in the long run, that an economic activity can be found in that cell.

$$(2.3)\quad P_{ij}=\lim_{n\to\infty} Pr\{n_{ij} \mid n_{..}=1'N1\}$$

Other than observed sample information, this specification of probability certainly does not include any underlying structure relating to the indivisual activity-location choice decision. In this sense, this approach is incomplete in the use of available information when analyzing the industrial location decision.

To an economist, the observed employment matrix is the realization of the individual choice decision of each economic agent who, at the beginning of each period, is assumd to be endowed with one unit of economic activity with homogeneous quality. The economic activity can be allocated to any industry and any region if zero transition cost is assumed. One unit of economic activity can be allocated in $a \cdot l$ different ways, hence the total of $n_{..}$ activities can be allocated in $\left[{n_{..} \atop a \cdot l}\right]$ different ways. All possible allocations of a given number of economic activities in the mutually exclusive activity-location pair form a sample space as in the standard probability theory. One element of the sample space is observed every period, and we call this an activity configuration. A collection of all possible activity configurations is a σ-algebra of subsets, of Borel field of the sample space. Upon this the economist can define aprobability measure, μ. The combination of the sample space and the σ-algebra of subsets of the sample space forms a measurable space for the industrial location choice.

The economist advances the hypotheses H_1 and H_2 that the employment configuration is from the statistical population with alternative probability measures, μ_1 for H_1 and μ_2 for H_2, say. He assumes that these probability measures are absolutely continuous with respect to one another such that the generalized probability densities corresponding to alternative hypotheses are obtained as the Radon-Nikodym derivatives of these measures with respect to a common probability measure(Halmos and Savage, 1949).

For example, the economist may advance the following hypothesis about the sequencing of the activity-location decision; the maintained hypothesis is that economic agents make the location decision first and then decide on the industry, conditional on the location decision. The alternative hypotheses is the opposite sequencing. The conditional and marginal distributions for different hypotheses have different probability measures but all are based on a common probability measure of the joint density. The comparison of alternative hypotheses to the economist is then the problem of comparing the posterior probabilities of alternative hypotheses following Bayes' theorem of the theorems on conditional probability based on the observed employment configuration n=vec (N), where vec( ) denotes column vectorization.

$$(2.4)\quad P(H_i \mid n)=\frac{P(H_i)P_i(n)}{P(H_1)P_1(n)+P(H_2)P_2(n)}$$

$$i=s1,2.$$

From the above equation we obtain the logarithm of the ratio of alternative probability distributions as the difference between the logarithm of the posterior odds of the hypotheses and that of the prior odds of the hypotheses:

$$(2.5)\quad \ln\frac{P_1(n)}{P_2(n)}=\ln\frac{P(H_1 \mid n)}{P(H_2 \mid n)}-\ln\frac{P(H_1)}{P(H_2)}$$

The activity configuration vector of a system of a industries and l regions at time t can be characterized by a column vectorization of the activity matrix N(t):

$$(2.6)\quad n(t)=vec(N(t))=[n_1(t),\cdots\cdots, n_X(t),\cdots\cdots, n_{a\cdot l}(t)]'$$

Where $x=a(j-1)+i$, $i=1,\cdots, a$, $j=1, \cdots l$ and the integer $n_X(t)$ is the number of activities in the i^{th} industry and the j^{th} region at time t. We assume $n_X(t)\geq 1$ using sufficient disaggregation and treat $n_X(t)$ as continuous variables. If the economy maintains the total number of activities at a constant level at each time such that retirement and frictional unemployment from the working generation is matched by the new entrants from the younger generation such that

$$(2.7)\quad \sum_x n_x(t)=n_{..}\quad \text{for all t.}$$

Then the change $n_X(t)$ per unit of time, $\frac{dn_X(t)}{dt}$, follows an intuitively clear equation of motion. The change depends on two factors; the flow of activities from all other industries and regions into industry i located in region j(the x^{th} position in the origin coordinate) and the flow of activities from the current x^{th} position into all other industries and regions(the y^{th} position in the destination coordinate, $y=a(j'-1)+i'$, $i'\neq i$ and $j'\neq j$) per unit of time. The former is denoted by $n_{IN,\ X}(t)$

and the latter by $n_{OUT,\ x}(t)$.

$$(2.8)\quad \frac{dn_X(t)}{dt} = n_{IN,\ x}(t) - n_{OUT,\ x}(t)$$

The collection of this equation of motion for all coordinates of the activity configuration gives a vector differential equation:

$$(2.9)\quad \frac{dn(t)}{dt} = n_{IN}(t) - n_{OUT}(t)$$

For a given time, the choice probability that an economic agent will perform one unit of economic activity in the industry i at region j is not independent of the other agents' decision and is also influenced by the economic attributes associated with industries, regions, or both. Region-specific effects like tax differentials, wage differences, levels of unionization, and input cost can be tempered or magnified by industry-specific effects in location decision(Schmenner et. al., 1987). In this section we constructed a general probability model based on the activity configuration vector and from it we can make inferences directly if the probability measure satisfies certain regularity conditions.

3. Random Profitability and the Sequential Decision in Industrial Location Choice.

The maximization of expected profitability by an individual decision maker in providing one unit of economic activity in the i^{th} industry, among a alternative industries, and the j^{th} region, among l alternative locations, can be modeled by a typical quantal choice model. We assume that the profitability that lead to a chioce deviates randomly from the systematic components specific to the activity's industrial-regional characteristics. Three assumptions are made about economic agents. First, it is assumed that the economic agents in the economy are indistinguishable in their preferences for the industry-region pair choice. Second, no individual has advantage over all other individuals in the form of accumulated human capital. Third, all agents have identical initial endowment at the beginning of each period. That is, the economist need not distinguish individuals from one another since they are indentical in taste, in quality of service, and in endowment. In another words, any variation in the observed number of activities after the individual's choice decision is made can be fully indentified by associated region specific and/or industry specific characteristics. We also assume that individual agents in the working generation must choose among mutually exclusive and exhaustive categorical industry-region pairs. That is, the economy is closed and all members in the working generation is fully employed.

Let a typical individual choice is denoted by

$$(3.1)\quad y_{ijm} = \begin{cases} 1 & \text{if the}(i,\ j)^{th} \text{ alternative is chosen} \\ 0 & \text{otherwise} \end{cases}$$

where $i=1, \cdots\cdots, a$, $j=1, \cdots\cdots, l$, and $m=1, \cdots\cdots, n..$ The activity distribution matrix at time t, denoted $N(t)$, is the $a \times l$ rectangular arrangement of the sum of chosen 1's for all n.. agents.

In our model the individual industry-region choice is based on the comparison of expected random profitability among alternatives based on the predicted earnings that will be received at the end of each period. The latent variable model for the random profitability is given by

$$(3.2)\quad \overset{\circ}{\pi}_{ij} = \pi_{ij} + \varepsilon_{ij}$$

whose errors are generalized extreme value distributed disturbances. Now we have a multivariate error component model for the column vectorization of the $a \cdot l$ elements of the expected profitability matrix $\overset{\circ}{\Pi}_{ij} = [\overset{\circ}{\pi}_{ij}]$

$$(3.3)\quad \pi^{\circ} = \mathrm{vec}(\Pi^{\circ}) = z\theta + \varepsilon.$$

This will provide a rich dynamic-spatial structure when the model is obtained from a reduced form of a simultaneous equation model, if the$(a \cdot l \times K)$ matrix of K predetermined variables including lagged values π, other industry-region specific attributes, and policy variables. We face two difficulties. One that of linking the unobservable profitability variable to the observed activity level vector. The other that of handling a multivariate extreme value distributed disturbances. We start from McFadden's work.

The generalized extreme value(GEV) model introduced by McFadden(1978, 1981, 1984) is defined in terms of a harmonic function $H(\cdot)$ which is a non-negative, linear homogeneous function of $w_{ij} = \exp(\prod_{ij})$, $i \in A$ and $j \in L$, where $A = \{1, \cdots\cdots, a\}$ and $L = \{1, \cdots\cdots, l\}$ denotes industrial and regional choice alternatives respectively,

$$(3.4)\quad H(w_{ij}\,;\ \forall(i, j) \in A \times L) = \left(\sum_{A \times L} w_{ij}^{\frac{1}{k}} \right)^{k}$$

with $0 < k \leq 1$. This H function has marvelous flexibility and complexity for applications in industrial-location choice models.

First, when $w_{ij} = \exp(\prod_{ij})$, the function H is a generating function for the choice probabilities as the first derivative of the logarithmic transformation of the H function

$$(3.5)\quad P_{ij} = \Pr\{ (i, j) \mid \prod_{ij} = z_{ij}\theta \} = \frac{\partial}{\partial \prod_{ij}} \ln H \{ e^{\prod_{ij}}, \forall(i, j) \in A \times L \}$$

This H function also has an interpretation as a measure of social utility, since H is the CES form (harmonic mean) of the nonlinear transformation of expected earnings.

Second, when $w_{ij} = \exp(-\epsilon_{ij})$ and $0 < k \leq 1$ the exponentiation of the negative of this function gives the cumulative distribution function of the disturbances which follow the generalized extreme value distribution function

$$(3.6)\quad F(\epsilon_{ij}, \forall(i, j) \in A \times L) = \exp[-H(e^{-\epsilon_{ij}}, i \in A, j \in B)] = \exp[-(\textstyle\sum e^{-\frac{1}{k}\epsilon_{ij}})^{k}]$$

where $0 < k \leq 1$ denotes the interdependence parameter and this F has the choice probabilities as its density function.

Third, if the parameter of dependence k takes values between zero and one, then the cross-elasticity of the response probability for (i, j), $i \in A$, $j \in L$ with respect to alternative (i', j') $i' \in A$ and $j' \in L$ is non-zero, such that the Independence from Irrelevant Alternatives (IIA) property implied by the multinomial logit model (MNL) is relaxed. The relaxation of this assumption is important when the decision is made sequentially by narrowing down the choice alternatives. We will discuss this later in detail.

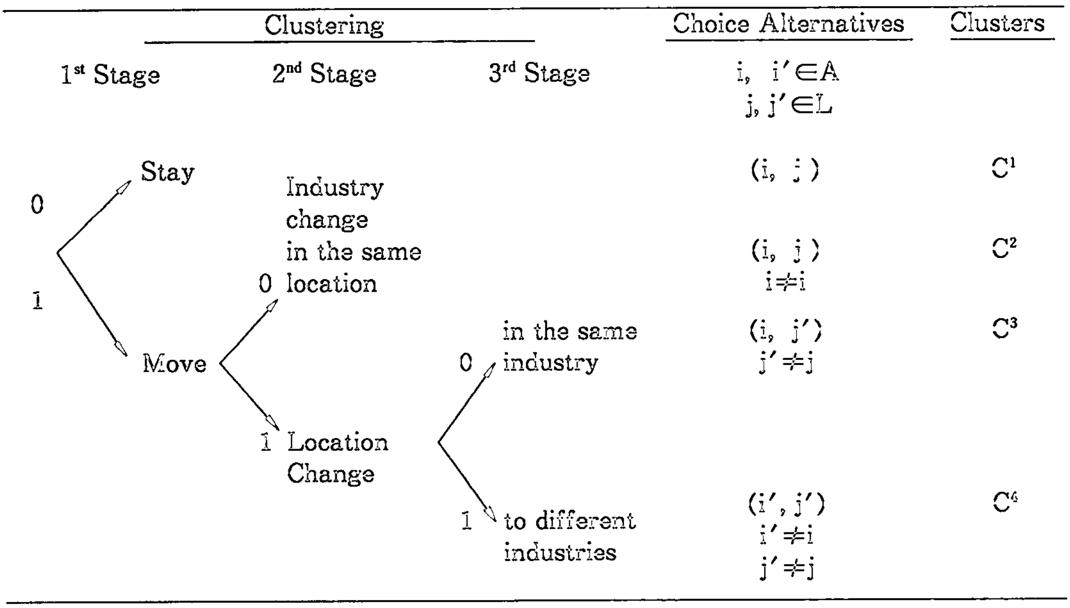
The industrial location choice problem usually involves a sequential decision process in which alternatives are grouped into clusters which are similar and then the decision is made to eliminate clusters until a single alternative remains.

Consider an agent currently providing one unit of economic activity to the i^{th} industry located at the j^{th} region. If a firm finds that its current profit is significantly lower than its expected earnings from other industries in other locations, then the firm will consider the relocating of economic activity. At the first stage the alternatives in the whole set, denoted by $A \times L$, are grouped into two clusters; one cluster consisting of the current industry-region pair (actually having one element in this set) and another cluster consisting of all the alternatives other than the current industry-region pair. This is actually a binary choice between 'move' and 'stay' decision.

When the firm's decision is in favor of

'move' then, at the second stage, it will compare the differences between the future profitability and the transport costs of moving to alternatives. Since the transport cose of locating the same industry in a different location or of changing to another industry in the same region would be much lower than that of changing both industry and region, the firm will eliminate the industry-region pair from the second cluster if neither of the pair is the same as the current one.

At the third stage, the remaining cluster will be seperated into two parts. One part has common geographic locations and the other requires common know-how in operating the activity. Now the transition in the decision tree of the industrial-location choice gives four mutually exclusive and exhaustive clusters C^1, C^2, C^3 and C^4 shown in the figure of the following page :



Region

Industry		1	j	1
	1	C^4 (i′, j′)	C^2 (i′, j)	C^4 (i′, j′)
Industry	i	C^3 (i, j′)	C^1 (i, j)	C^3 (i, j′)
	a	C^4 (i′, j′)	C^2 (i′, j)	C^4 (i′, j′)

$i' \neq i \in A$, $j' \neq j \in L$

$$C^1 = \{(i, j) \in A \times L\}$$

$$C^2 = \{(i', j) \in A \times L,\ i' \neq i\}$$

$$C^3 = \{(i, j') \in A \times L,\ j' \neq j\}$$

$$C^4 = \{(i', j') \in A \times L,\ i' \neq i\ \ j' \neq j\}$$

The transition in the decision tree involves the process of narrowing down the cluster size until it contains a single industry-region pair. This we call a top-down process, since $A \times L = \bigcup_{d1} C^{d1}$ and $\bigcap_{d1} C^{d1} = \phi$ at the first stage, $C^{d1} = \bigcup_{d12} C^{d12}$ and $\bigcap_{d12} C^{d12} = \phi$ at the second stage, and $C^{d12} = \bigcup_{d123} C^{d123}$ and $\bigcap_{d123} C^{d123} \neq \phi$ at the third stage, where $d1 = \{0, 1\}$, $d12 = \{(1, 0), (1, 1)\}$ and $d123 = \{(1, 1, 0), (1, 1, 1)\}$ denotes a choice pairs conditional on the previous decision.

In a similar way, we may consider another mutually exclusive and exhaustive clustering process; the expected profitability of the current activity is compared, first, with the expected earnings of alternative industries in the same region, second, with those of alternative regions in the same industry, and, finally, with those of alternative regions in alternative industries. We call this a bottom-up process. The information cost involved in the search process would determine the choice between the processes. Here we examine the decision process in the context of the former.

The nesting of clusters $C^{d123} \subseteq C^{d12} \subseteq C^{d1} \subset A \times L$ can be incorporated into the generalized extreme value structure such that the choice probability is represented in terms of transition probabilities from the choice set $A \times L$ to the nested clusters

$$\text{(3.7)}\quad P((i, j) \in C^{d123} \subseteq C^{d12} \subseteq C^{d1} \subseteq A \times L | Z_{ij}, \theta) = P((i, j)|C^{d123}) \cdot P(C^{d123}|C^{d12}) \cdot P(C^{d12}|C^{d1}) \cdot P(C^{d1}|Z_{ij}, \theta)$$

In order to have explicit functional forms for each probability involved, we need to generate vectors of cluster's attributes representing decision maker's search sequence. Now the reduced form for the expected profitability for the (i, j) alternative has the form:

$$\text{(3.8)}\quad \prod_{ij} = f(z_{ij}, \theta)$$

where $z'_{ij} = [x'_{ij}\ x'_{d123}\ x'_{d12}\ x'_{d1}]$, and $\theta' = [\beta'\ \beta'_{d123}\ \beta'_{d12}\ \beta'_{d1}\ k_{123}\ k_{12}\ k_1]$ in which $x_{d.}$ are vectors of attributes and $\beta_{d.}$ are associated coefficient vectors for each cluster, and $o < k_1, k_{12}, k_{123} \leq 1$ are associated interdependency parameters of alternatives among clusters. Now the transition probabilities along the top-down decision process is given by:

$$\text{(3.8)}\quad P\left((i, j)|C^{d123}\right) = \frac{1}{c_{123}} \exp(x'_{ij}\beta)$$

$$P(c^{d123}|c^{d12}) = \frac{1}{c_{12}} \exp(x'_{d123}\beta_{d123} + k_{123} \ln c_{123})$$

$$P(c^{d12}|c^{d1}) = \frac{1}{c_1} \exp(x'_{d12}\beta_{d12} + k_{12} \ln c_{12})$$

$$P(c^{d1}|z'_{ij}, \theta) = \frac{1}{c} \exp(x'_{d1}\beta_{d1} + k_1 \ln c_1)$$

where $c_{123} = \sum_{(i, j) \in c^{d123}} \exp(x_{ij}\beta)$

$$c_{12} = \sum_{(i, j) \in c^{d12}} \exp(x_{d123}\beta_{d123}) c_{123}^{k123}$$

$$c_1 = \sum_{(i, j) \in c^{d1}} \exp(x_{d12}\beta_{d12}) c_{12}^{k12}$$

$$c = \sum_{(i, j) \in A \times L} \exp(x_{d1}\beta_{d1}) c_1^{k1}$$

Following McFadden (1984) we can define a H function for this nested clustering decision process as:

$$\text{(3.9)}\quad H(\exp(\pi_{ij}), (i, j) \in A \times L) = \sum_{A \times L}\left[\sum_{c^{d1}}\left[\sum_{c^{d12}}\left[\sum_{c^{d123}}\left(e^{\pi_{ij}}\right)^{\frac{1}{k_{123}k_{12}k_1}}\right]^{k_{123}}\right]^{k_{12}}\right]^{k_1}$$

Now the expected profitablility under this clustering has the form:

$$(3.10)\quad X_{ij}=X_{ij}'\beta k_{123}k_{12}k_1+x_{d123}'\beta_{d123}k_{12}k_1+x_{d12}'\beta_{d12}k_1+x_{d1}'\beta_{d1}$$

which is nonlinear in parameters. It is simple to verify that the first order derivative of the logarithm of H with respect to [illegible]$_{ij}$ gives the choice probability under this hierarchical elimination process.

$$(3.11)\quad P((i,j)\in c^{d123}\subseteq c^{d12}\subseteq c^{d1}\subseteq A\times L)=\frac{\exp\left(\frac{[illegible]_{ij}}{k_1k_{12}k_{123}}\right)}{c\ c_1^{1-k_1}\ c_{12}^{1-k_{12}}\ c_{123}^{1-k_{123}}}$$

If $k_1=k_{12}=k_{123}=1$, then the nested multinomial logit model reduces to the usual multinomial logit model with the IIA property.

This is a generalization of the nested multinomial logit model for the random profitability reduced form model with generalized extreme value disturbances. Therefore, this model is consistent with the assumption of optimizing economic agents who make the industrial location decision based on the comparison of the expected profitability of the chosen industry-region pair with those of all alternatives, and the migration decision is made on the industry–location pair which gives the highest expected profitability based on all available information at time t.

Econometric applications of the industrial location choice model involve observations through time in a combined cross-section/time-series framework. The underlying random profitability describes economic agents formation of expectations about realized earnings at the end of period. This latent variable model then has a variance component structure, with industry-specific effects(activity heterogeneity) and region-specific effects, as well as the usual autocorrelation and spatial-correlation of disturbances.

Dynamic qualitative choice models are special cases of systems of non-linear simultaneous equations, and their econometric analysis for practical applications needs simplifying assumptions so that their multivariate random disturbances of the latent variable model have an additive structure. We consider the unobservable random profitability model of the form :

$$(3.12)\quad \pi_{ijmt}^{*}=x_{ijt}\beta+\epsilon_{ij}t+\alpha_i+\lambda_j \text{ and}$$

$$y_{ijmt}=\begin{cases}1 \text{ if } \pi_{ijt}^{*}>\pi_{i'j't}^{*} \text{ for all } i\neq i'\in A \\ \quad \text{and } j\neq j'\in L \\ 0 \text{ otherwise}\end{cases}$$

and $m=1,\cdots,n_{..}(t)$ and $t=1,\cdots,T$. α_i is an industry-specific random effect which persists over location and time, and λ_j is a region-specific random effect which persists over location and time, and λ_j is a region-specific random effect which persists over industry and time. ϵ_{ijt} are disturbances independent of each other and of industry-region specific random effects. Note that the model does not have an individual specific component due to our assumptions of homogeneity and of integra-bility of individuals involving in the same activity. Neither does the model have a time specific effect assuming that the macro part of the model absorbs all time effects. Therefore this part of the model concentrates on the micro unit choice behavior within the macroeconomic aggregate constraint. Economic agents considers their relative share in the economy as their reference point and all the variables in the information set are normalized to the share distribution whose sum remains constant over time.

In order to explain the behavior of the activity variable over time, we construct the macro-micro composite model consisting of two components : the macro aggregate component of the activity variable follows a typical covariance stationary time series process when sources of systematic components(non-zero mean, trend drift, autoregressive cyclical behavior) are taken out. The micro disaggregates over industry and region follow the probability-like share distribution under the nonnegativity and normalization constraints. Therefore the prediction for the observed activity distribution is the composition of the predicted macro aggregates based on time series modeling and the predicted micro disaggregates are based on the multivariate error component model of the probability-like share distribution over a biproportional industry-region share matrix. In the absence of a priori non-sample information on the parameters, any prediction of the variable under investigation is a linear or nonlinear projection of the observation onto the data space spanned by independent components of the design matrix. We apply the same transformation on the explanatory or policy variables involved in the econometric analysis, and then functional relation and parameter restrictions are imposed on these transformed variables. As a result, we will have a dynamic simultaneous equation model for macro-aggregates, a model which considers joint endogeneity among variables that are econometrically non-exogenous, and a multivariate error component model for micro-disaggregates, a model which considers a probability-like distribution chan-ge on a normalized space.

The product of the time-varying fluctuation of macro aggregates with individual share fluctuations of micro disaggregates will give the model prediction of these aggregates in the original unit of measurement. The basic philosophy behind this approach is the consideration that information processing should prevent the loss of information and the employed procedure should minimize the sources of uncertainty caused by inappropriate statistical assumptions about random components and/or misspecified theoretic postulates about systematic components.

We adopt a Vector Autoregressions (VAR) model to approximate the behavior of total activity and other relevant variables. The use of VAR is not just because of its convenience, rather because of two important properties which are consistent with the behavior of macro-aggregates. First, VAR treats all the variables as endogenous variables in the simpleunstructured dynamic simultaneous equations. Second, without coefficient constraint, VAR have identical regressors, such that the least squares estimator is as efficient as any other estimators whose residuals alsohave the minimum variance property. However, this method ignores available structural information and, hence, is not appropriate if the purpose of analysis is simulation of policy alternatives.

We adopt a multivariate error component model whose systematic component is exponentially related to the dependent variable, and the error decomposition follows a multivariate normal time component and a region specific component and the disturbance term :

$$(3.13)\quad \underset{(a\times 1)}{n_{jt}} = g(x_{jt}\beta) + u_{jt} \qquad \begin{matrix} j = 1,\cdots\cdots,l \\ t = 1,\cdots\cdots,T \end{matrix}$$

$$\underset{(a\times 1)}{u_{jt}} = e_t + \rho_j + \epsilon_{jt}$$

where the error component have multivariate normal distributions $e_t \sim MN(O, \Gamma)$, $\rho_i \sim MN(O, Z)$ and $\epsilon_{it} \sim MN(O, \triangle)$, and where Γ and Z are a×a positive semidefinite matrices and $\triangle$ is a a×a positive definite variance covariance matrix. Assuming that all cross terms have zero expectation, a column vectorization of all disturbances u_{jt}, $i = 1,\cdots\cdots,l$ and $t = 1,\cdots\cdots,T$ gives a $alT\times 1$ vector in the form :

$$(3.14)\quad \underset{(alT\times 1)}{U} = \begin{bmatrix} 1_l \otimes e_l \\ 1_l \otimes e_T \end{bmatrix} + 1_T \otimes \rho + \epsilon$$

where 1_l denotes a $l\times 1$ vector of ones, $\otimes$ denotes the Kronecker product defined by $A \otimes B = [a_{ij}B]$, and ρ is a $a l \times 1$ vector of region specific error components.

Let $M_T \equiv \frac{1}{T} 1_T 1'_T$ and $M_l = \frac{1}{l} 1_l 1'_l$ be matrices of $\frac{1}{T}$ of dimension $T \times T$ and of $\frac{1}{l}$ of dimension $l \times l$ respectively, then we have a complete covariance structure as :

$$
\begin{aligned}
(3.15)\quad \Omega^{*} &= E[uu'] \\
&= E\left[\begin{bmatrix} 1_l \otimes e \\ \vdots \\ 1_l \otimes e \end{bmatrix}\begin{bmatrix} 1_l \otimes e \\ \vdots \\ 1_l \otimes e \end{bmatrix}'\right] \\
&\quad + E[(1_T \otimes \rho)(1_T \otimes \rho)'] + E[\epsilon\epsilon'] \\
&= I_T \otimes 1_l 1'_l \otimes \Gamma + 1_l 1'_l \otimes I_l \otimes Z + I_T \otimes I_l \otimes \triangle \\
&= I_T \otimes M_l \otimes l\Gamma + M_T \otimes I_l \otimes TZ + I_T \otimes I_l \otimes \triangle \\
&= (J_3 + J_1) \otimes l\Gamma + (J_2 + J_1) \otimes TZ + (J_4 + J_3 + J_2 + J_1) \otimes \triangle \\
&= J_1 \otimes (\triangle + TZ + l\Gamma) + J_2 \otimes (\triangle + TZ) + J_3 \otimes (\triangle + l\Gamma) + J_4 \otimes \triangle \\
&= \sum_i J_i \otimes W_i
\end{aligned}
$$

where, $J_1 \equiv M_T \otimes M_l$, $J_2 \equiv M_T \otimes (I_l - M_l)$, $J_3 \equiv (I_T - M_T) \otimes M_l$ and $J_4 \equiv (I - M_T) \otimes (I - M_l)$ are idempotent matrices of rank $\gamma_1 = 1$, $\gamma_2 = l - 1$, $\gamma_3 = T - 1$, and $\gamma_4 = (l-1)(T-1)$, respectively and W_i are covariances defined appropraitely.

Magnus(1984, p242) obtains the determinant and the inverse of the complete covariance matrix Ω^{*} as some function of determinants and inverses of W's

$$(3.16)\quad |\Omega^{*}| = \prod_{i=1}^{4} |W_i|^{\gamma_i}$$

$$(3.17)\quad \Omega^{*-1} = \sum_{i=1}^{4} J_i \otimes W_i^{-1}$$

Now the log-likelihood function is obtained as :

$$(3.18)\quad L = -\frac{1}{2} Tla \ \ln\ 2\pi - \frac{1}{2}\sum_{i=1}^{4} \gamma_i \ln\ |W_i| - \frac{1}{2}\sum_{i=1}^{4} tr\ UJ_iU'W_i^{-1}$$

where $U = [u_{jt}]$ and $u_{jt} = n_{jt} - g(x'_{jt}\beta)$.

This function is maximized to obtain the parameters of the activity equation using the Newton Raphson algorithm.

Ⅳ. Markovian Industrial Migration :

Let us consider the migration of an economic activity consisiting of n.. activities between a industries and l regions. The particular quantity of interest is the $(a \cdot l \times 1)$ migratory configuration vector $n(t)$ of the $(a \times l)$ activity distribution matrix $N(t)$:

$$(4.1)\quad n(t) = vec(N(t)), \quad n(t) = [n_x(t)] \text{ and } N(t) = [n_{ij}(t)]$$

where vec( ) denotes the column vectorization of a matrix, $n_{ij}(t)$ is the number of activities in industry i and region j at time t, and $n_x(t)$ is the same number found in the $x(=a(j-1)+i)^{th}$ position in the vector $n(t)$. Here $n_{..} \equiv \sum_i\sum_j n_{ij}(t)$, $n_{i.} \equiv \sum_j n_{ij}(t)$, and $n_{.j} \equiv \sum_i n_{ij}(t)$ denotes the total number of activities in the economy, in the i^{th} industry and in the j^{th} region, respectively for a given time. The evolution of these numbers is a stochastic process, because the migration of an economic activity depends on the individual economic agents' probabilistic choice decision. The random fluctuations of these numbers identified in the activity-location coordinate over time are of interest to economists and policy decision makers.

To an economist, the random fluctuation of economic activities are observed in the form of employment data after individual choice decisions are made and economic activity is realized. Therefore, the fully adequate treatment of this stochastic process for the economist is to provide the probability distribution :

$$(4.2)\ P(n(t)) \equiv P\Big([n_{11}(t), n_{21}(t), \cdots\cdots, n_{al}(t)]'\Big)$$

to the economic activity configuration vector n(t) and to make inference about the first two moments of this random vector process. The mean of this process is given by :

$$(4.3)\ \mu(t) \equiv E[n(t)] = \sum_{n(t)} n(t)\ P(n(t))$$

and the variance covariance matrix is given by :

$$(4.4)\ \Sigma(t) \equiv E\Big[(n(t) - E[n(t)])(n(t)) - E[n(t)])'\Big]$$

assuming that this vector process is strictly covariance stationary. By a strictly stationary vector process n(t), we mean the process for which, for all $T = t_i, \cdots\cdots, t_N$ and τ, the distribution of $n(t_1), \cdots\cdots, n(t_T)$ and of $n(t_1+\tau), \cdots\cdots, n(t_T+\tau)$ are the same. This means that its auto-covariance matrix function depends only on time displacement τ, but not on real time t, if the n(t) have finite mean square. Therefore, we further assume that this vector process converges in mean square to its limiting process $\bar{n}(t)$, $n(t) \Rightarrow \bar{n}(t)$.

Under these assumptions the observed random fluctuation of economic activity around $\bar{n}(t)$ has the mean squared error module :

$$(4.5)\ \| n(t) - \bar{n}(t) \| = \Big(E[(n(t) - \bar{n}(t))'(n(t) - \bar{n}(t))]\Big)^{\frac{1}{2}}$$

which is finite and the square of this module becomes more negligible as we observe indefinitely. The necessary and sufficient condition that the limiting vector process $\bar{n}(t)$, to which n(t) converges in mean square, exist is the Cauchy condition :

$$(4.6)\ \lim_{\substack{t\to\infty \\ s\to\infty}} \| n(t) - n(s) \| = 0$$

meaning that the limiting sequence is uniquely defined in the sense that any two random vectors in the convergence sequence differ only on a set of measure zero. We provide the same assumptions on the vector process e(t) of a random vector, i.e., the earnings from economic activity. Then the Cauchy condition guarantees that the cross covariance matrix of the activity vector process and the earnings vector process converge to that of the two limiting processes, because the covariance matrix is a continuous function of two limiting processes, $\bar{n}(t)$ and $\bar{e}(t)$.

The mean-square random processes, as well as random processes whose increments are mutually orthogonal, are imbedded in the space of the generalized random process (Hannan, 1970). Therefore, the stochastic vector process n(t) and the projection of n(t) onto the space of another stochastic vector process e(t), under the regression mean square module, must converge to the same limiting vector process, if the economist to make proper inference.

An equation of motion for the mean process $\mu(t)$ of n(t) is obtained from its time derivative :

$$(4.7)\ \frac{d\mu(t)}{dt} = \sum_{n(t)} \frac{dP(n(t))}{dt} n(t) + \sum_{n(t)} P(n(t)) \frac{dn(t)}{dt} = \sum_{n(t)} \frac{dP(n(t))}{dt} n(t)$$

by assuming that the second term in the first line is zero, that is, the average of random change in economic activity change over all activity configurations is zero. The time evolution of the probability of a stochastic vector process is in general difficult to find unless we know the transition probabilities from each coordinate of the activity configuration at the origin to all other coordinates of the activity configuration at the destination for all different combination of activity configurations. Moreover, this transition probability depends on real time. For the given transition probability, the evolution of the probability of the activity configuration has the following form of the equation of motion:

$$(4.8)\quad \frac{dP(n(t))}{dt} = \sum_{\Delta n(t)} W(n(t);n(t)-\Delta n(t))\, P(n(t)+\Delta n(t)) - \sum_{\Delta n(t)} W(n(t)+\Delta n(t);n(t))\, P(n(t)$$

where $\Delta n(t)$ represents the change in the activity configuration from the current $n(t)$ to the neighboring activity configuration $n(t)+\Delta n(t)$.

Now, $W(n(t)+\Delta n(t);\ n(t))$ represents the transition matrix per unit of time from $n(t)$ to $n(t)+\Delta n(t)$ via industrial migration of $\Delta n(t)$ activities from any member in the $n(t)$ activities from any coordinate in the origin to any coordinate to the destination. This probability evolution equation consists of $[{n_{..} \atop al}]$ coupled, linear differential equations for the probabilities $P(n(t))$ of the $[{n_{..} \atop al}]$ possible configuration of $n(t)$ over the activity−location coordinate for $n_{..} = \{\sum_i \sum_j n_{ij}(t)\}$ total activities at given time. The first term in the evolution equation represents the probability change due to the inflow of economic activities into the activity configuration and the second term represents the probability cha-nge due to outflow out of the activity configuration $n(t)$, summed over all possible configurations of $\Delta n(t)$, which has $[{\Delta n_{..} \atop al}]$ possible compinations. Several maxima of the limiting probability, $\lim_{t\to\infty} P(n(t))$, give the final steady state activity configurations of the migratory system at the condition $\frac{dP(n(\infty))}{dt} = 0$. This condition represents the individual on in− and out− flux of activities in all industry and in all regions.

For an economist as a passive observer of the non-experimental economic system, perfect understanding of multi-industry and multi-regional activity migration could be incurred only at the infinite information cost. We assume, only in the probabilistic sense, that the economic sysyem is a conservative dynamic system, which has the property that if an economist finds an economic activity at the x^{th} coordinate (i.e. in the i^{th} industry and the j^{th} region) at time 0 that moves to the y^{th} coordinate at time τ, then the same activity he finds at the x coordinate at any time t, can always be found at the y coordinate at time $t+\tau$. In other words, the economic system evolves in a similar way, not in exactly the same way, from a given activity configuration, irrespective of initial time it started from that configuration. An extreme form of the corresponding property for an isolated Markovian dynamic system is that the transition probabilities do not depend on real time. a series of papers by Haag and Weidlich(1984, 1986, 1987) develop a dynamic residential migration theory under the Markovian assumption. Constancy of the transition probability under the Markov system is convenient for analytic study, but this approach lacks empirical utility. On the other hand, the transition probability matrix constructed from observations usually fails to meet the stability requirement and the use of means of sample data disregards the information about the underlying structure. The avenue taken in this study constructs the transition probability matrix from the prediction of the earnings vector, conditional on the activity−specific, region-specific and both−specific information available at the time the probabilistic choice decision is made.

The transition probability matrix W(t) empirically obtained along this line is time varying and information specific, but exhibits conserbative dynamism with high probability.

We assume that the transition probability is a function of the predictions, $\hat{e}(t)$, of another relevant earnings stochastic vector process e(t), which is independent of increments of the activity configuration $\triangle n(t)$. Due to its relevance to the corresponding coordinate, the transition probability in terms of $\hat{e}(t)$ is in matrix from, rather than a scalar:

(4.9) $W(t)=W(\hat{e}(t)),\ 1'_{a\cdot l}\ W(t)1_{a\cdot l}=1$

where the character $1_{a\cdot l}$ represents a $(a\cdot l \times 1)$ vector of ones and the condition requires that the sum of all transition probabilites equals one. The latter condition is not necessary for the Markovian system. However, we require this for two reasons, first, because of its dependence on $\hat{e}(t)$, which is a nonlinear function of some other stochastic processes, second, because of its assumed relation to the mean process. Now, the evolution of the mean process with the activity−change−independent transition probability matrix is given by:

$$(4.10)\ \frac{d\mu(t)}{dt}=\sum_{n(t)}\sum_{\triangle n(t)} W(t)P(n(t)+\triangle n(t))n(t)-\sum_{n(t)}\sum_{\triangle n(t)} W'(t)P(n(t))n(t)$$
$$=W(t)\mu_{IN}(t)-W'(t)\mu_{OUT}(t)$$

where $\mu_{IN}\equiv\sum_{n(t)}\sum_{\triangle n(t)} P(n(t)+\triangle n(t))\ n(t)$ and

$\mu_{OUT}\equiv\sum_{n(t)} P(n(t))n(t)$

Now we introduce the relative frequency vector p(t) to find $\mu_X(t)$, the number of activities at the coordinate x, by normalizing $\mu(t)$ to a unit length. This p(t) is by definition a vectorization of the activity share distribution matrix. By assuming the transition matrix W(t) is invariant under this normalization, we have equivalent equation of motion:

$$\frac{dp(t)}{dt}=W(t)p_{IN}(t)-W'(t)p_{OUT}(t)$$

The x^{th} coordinate number $p_X(t)$ in the share vector p(t) can be interpreted as a probability (relative frequency) that an activity can be found after its realization in the x^{th} coordinate(ie : the i^{th} industry and the j^{th} region if $x=a(j-1)+i$).

Now, the x^{th} coordinate numbers $p_{IN,X}(t)$ in the vectors $p_{IN}(t)$ and $p_{OUT}(t)$ denote the probability that an economic activity is added to or subtracted from that coordinate, respectively. In this setup, the W(t) matrix has the interpretation of an instantaneous transition probability that one unit of an economic activity will move from the x^{th} coordinate(corresponds to columns in W(t)) to the y^{th} coordinate(corresponds to rows in W(t)). In terms of the conditional probability notation, the y^{th} row and x^{th} column element of the W matrix:

$$(4.12)\ W_{yx}=\lim_{\triangle t\to 0}\frac{P(y,t+\triangle t\mid x,t)}{\triangle t}=p_{yx}(t)$$

where x is the coordinate of the origin and y is the coordinate of the destination, representing the transition probability that one unit of economic activity located in (i, j), $x = a(j-1) + i$, will move to another industry-region pair (i′, j′), $y = a(j'-1) + i'$. While the transition probability is the probability per unit of time the conditional probability P(y, t | x, s) is a dimensionless number which records the historical change of the probability $p_y(t)$ at the destination from the arbitrary origin at the initial time :

$$(4.13)\quad p_y(t) = \sum_x P(y, t \mid x, o)\, p_x(o)$$

The conditional probability also satisfies the well-known Chapman-Kolmogorov equation :

$$(4.14)\quad P(y, t \mid x, t') = \sum_z P(y, t \mid z, s)\, P(z, s \mid x, t')$$

which means the probability to find an activity in y at time t, originally in x at time t′, can be composed of all conditional probabilities that the same activity can found at any intermediate state z at time s, from the origin x at time t′ to reach the destination y at time t. In terms of transition probabilities, the probability evolution equation for the x^{th} coordinate activity share is given by :

$$(4.15)\quad \frac{dp_x(t)}{dt} = \sum_i p_{xi}(t) p_i(t) - \sum_j p_{jx}(t) p_x(t)$$

whose first term represents an activity share change due to in-flux from all other coordinates and the second term represents an activity change due to out-flux from the x^{th} coordinate to all other coordinates.

In order to find the empirical connection of this transition probability matrix, we introduce a simple assumption that the transition probability depends on the deviation of the expected profitability at the destination π_y from that at the origin π_x. In order to assure non-negativity of the rransition probability we use the exponential transformation, and to introduce the concept of transition cost from one industry or location to another industry or/and location, we explicitly introduce the transition cost representing economic and geographical ‘distance’.

$$(4.16)\quad p_{yx}(t) = \exp(\alpha_y + \lambda_y - \alpha_x - \lambda_x)\, \exp(\pi_y(t) - \pi_x(t))$$

where α_y and α_x represents industry-specific mobility factorsinversely related to economic transition cost, and λ_y and λ_x represents region-specific mobility factors inversely related to geographical transition cost. As we may expect, the transition probability p_{yx} from x to y increases as the expected profitability at the destination exceeds the expected profitability of the origin, after considering the transition cost incurred by the mobility decision.

In vetor notation, the share evolution equation has the form :

$$(4.17)\quad \frac{dp(t)}{dt} = P(t)p(t)$$

where $P(t) = [P_{ij}(t)]$

$$= [\sum_x P_{ix}(t) - \sum_y P_{yj}(t)], \text{ and}$$

$$(4.18)\quad P_{ij}(t) = \exp(\pi_i(t) + \alpha_i + \lambda_i - \pi_j(t) - \alpha_j - \lambda_j) - \delta_{ij}[\sum_k \exp(\pi_k(t) + \alpha_k + \lambda_k)]\, \exp(-\pi_j(t) - \alpha_j - \lambda_j)$$

where δ_{ij} denotes a Kronecker delta having values 1 if i=j, and 0, otherwise. Note that the matrix P is not symmetric and hence cannot be a Hermitian.

In order to obtain eigen solutions for this first order matrix differential equation, we use a simplifying notation

$$(4.19)\quad a_x = b_x = \exp(\pi_x + \alpha_x + \lambda_x)$$

such that

$$p_{yx} = \frac{a_y}{b_x} = \exp(\pi_y - \pi_x - \alpha_y - \alpha_x + \lambda_y - \lambda_x)$$

under the time invariant assumption on the transition probability ; then the P matrix has a manageable form

(4.20) $p=[p_{yx}]$

$$p_{yx}=p_{yx}-\delta_{yx}\sum_k p_{kx}$$
$$=\frac{a_y}{b_x}-\delta_{yx}\frac{\sum_k a_k}{b_x}$$

An analytic solution for this type of transition matrix for the first-order differential equation is done in Haag-weidlich (1986). We will briefly explain their results to be used for our purpose. First, using a nonsingular transformation matrix $Q=[\delta_{yx}\ b_y]$ a transformed vector r is obtained as an inverse transformation

(4.21) $r=Q^{-1}\,p$

and the resulting differential equation has the form

(4.22) $\frac{dr}{dt}=Lr$

where

$$L=Q^{-1}PQ=[L_{yx}],$$

and

$$L_{yx}=\frac{a_y}{b_y}-\delta_{yx}\frac{\sum_k a_k}{b_x}$$

Second, define the eigen solutions of the transformed model

(4.23) $r^{(\lambda)}(\tau)=r^{(\lambda)}e^{-\lambda\tau}$

with

$$Lr^{(\lambda)}=(-\lambda)\ r^{(\lambda)}$$

Third, explicitly determine the eigenvaluse λ, and associated eigenvectors $r^{(\lambda)}$ from the transormed model

(4.24) $\left[\frac{a_y}{b_y}-\delta_{yx}\frac{\sum_k a_k}{b_x}\right]\ r^{(\lambda)}=(-\lambda)\ r^{(\lambda)}$

or

(4.25) $\frac{a_y}{b_y}[\sum_j r_j^{(\lambda)}]-\frac{\sum_k a_k}{b_x}r_y^{(\lambda)}$
$=-\lambda\, r_y^{(\lambda)} \qquad y=1,\cdots,a\cdot l.$

which gives the y^{th} component of eigenvector $r^{(\lambda)}$ as

(4.26) $r_y^{(\lambda)}=\dfrac{\frac{a_y}{b_y}\ (\sum_j r_j^{(\lambda)})}{\frac{\sum_k a_k}{b_y}-\lambda}$

Summing over Y and cancelling $r_y^{(\lambda)}$ gives the equation

(4.27) $1=\sum_y\dfrac{\frac{a_y}{b_y}}{\frac{\sum_k a_k}{b_y}-\lambda}$

from which we can solve for the exact values for the eigenvalue λ and the y^{th} component of the associated eigenvector,

(4.28) $r_y^{(\lambda)}=\dfrac{g^{(\lambda)}\frac{a_y}{b_y}}{\frac{\sum_k a_k}{b_y}-\lambda}$

where $g^{(\lambda)}$ is a correct normalization factor for the unit length. Fourth, the inverse transformation $p=Q_r$ gives the values for eigenvectors for the original differential equation

(4.29) $p_y^{(\lambda)}=b_y r_y^{(\lambda)}=\dfrac{g^{(\lambda)}\ a_y}{\frac{\sum_k a_k}{b_y}-\lambda}$

for the eigen solutions

(4.30) $p^{(\lambda)}(\tau)=p^{(\lambda)}e^{-\lambda\tau}$

with

(4.31) $Pp^{(\lambda)}=(-\lambda)p^{(\lambda)}$

Finally, any time dependent share distribution, which is a solution of the first order differential equation for the share vector evolution

$$(4.32)\quad \frac{dp(t)}{dt}=Pp(t),$$

can be expanded on the space spanned by eigen solutions : eigenvalues λ's and associated eigenvectors $p^{(\lambda)}$ will satisfy the equation

$$(4.33)\quad p(\tau)=\sum_{\lambda} c^{(\lambda)}\, e^{-\lambda\tau} p^{(\lambda)}$$

The expansion coefficients $c^{(\lambda)}$ are determined such that the initial condition of the conditional probability satisfies

$$(4.34)\quad p(y,0 \mid x,0)=\delta_{xy}$$

where the conditional probability is defined by

$$(4.35)\quad p(y,\tau_2 \mid x,\tau_1)=\sum_{\lambda} e^{-\lambda(\tau_2-\tau_1)}\, p_y^{(\lambda)}\, c_x^{(\lambda)}$$

Several remarks are in order. The conditional probability $p(y,\tau \mid x,o)$ approaches the stationary probability as $\tau\rightarrow\infty$

$$(4.36)\quad \lim_{\tau\rightarrow\infty} p\,(y,\tau \mid x,0)=p_y^{(\lambda_0)}=\bar{p}_y$$

for the system to be stable. This is true for $\lambda_i>0\ \forall\ j$, such that the exponential factor $e^{-\lambda\tau}$ dies out with the exception of the smallest eigen value obtained as $\lambda_0=0$.

From the relation that we solve for eigenvalues it is interesting to see that the denominator $\left[\frac{\sum_k a_k}{b_y}-\lambda\right]$ gives as many intervals as the length of the vector and each interval will include exactly one eigenvalue.

5. Summary and Conclusion

The reshuffling of activity share among different industrial locations is modeled as the change in the probabilistic distribution of a biproportional matrix. The stochastic activity share in each cell in the matrix is indexed by a mutually exclusive categorical industry-region pair for a given time. If the profitability of another location exceeds that of the region where the firm is currently located, it will consider relocationing its activity. Conditional on the migration decision, the excessive transition cost over profitability margin would put the firm in the search process not just for the location but also for the activity which would secure the firm's long-term profitability.

one feature of the probabilistic industrial location choice model that distinguishes it from the general contingency table analysis is the postulate that the choice probability belongs to a known parametric family. Now the underlying structural link between output share and attributes can be explicitly region-specific, industry-specific, or both. Extending McFadden's approach to matrix alternatives, the probability matrix of industry-location choice was obtained from the generalized extreme value. The maximum likelihood estimation of the multivariate error components model was also considered to handle the nature of panel data, i.e., the pooling of the cross-sectional industry-region pair and the time-series data.

With explicit incorporation of the a priori known structural information, we can now examine the effect of industry-specific and region-specific intervention or deregulation. The expected information gain as a result of the policy change is measured as the percentage reduction of uncertainty (entropy). In order to examine the dynamic transition of activity sharein the context of multiregional industrial migration, the equation of motion for the probability evolution was developed under the Markovian assumption. This model can be applied to interesting practical problems such as the plant location choice of the multinational enterprise, the government industrial policy to attract the international migration of firms, and the local government tax-transfer strategy to influence the industrial location choice. The latter was examined in this study using the non-farm employment data for the United States in ten industries and in seventeen states over the years 1969–1986.

In this paper the industrial location choice and its industrial migration are modeled as the combined effect of the probability-like share distribution change for internal adjustment at the micro level, and the dynamic reduced form for the time evolution at the macro aggregate level. This approach, which I would like to call the Micro-Macro Composition Approach, helps our modeling process in two ways. First, the decomposition of observed industrial activity data into two components, one for the macro level aggregation representing the size of the activity distribution on the industry-region array and the other for the micro level disaggregation representing the probability-like biproportional share allocation, maximizes the use of information in the sense that any prediction from two sources of behavioral conjecture can be composed back to the level distribution based on the macro level aggregate predictions.

Second, the decomposition (and then recomposing) of industrial location choice behavior into two components ; one that governs the interaction among macro aggregates which cannot convey necessary information about individual agent's optimizing behavior and the other that governs the individual choice behavior whose behavioral adjustment is mainly etermined based on each individual's 'economic location' and 'relative distance' in the share distribution, minimizes the sources of uncertainty in the sense that two behavioral aspects, external adjustment and internal optimization, are not treated as the same phenomena.

This decomposition for the pupose of analysis and then recomposing afterwards for the pupose of policy simulation allows more freedom to conceptual models seeking data evidence and gives more flexibility in assessing the probability distribution from economic data which doesn't seem to have any regularity to the naked eye.

Data Description

The data used in this study was obtained from the Data Center of the Institute for Policy Research at the University of Cincinnati. The original data tape was released in April 1988 by the Reginal Economic Information System, at the Bureau of Economic Analysis in the U.S. Department of Commerce. The data files on the magnetic tape consist of five tables : (1) Personal Income by Major Source and Major Industry, (2) Full-time and Part-time Employment by Major Industry, (3) Regional Economic Profile (4) Transfer payments, and (5) Farm Income and Expenses. They also include two titles and footnote files for each of the five data files. They are yearly estimates covering 1969 through 1986. This study uses the first four data files which contains 228 lines of data information for 3184 sets of classified regions (county, state, region and the U.S) by metropolitan and nonmetropolitan, MSA's, DMSA's and CMSA's, over 18 years. The data dump file goes up to 311 columns. The annual estimates of State Personal income for 1969–86 is reported by Regional Economic Measurement Division, "State Personal Income, 1969–86 : Revised Estimates", *Survey of Current Business,* August 1987. 43–57, and the quarterly estimates of

State personal income for 1984 : I −89 : II, *Survey of Current Business*, January 1988, April 1989, July 1989, and October 1989 (Quarterly estimates for the years 1969−83 are available from the regional Economic Information system). This study uses the annual data at the aggregated regional level for the years 1969−86 from the tables (1) through (4). The total aggregates of the United States is disaggregated into four regions (New England, Mideast, Great Lakes, and the rest of the United States) and the non-farm employments and earnings are disaggregated into six industrial sectors (Primary, Manufacturing Transportation and Public Utilities, Wholesale and Retail Trade, Finance and Services, and Government and Government Enterprises). Primary sector includes Agricultural services, forestry, fisheries and others, Mining and Construction which are location specific. The detailed industries of other sectors are described in the above cited government reports. Non-farm employment and earnings data are obtained from tables (1) and (2), and regional profiles are obtained from tables (3) and (4) from the tape.

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ABSTRACT

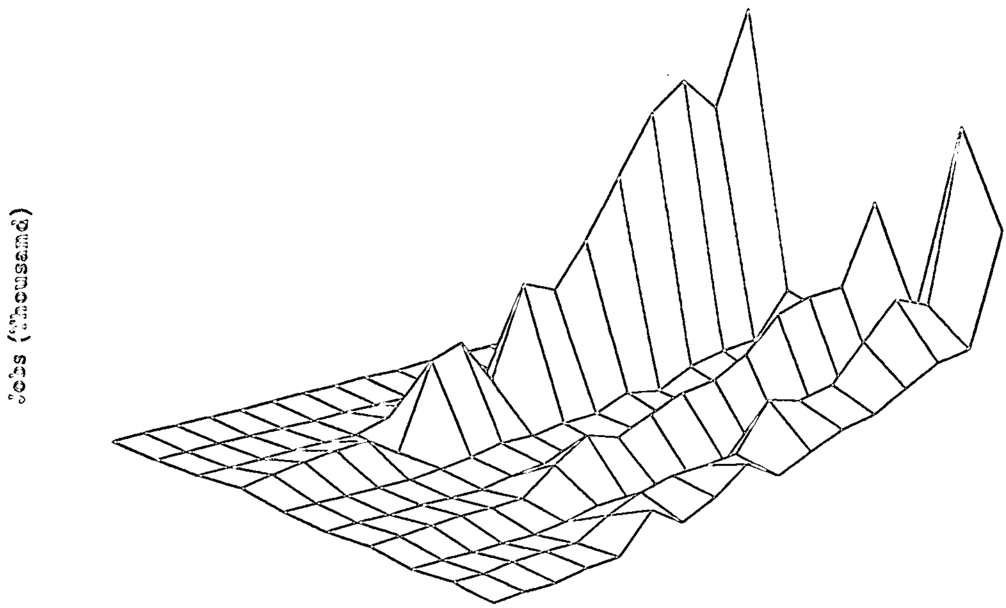
The distribution of economic activity over a mutually exclusive and exhaustive categorical industry−region matrix is modeled as a composition of two random components: the probability−like share distribution of jobs and the dynamic evolution of absolute aggregates. The former describes the individual activity location choice by comparing the predicted profitability of the current industry−region pair against that of all other alternatives based on the available information on industry−specific, region specific, or activity specific attributes. The latter describes the time evolution of macro−level aggregates using a dynamic reduced form model.

With this seperation of micro choice behavior and macro dynamic aggregate constraint, the usual independence and identicality assumptions of adopted parametric distributions become consistent with the activity share distribution, hence multi−regional industrial migration can be represented by a set of probability evolution equations in a conservative Markovian form. We call this a Micro−Macro Composition Approach since the product of the aggregate prediction and the predicted activity share distribution gives the predicted activity distribution which explicitly considers the underlying individual choice behavior.

The model can be applied to interesting practical problems such as the plant location choice of multinational enterprise, the government industrial ploicy to attract international firms, and the optimal tax−transfer mix to influence activity location choice. We consider the latter as an example.

[Figure 1.a]

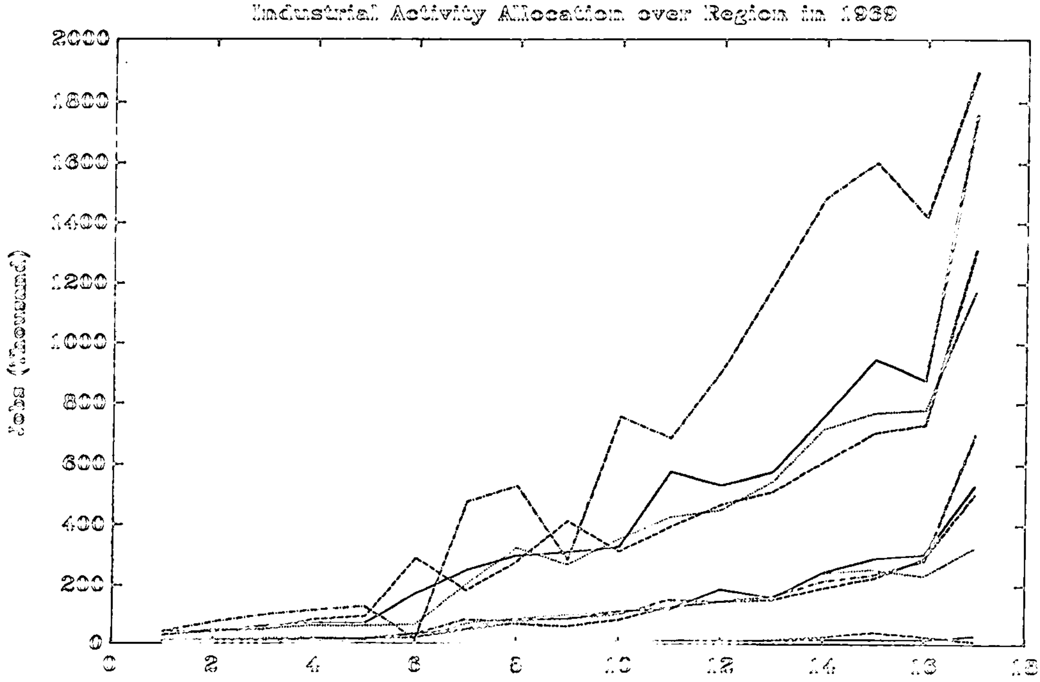
Activity Surface over Industry and Region in 1969



Employment over 10 Industries and 17 States

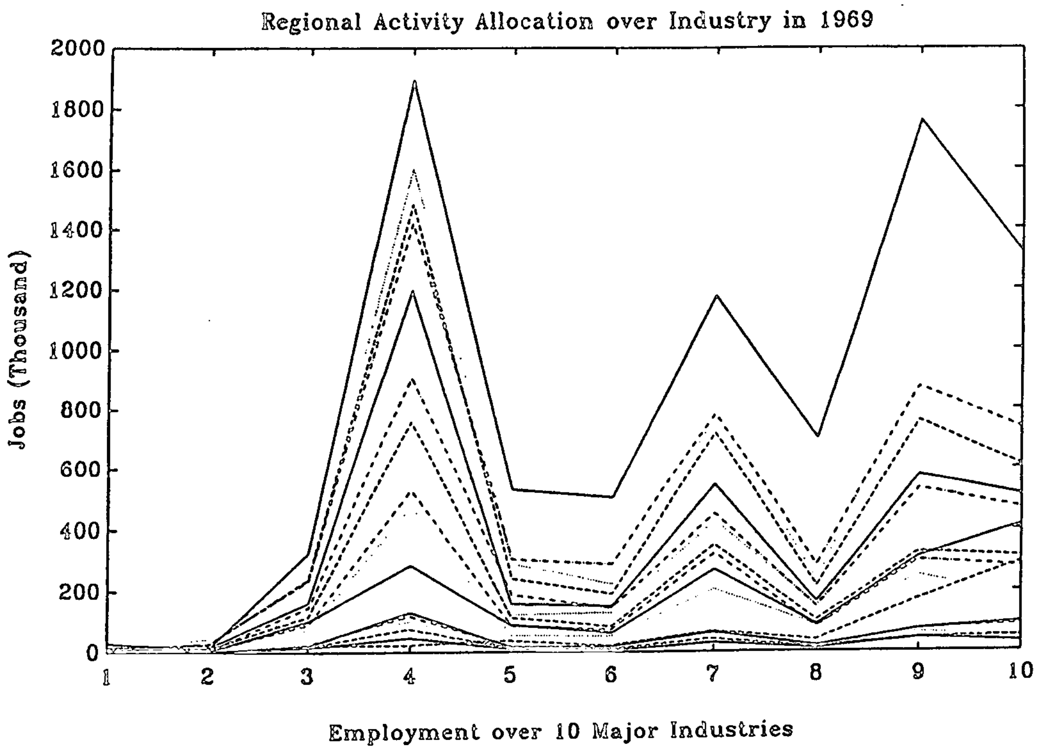
[Figure 1.b]

Industrial Activity Allocation over Region in 1969

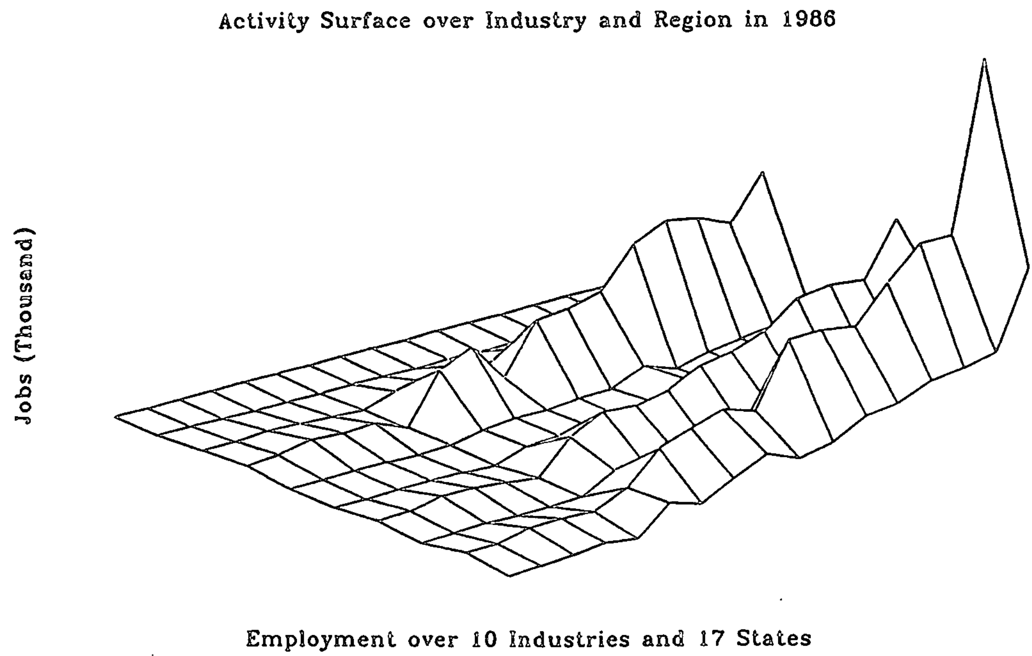


Employment over 17 Northeast States

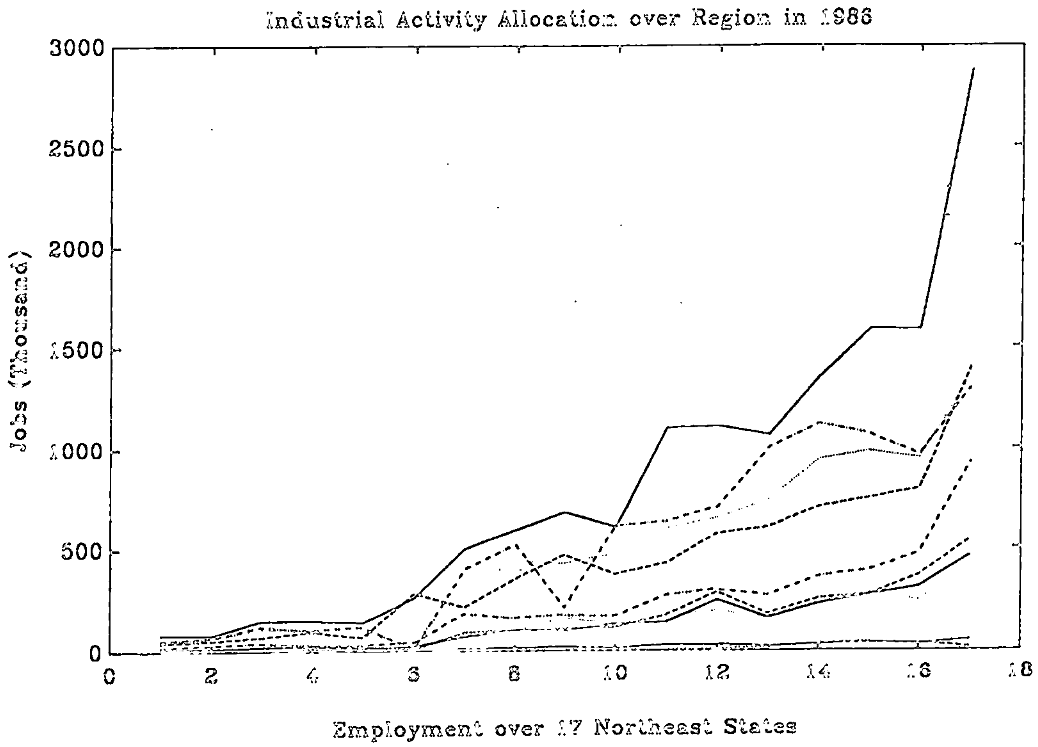
[Figure 1.c]



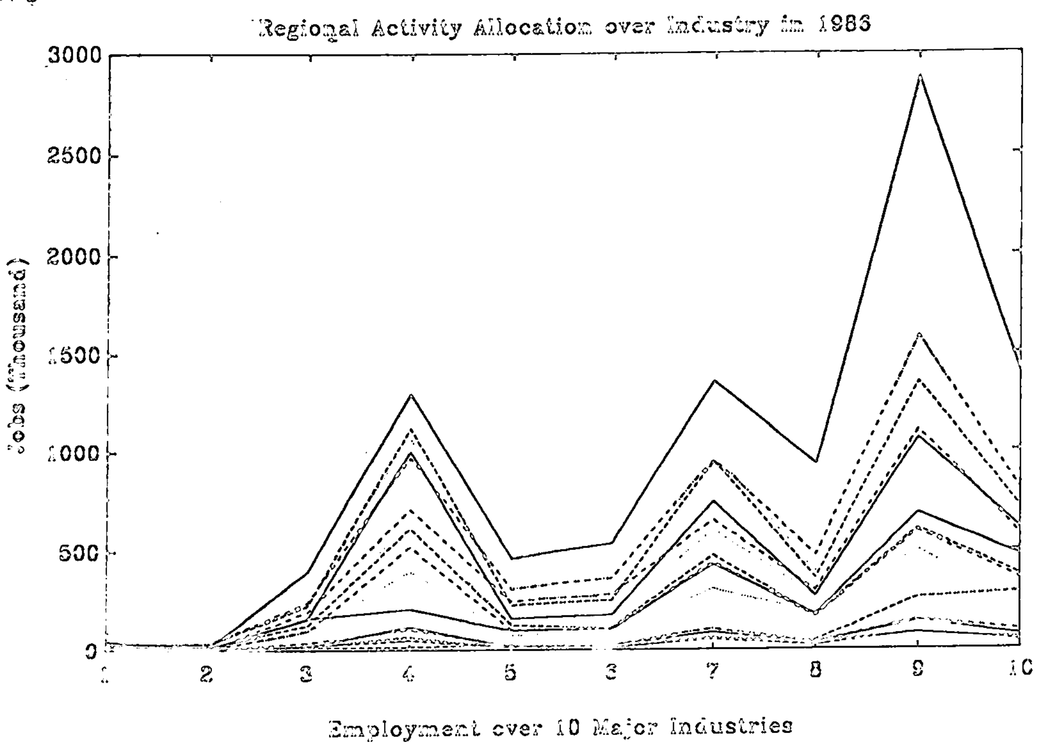
[Figure 2.a]



[Figure 2.b]

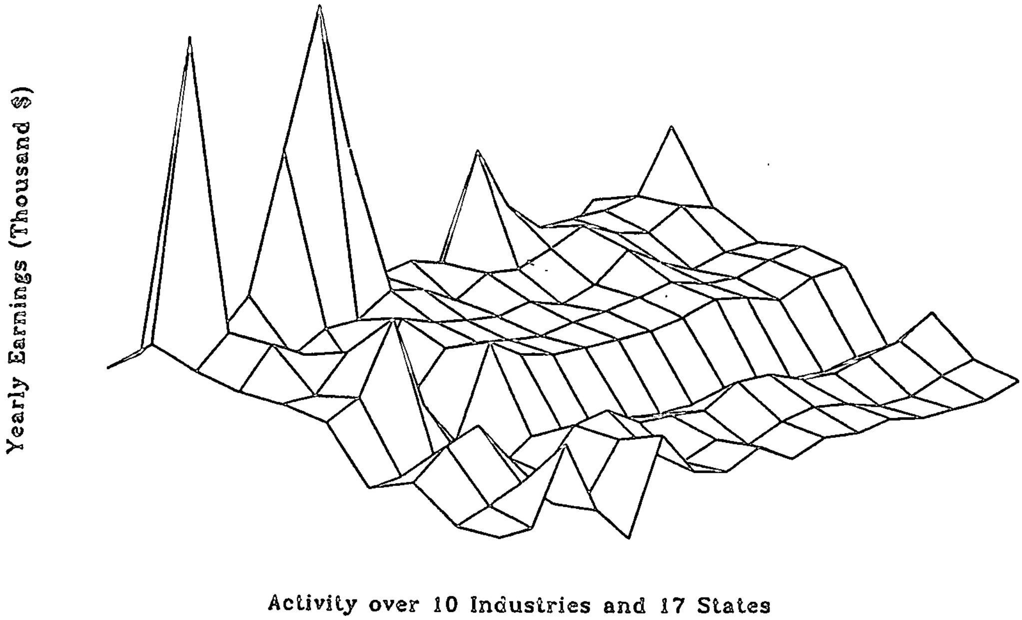


[Figure 2.c]



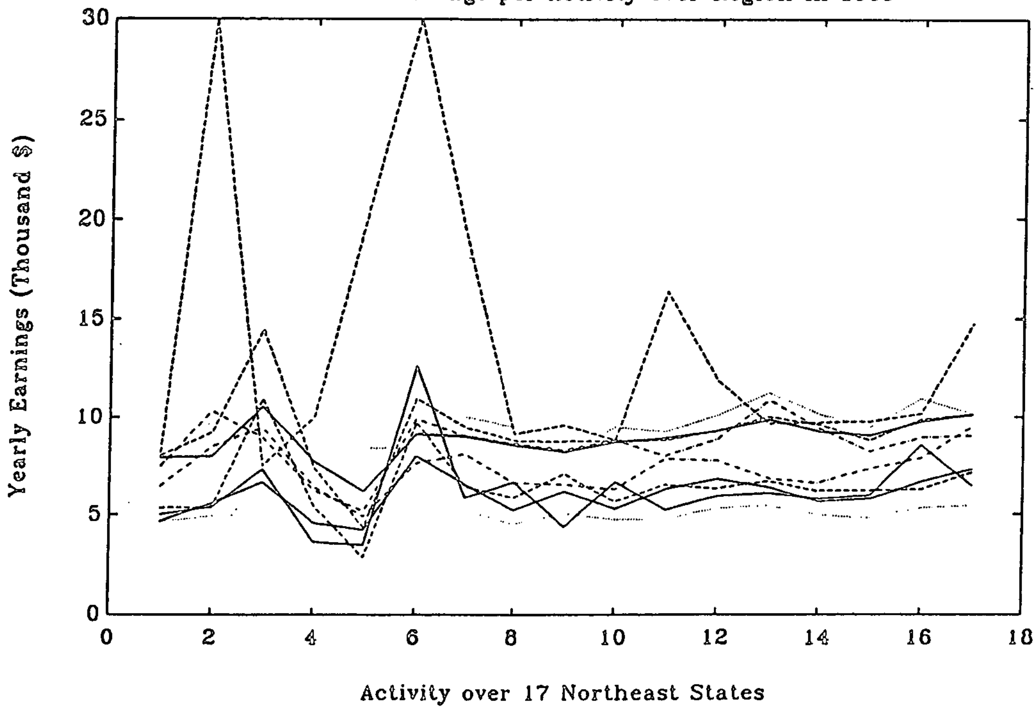
[Figure 3.a]

Yearly Earnings Surface over Industry and Region in 1969



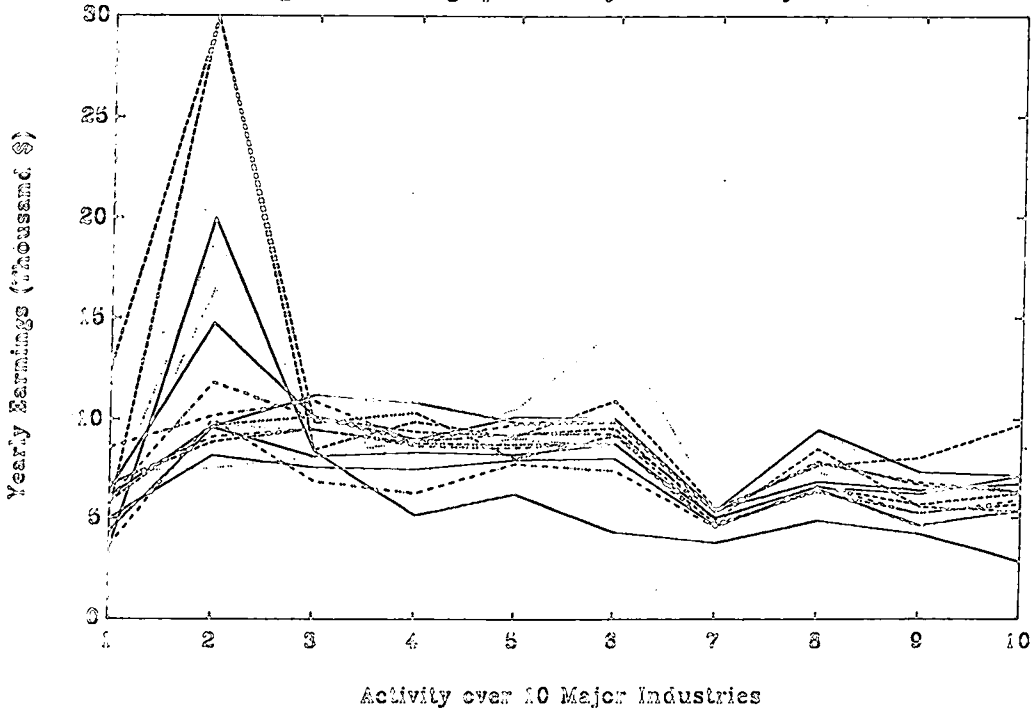
[Figure 3.b]

Industrial Earnings per Activity over Region in 1969



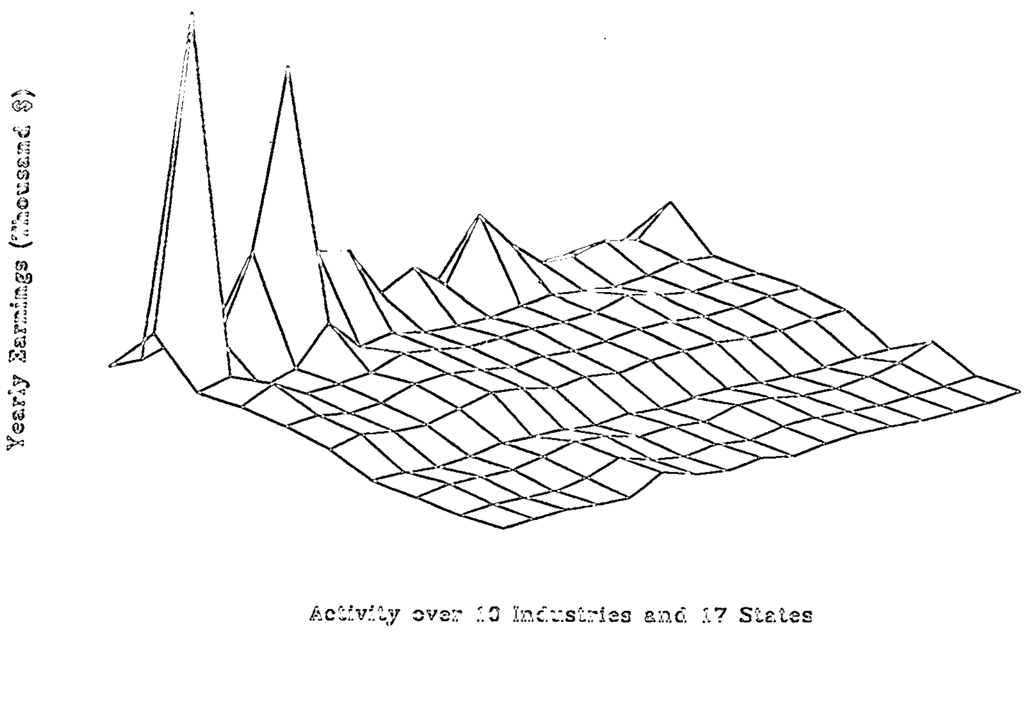
[Figure 3.c]

Regional Earnings per Activity over Industry in 1969

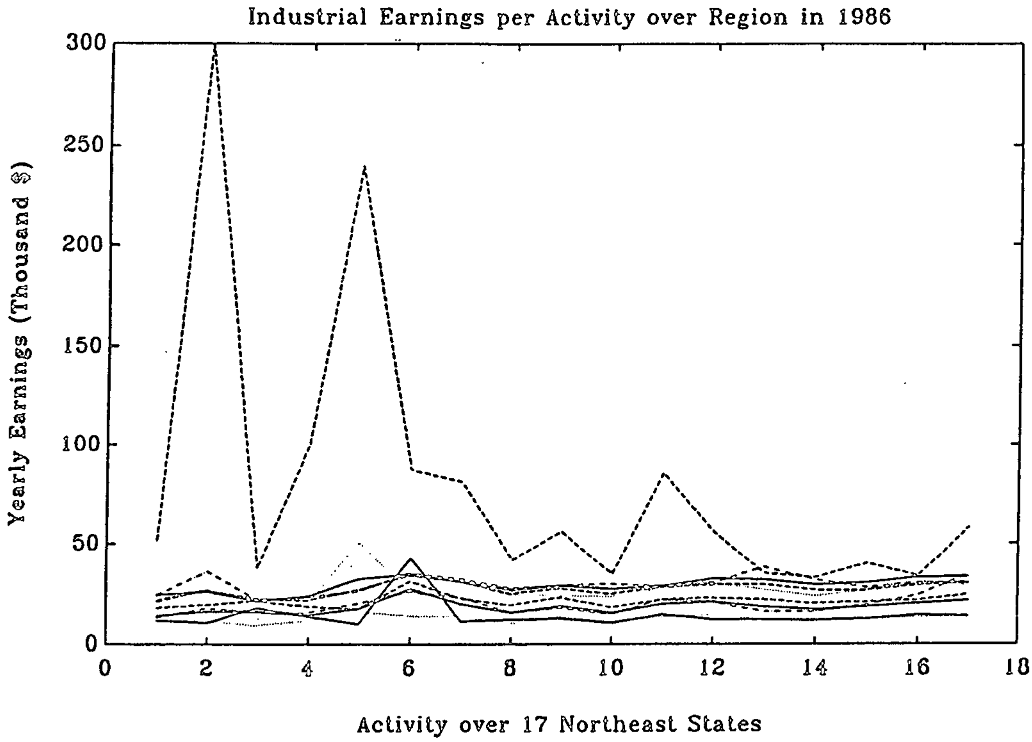


[Figure 4.a]

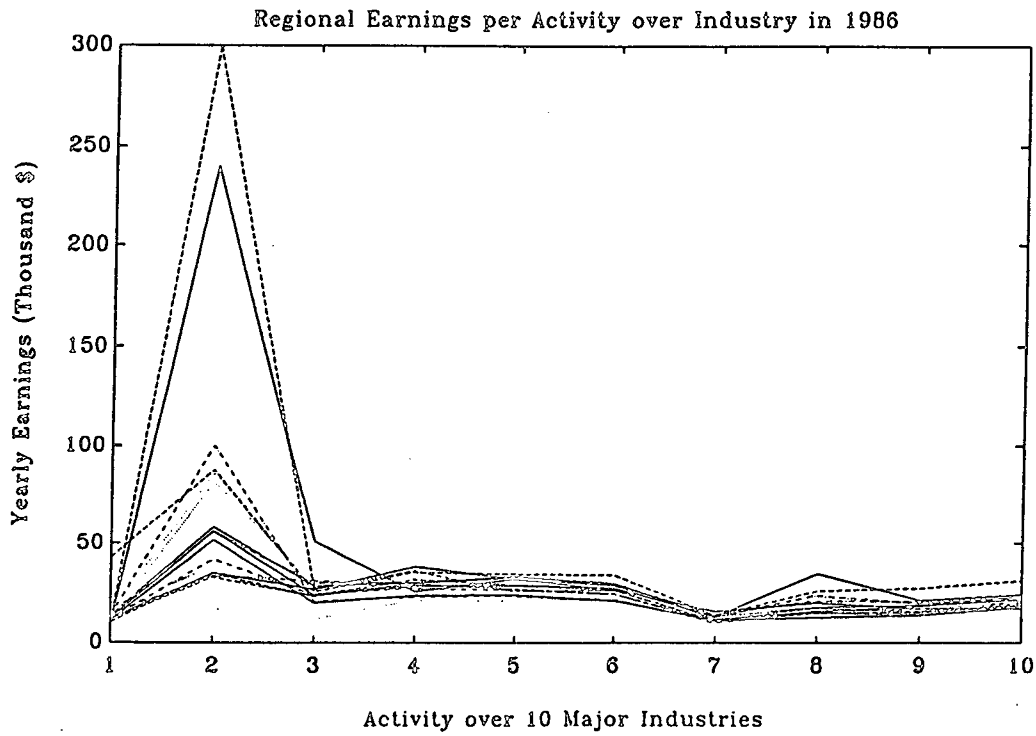
Yearly Earnings Surface over Industry and Region in 1986



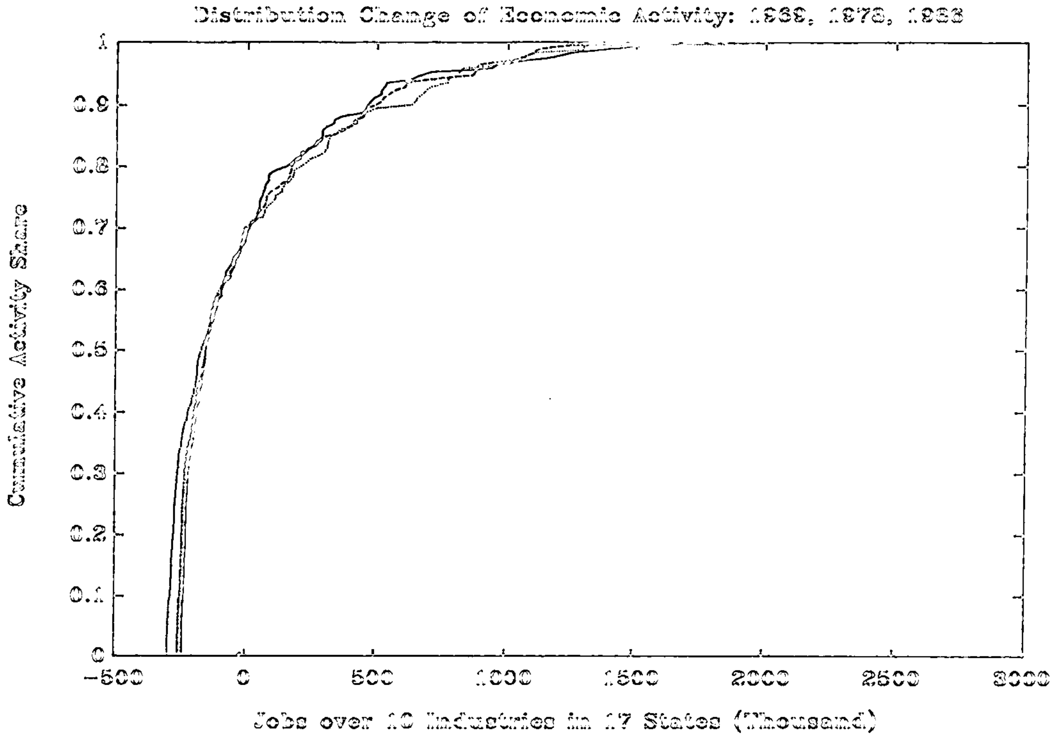
[Figure 4.b]



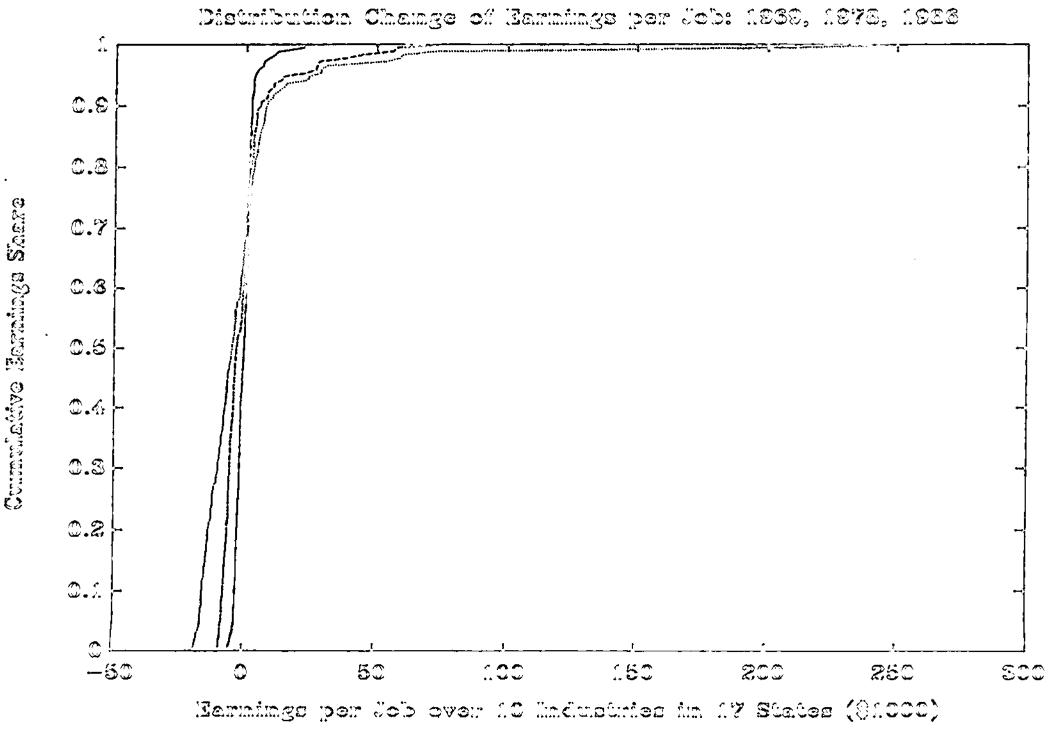
[Figure 4.c]



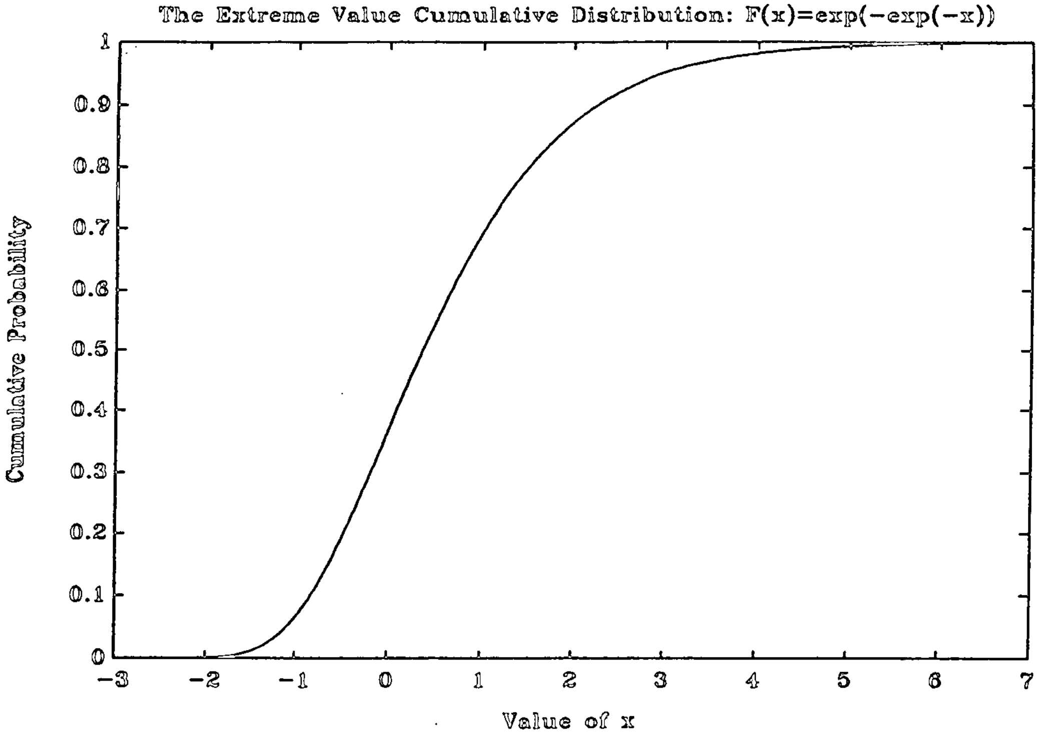
[Figure 5.a]



[Figure 5.b]



[Figure 6.a]



[Figure 6.b]