

# 다층건물의 비선형 반응해석을 위한 반응수정계수

## Response Scaling Factors for Nonlinear Response Analysis of MDOF System

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### 요 약

지진하중과 같은 동적인 하중에 대한 다자유도 구조물의 비선형 해석은 많은양의 계산을 요구한다. 이런 계산상의 어려움을 감소시키기 위하여 다자유도를 가진 복잡한 구조물의 비선형해석을 간략화된 동위 구조물(Equivalent Nonlinear System(ENS))을 이용해 구할 수 있는 약산법을 제시한다. 간단한 동위 구조물은 원 구조물의 가장 중요한 구조물의 성질을 가지고 있는데 구조물의 처음 두 개의 주기(natural periods)의 동적 특성 및 전체 항복변위(global yield displacement)를 가진다. 구조체 반응(response)으로 이 논문에서는 구조체의 전체변위 및 층간변위가 고려된다. 구조체의 전체 변위 및 층간변위를 얻기 위하여 전체 반응수정계수(global response scaling factor)  $R_G$ 와 국부 반응수정계수(local response scaling factor)  $R_L$ 을 동위 구조물로부터 얻어진 변위에 적용한다. 이 반응수정계수는 다자유도 구조물의 비선형 해석을 통하여 얻어진 변위들과 동위 구조물을 이용해 얻어진 변위들을 이용해 광범위한 회귀분석(regression analysis)을 통하여 구조물의 연성과 첫번째 두 모드의 질량참여계수의 함수형태로 얻는다. 반응수정계수를 가진 동위구조물을 만들기 위하여 철골 모멘트 연성 골조 방식의 구조물(Special Moment Resisting Steel Frames(SMRSF))을 이 논문에서는 고려한다. 함수형태로 표현된 반응수정계수는 동위구조물의 반응에 적용되어 복잡한 구조물의 비선형 반응을 얻을 수 있다.

### Abstract

Evaluating nonlinear response of a MDOF system under dynamic stochastic loads such as seismic excitation usually requires excessive computational efforts. To alleviate this computational difficulty, an approximation is developed in which the MDOF inelastic system is replaced by a simple nonlinear equivalent system(ENS). The ENS retains the most important properties of the original system such as dynamic characteristics of the first two modes and the global yielding behavior of the MDOF system. The system response is described by the maximum global(buliding) and local(interstory) drifts. The equivalency is achieved by two response scaling factors, a global response scaling factor  $R_G$ , and a local

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이 논문에 대한 토론을 1996년 3월 31일까지 본 학회에 보내주시면 1996년 9월호에 그 결과를 게재하겠습니다.

response scaling factor  $R_L$ , applied to the responses of the ENS to match those of the original MDOF system. These response scaling factors are obtained as functions of ductility and mass participation factors of the first two modes of structures by extensive regression analyses based on results of responses of the MDOF system and the ENS to actual ground accelerations recorded in past earthquakes. To develop the ENS with two response scaling factors, Special Moment Resisting Steel Frames are considered. Then, these response scaling factors are applied to the response of ENS to obtain the nonlinear response of MDOF system.

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## 1. INTRODUCTION

In seismic risk analysis of buildings and structures, the computational efforts required can become excessive since the seismic risk is usually evaluated based on repeated nonlinear response analyses of MDOF systems which are time consuming. In order to reduce the computational cost, many researchers tried to develop methods of estimating the nonlinear response of a MDOF system using a simple equivalent system.

Bazzurro and Cornell(1992), and Inoue and Cornell(1990, 1991) proposed an equivalent linear SDOF system(ELSS) which incorporated a nonlinear spectral reduction factor(F) and a MDOF response factor(C). This equivalent system has the same structural period as that of the first mode of the MDOF system being considered. The nonlinear spectral reduction factor(F) was originally developed by Kennedy, et al(1984). Given an earthquake which causes incipient yield in a structure, the factor F is the amount by which this earthquake must be scaled up in order to attain a specified damage level(e.g., a ductility ( $\mu$ ) of 3.). Several other researchers have also developed nonlinear spectral reduction factors using a slightly different procedure[for example, Riddell and Newmark(1979), Bertero(1986), and Nassar and Krawinkler(1991)]. Using an equivalent linear system with the nonlinear

spectral reduction factor F, nonlinear response spectral values for a MDOF system at a given damage level can be obtained. However, this response only includes the effect of the 1st mode of a MDOF system. In order to capture the effects of all the modes of a MDOF system, the use of a MDOF response factor(C) has been proposed by Inoue and Cornell(1990, 1991). This factor accounts for the difference between the linear response of MDOF system and that of the ELSS. The factor C is defined as the ratio of the maximum elastic response of the ELSS to the maximum elastic response of the MDOF system; the maximum MDOF and ELSS responses are normalized by their respective story drift capacities. Cornell and others(1989, 1992) also found that the mean values of the factors F and C are only slightly dependent on earthquake magnitude and source distance. Furthermore, they observed that the record to record variability is also small compared to the variabilities in spectral response quantities such as spectral acceleration. These findings imply that a moderate sample size of earthquake records is sufficient to evaluate these factors for any given structure, and these factors can be treated as deterministic values. Therefore, only the mean value or median value is needed to evaluate the risk of a structural limit state.

The analysis of the ELSS with the use of factors C and F, however, may not be satisfac-

tory in predicting the seismic risk associated with a nonlinear multi-degree-of-freedom system(NMS) if good accuracy is required. To improve the accuracy, more of the inelastic and dynamic response characteristics of the MDOF system need to be considered; these include the yielding displacement, the modal mass participation factors and mode shapes, etc. The use of the nonlinear spectral reduction factor( $F$ ) to scale and earthquake to attain a target damage level of different intensity is questionable since scaling a ground motion does not account for variations in ground motion characteristics(e.g. frequency content) which change with intensity. Furthermore, the use of the MDOF response factor( $C$ ) is also questionable for NMS since it is derived based on a linear elastic MDOF system. In risk analysis of a large number of structures, total computational effort for evaluating the factors  $F$  and  $C$  for each individual structure can be significant. In a reliability-based calibration of current seismic codes and provisions, such analyses are required; therefore, there is still a need for an equivalent system which is simple to use, accurately represents a NMS, and does not require excessive computational effort in response analysis.

In this paper, the concept of an Equivalent Nonlinear System(ENS) is used in conjunction with a global response scaling factor( $R_G$ ) and a local response scaling factor( $R_L$ ) for evaluating the seismic risk associated with a MDOF structure. The response of the ENS is obtained by considering the first two modes and the yield displacement of the MDOF structure. Response scaling factors can be defined as the correction factors needed to obtain the response of a NMS from that of an Equivalent Nonlinear System(ENS). To include the dependence of these factors on the

response level and structural system properties into consideration, the  $R_G$  and  $R_L$  are functions of the ductility ratio and ratio of sum of modal mass participation factors of the first two modes(RMP) to that of all modes of a structure. Their relationships are established by regression analyses of dynamic responses of MDOF system versus ENS under excitation of actual earthquake ground motions. To establish the functional relationships, seven structures and eighty eight real earthquake records are used.

## 2. EQUIVALENT NONLINEAR SYSTEM

An ENS is defined as the system which retains the first two modes of a MDOF structure(i.e., having the same natural periods, mass participation factors, and mode shapes). The ENS has a yield displacement equal to the global yield displacement associated with the MDOF structure. Global yield displacement is determined based on the results of a static nonlinear push-over analysis. The vertical distribution of lateral force used in the analysis is that used in Uniform Building Code(UBC). In the push-over analysis the lateral forces are proportionally increased, and the displacement at the top of the structure is monitored. A force-deflection diagram is then constructed as shown in Figure 1. The resulting nonlinear force-deflection relationship is approximated by a bilinear one, and the yield displacement is the displacement corresponding to the intersection point of the two lines. DRAIN-2DX, developed by R. Allahabadi and G. H. Powell (1988) for the analysis of inelastic two dimensional structures under static and dynamic loadings, was used to perform this analysis. Figure 1 also shows how the global yield displacements were determined for a

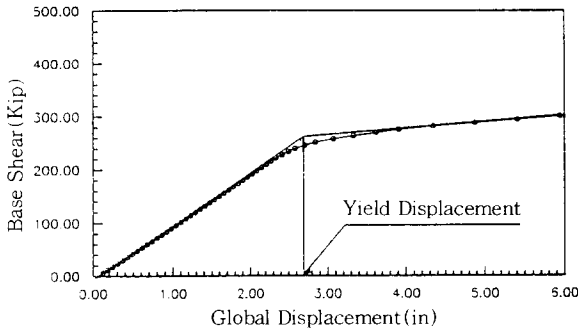


Fig. 1 Global yield displacement for two story SMRSF

structure.

Using the first two modes of a MDOF structure, two equations of motion, which allow for inelastic responses, can be established. Both modes have the same global yield displacement and restoring force model (e.g., elasto-plastic model). An analytical restoring force model is determined based on the type of structural systems, the type of material, etc. For example, an elasto-plastic or bi-linear model is often used to represent the restoring force characteristics of steel frames (Wakabayashi, 1986). Given an earthquake time history, the displacement is calculated from the equation of motion of each mode at each time step. The displacements of two modes are then combined by modal superposition. Although modal superposition is only valid for linear system, it can be used to approximate the displacement of NMS. The accuracy of this procedure is discussed in the companion paper. To evaluate the maximum global displacement  $U_E$  using ENS, the following procedure is used.

- 1) Design the MDOF structure based on a seismic code (e.g., UBC)
- 2) Calculate the natural periods ( $T_1, T_2$ ), mass participation factors ( $\Gamma_1, \Gamma_2$ ), and the normalized mode shapes ( $\phi_1, \phi_2$ ) of the first two

modes. Note that the mode shapes are normalized so that  $\phi_i M \phi_i = 1$ . ( $M$  is the mass matrix.)

3) Find the global yield displacement ( $U_y$ ) of the MDOF system using static nonlinear push over analysis

4) Evaluate the displacement of the first two modes of the MDOF system using the following nonlinear dynamic equation :

$$\ddot{q}_i(t) + f[q_i(t), \dot{q}_i(t), \alpha, \zeta] = -\Gamma_i \ddot{u}_g(t) \quad i=1, 2 \quad (1)$$

where  $\alpha$  is the strain hardening ratio,  $\zeta$  is the viscous damping ratio, and  $\ddot{u}_g$  is the ground acceleration. (note that each mode has same global yield displacement.)

5) Evaluate the maximum global displacement by combining responses obtained from solving equation (1) as follows :

$$U_E = \max \left( \sum_{i=1}^2 \phi_{in} x q_i(t) \right) \quad (2)$$

where  $\phi_{in}$  is the element of the mode shape vector corresponding to the horizontal displacement of the top floor in the  $i$ th mode. [Note : Since axial deformation of the beams was not considered, all nodes at the top floor have the same horizontal displacement]

### 3. DESIGNING REPRESENTATIVE STRUCTURES FOR CALIBRATING $R_G$ AND $R_L$

In this paper, seven typical office buildings (Occupancy importance factor is 1.0) are designed as representative structures. These structures are assumed to be located at the highest seismic zone in UBC (Zone 4). The soil condition on which buildings are located is the soil profile with deep cohesionless or stiff clay conditions where soil depth exceeds 200

feet(Soil type 2). The ranges of the dimensions of a structure are limited to one to four bays and one to twelve stories. The design seismic force is determined in accordance with the provisions of UBC. This study concentrates on buildings having lateral resistance provided by special moment resisting steel frames(SMRSF) located on the perimeter of a structure. These SMRSFs are designed according to UBC and the AISC Allowable Stress Design manual using IGRESS-2(computer software for analysis and design developed at University of Illinois at Urbana-Champaign). The properties of representative structures are shown in Table 1.

Table 1. Properties of representative structures

Structure No.	No. of Stories	No. of Bays	Span Length(m)	Height of 1st Story(m)	Height of stories(m)
1	1	1	8	5	
2	2	3	8	5	4
3	5	4	9	5	4
4	5	3	9	5	4
5	5	5	12	5	4
6	9	3	8	5	4
7	12	3	9	5	4

Table 2. Properties of representative structures

Structure No	$T_1$	$T_2$	$\phi_{1n}$	$\phi_{2n}$	$\Gamma_1$	$\Gamma_2$	RMP	$\xi_{\%}$	$U_{yin}$	$h_n$
1	0.619	0.059	1.13	0.00	0.88	0.00	1.00	5	1.3	15
2	0.914	0.355	-0.74	0.45	-1.66	-0.51	0.99	5	2.7	28
3	1.550	0.557	-0.42	-0.42	-3.26	1.31	0.71	5	6.4	67
4	1.540	0.558	-0.43	0.42	-3.25	-1.32	0.69	5	6.6	67
5	1.560	0.526	-0.43	0.42	-3.29	-1.31	0.68	5	6.8	67
5	2.330	0.831	0.46	0.46	-3.39	-1.29	0.61	5	9.1	119
7	2.700	0.962	0.39	0.39	-3.91	-1.53	0.58	5	12.4	158

In the dynamic analysis of each of these structures, two important parameters which must be specified are the damping ratio and strain hardening ratio. Five percent damping is used in all dynamic analysis. This appears to be consistent with the damping levels referenced in current seismic codes and pro-

visions for buildings(SEAOC, UBC, NEHRP). In the dynamic analysis involving the modeling of individual members of the frames, a strain hardening ratio of 5% is assumed. However, in the ENS analyses, as strain hardening ratio of 10% is used to represent the strain hardening effects on a global, or system level. Osteraas and Krawinkler(1990) note that a strain hardening ratio of 10% for a structural system may be on the conservative side for many structures.

#### 4. EARTHQUAKES USED FOR CALIBRATING $R_G$ AND $R_L$

A suite of eighty-eight real earthquake records are used in calibrating  $R_G$  and  $R_L$ . Eighty two of the records were recorded in North America, and six earthquakes were recorded in Japan. Among the earthquake records from North America, 10 records are obtained from North Ridge earthquake (California Department of Conservation, 1994) and 66 earthquake records are obtained from the U.S. Geological Survey digital data series, DDS-7, CD-Rom(1992). This USGS data base provides uncorrected accelograms of earthquakes which occurred in North America and Hawaii from 1933 to 1986. Basic Strong Motion Accelogram Processing Software (BAP) is used for correcting the earthquake records extracted from the CD-Rom. The correction procedures were similar to those outlined in the report by Naeim and Anderson (1993). The magnitudes of the earthquakes range from 4.4 to 8.1, and the peak ground acceleration range from 0.03g to 1.17g. The source distances range from 0km to 400km.

#### 5. RESPONSE SCALING FACTORS( $R_G$ AND $R_L$ )

The response scaling factors can be con-

sidered as correction factors to be applied to the response of the ENS to obtain the comparable response of a NMS. In a reliability-based code calibration, many prototype structures need to be considered when evaluating the risk associated with current seismic code designs or calibrating the coefficients in these codes based on reliability. In order to reduce this computational burden, this study aims at establishing a functional form of the response scaling factors based on regression analysis ; the response scaling factors are assumed to be functions of ductility and RMP. Seven(7) representative structures and eighty eight(88) real earthquake records mentioned earlier are used for this purpose. Once the functional form of the factors  $R_L$  and  $R_G$  are established, the response of a NMS can be evaluated without performing a nonlinear dynamic response analysis of a MDOF system.

6. GLOBAL RESPONSE SCALING FACTOR( $R_G$ )

The global response scaling factor( $R_G$ ) is defined as the ratio of the maximum displacement of ENS( $U_E$ ) to the maximum global displacement(at the top) of a MDOF structure ( $U_G$ ) :

$$R_G = \frac{U_E}{U_G} \tag{3}$$

Figures 2-4 show  $R_G$  vs. ductility( $\mu_E = U_E/U_y$ ) for ENS corresponding to 2, 5, and 12 story structures. In each figure, there are eighty eight  $R_G$  versus  $\mu_E$  data points for each representative structures since eighty eight different earthquakes are used. Hence, 616  $R_G$  versus  $\mu_E$  data points are used for establishing the functional form of the global response scaling factor,  $R_G$ . As shown in Figures 2-4, for

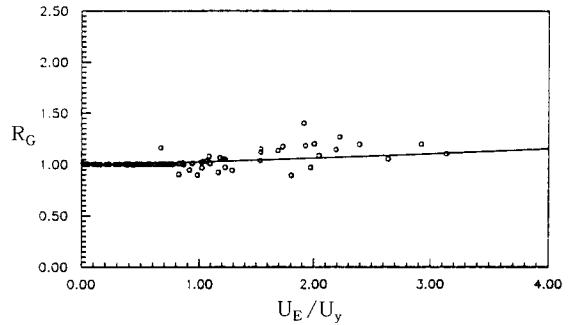


Fig. 2 Global response scaling factor  $R_G$

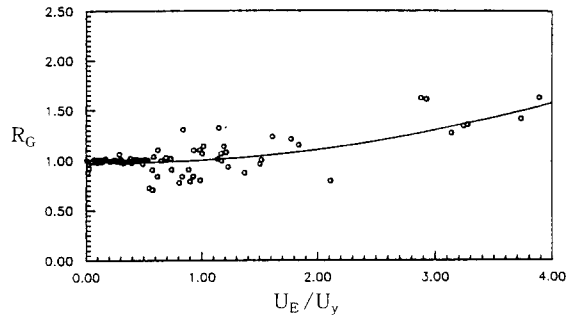


Fig. 3 Global response scaling factor  $R_G$  for five story SMRSF 1

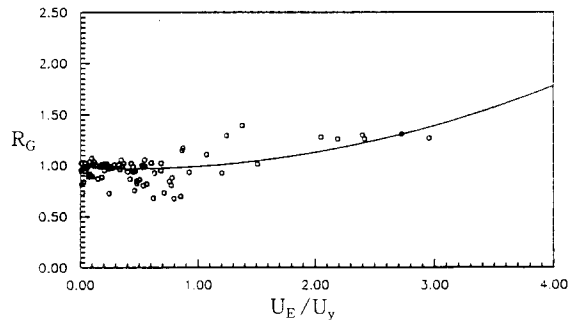


Fig. 4 Global response scaling factor  $R_G$  for twelve story SMRSF

a given value of ductility( $\mu_E$ ), there appears to be only a small amount of scatter. However, considering the entire range of ductility values, there is significant scatter. This suggests that  $R_G$  is a function of ductility( $\mu_E$ ). As two stage regression analysis is carried out in 2D domain. In the first stage, the function

for  $R_G$  vs. ductility( $\mu_E$ ) is regressed for discrete values of RMPs(Seven discrete points for seven representative structures), and then the effect of the RMP is evaluated at the second stage. The dependence of  $R_G$  on  $\mu_E$  is modeled by second order polynomial as follows :

$$R_G = C_0 + C_1\mu_E + C_2\mu_E^2 \quad (4)$$

in which coefficients  $C_0$ ,  $C_1$  and  $C_2$  are functions of the RMP to be determined by linear regression analysis. The results of the two-stage regression analysis are :

$$C_0 = 0.9695 + 0.0178\Gamma \quad (5)$$

$$C_1 = -0.1664 + 0.2016\Gamma \quad (6)$$

$$C_2 = 0.1473 - 0.1467\Gamma \quad (7)$$

$$\Gamma = \frac{|\Gamma_1| + |\Gamma_2|}{\sum_{i=1}^{\#nodes} |\Gamma_i|} \quad (8)$$

where  $\Gamma$  represents the RMP. For the buildings considered in this study, RMP decreases as the number of stories of a structure increases since higher mode effects become more significant for taller buildings.  $\Gamma_i$  is the mass participation factor of the  $i$  th mode of a structure. In Figures 2-4, the solid line represents the values from the regressed function of global response scaling factor(Eq. (4)). For ductilities( $\mu_E$ ) less than 0.5, the coefficient of variation(COV) of the factor  $R_G$  is small ; hence, the variability is neglected as an approximation. For ductility values larger than 0.5, COV of  $R_G$  for 1, 2, 5, 9 and 12 story structures are evaluated, which are 5%, 6%, 10%, 11% and 10% respectively.

### 7. LOCAL RESPONSE SCALING FACTOR( $R_L$ )

In addition to achieve equivalence in global response, the local response which is closely related to structural and non-structural damage is also important and needs to be considered. The local limit state corresponding to the exceedence of an interstory drift threshold of a structure is used for this purpose. Local limit states are more likely to occur than global limit states, since damage does not spread throughout the entire structure during an earthquake. Certain stories may be damaged more than others even if a structure is well designed according to current seismic codes. This phenomena is accounted for by the local response scaling factor,  $R_L$ . The factor  $R_L$  is the ratio of the global ductility( $\mu_G$ ) to maximum local ductility( $\mu_{Li}$ ). Local ductility( $\mu_{Li}$ ) is defined as the ratio of the maximum inter story drift of  $i$ th floor to the story yield displacement. The story yield displacement of  $i$ th floor(local yield displacement) is obtained by the same procedure as used for the global yield displacement( $U_y$ ). The factor  $R_L$  can be written as follows :

$$R_L = \frac{(U_G / U_y)}{\max_i(U_{Li} / U_{yi})} = \frac{\mu_G}{\max_i(\mu_{Li})} \quad (9)$$

where  $U_{Li}$  is the maximum inter story drift and  $U_{yi}$  is the yield displacement of the  $i$ th story (local yield displacement).  $U_G$  is the maximum displacement at the top of the structure and  $U_y$  is global yield displacement.

Figures 5-7 show  $R_L$  vs. global ductility( $\mu_G = U_G / U_y$ ) for each of 2, 5, and 12 story structures subjected to the 88 earthquakes. (For one story SMRSF,  $R_L$  factor is not needed.) Hence, for each representative structure, there are eighty-eight data points. for

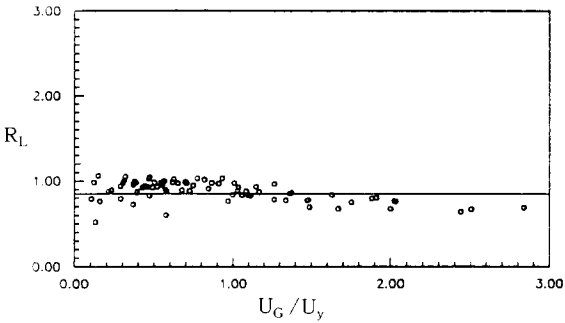


Fig. 5 Local response scaling factor  $R_L$  for two story SMRSF

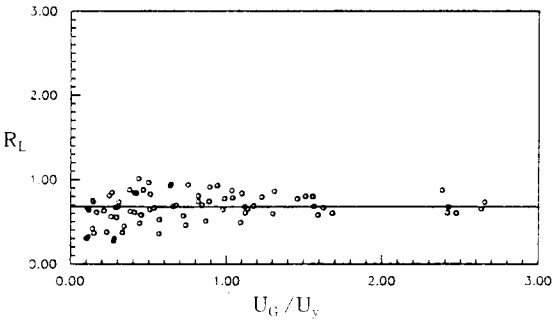


Fig. 6 Local response scaling factor  $R_L$  for five story SMRSF

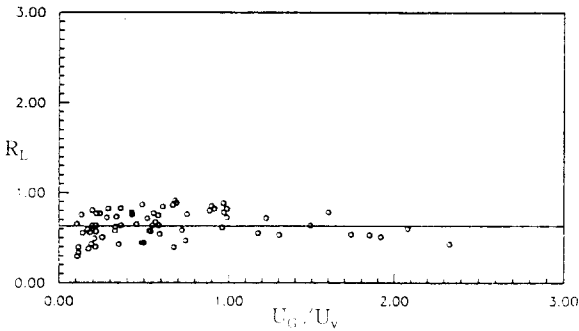


Fig. 7 Local response modification factor  $R_L$  for twelve story SMRSF

establishing the functional form of the local response scaling factor,  $R_L$  by regression analysis, 616 data points were evaluated. As shown in Figures 5-7,  $R_L$  does not appear to be strongly dependent on ductility( $\mu_G$ ). However,

Figure 3.27 indicates that  $R_L$  does vary with respect to RMP. The following regressed function for the local response scaling factor is obtained by a two step regression analysis :

$$R_L = 0.3627 + 0.4774\Gamma \quad (10)$$

where  $\Gamma$  represents the RMP. In Figures 5-7, the solid line represents the values from the regressed function of RMP(Eq.(10)).

### 8. CONCLUSIONS

In this paper, the equivalent nonlinear system(ENS) is developed to evaluate the responses of nonlinear MDOF systems. In order to obtain the comparable global and local responses with those of nonlinear MDOF systems, global and local response scaling factor is derived which is the functional form of RMP and ductility. Figures 2-4 show that the functional form of global response scaling factor has been derived with good precision(see solid line in Figures 2-4). Also, figures 3-5 show the same conclusion for the functional form of local response scaling factor. Specially when one tries to evaluate the seismic risk associated with current seismic code procedures and calibrate the design parameters in the seismic codes and provisions, this equivalent nonlinear system with response scaling factors eliminate the computational difficulties. The validity of ENS using global and local response scaling factors will be verified in the companion paper.

It is noted that only regular SMRSF are considered in this study. The methodology may be extended to other structural systems, such as Ordinary Moment Resisting Frame (OMRF), Concentric Braced Frame(CBF), and Eccentric Braced Fame(EBF), etc. Also, ENS may need to be modified when it is ap-



plied to irregular structures.

#### REFERENCES

- [ 1 ] Allahabadi, R., and Powell, G. H., *DRAIN-2DX, Computer-Program*, UCB /EERC-88 /06, EERC, College o Engineering, University of California, Berkeley, California, 1988.
- [ 2 ] Bazzurro, P., and Cornell, C. A., "Seismic Risk: Nonlinear MDOF Structures," *Proceedings of 10th World Conference of Earthquake Engineering*, Balkema, Rotterdam, Vol.1, 1992, pp.563-568.
- [ 3 ] Bertero, V. V., *Lessons Learned from Recent Earthquakes and Research and Implementation for Earthquakes-Resistant Design of Building Structures in United States*, Report No. UCB /EERC 86-03, EERC, University of California, Berkeley, California, 1986.
- [ 4 ] Inoue, T., and Cornell, C. A., "Seismic Hazard Analysis of MDOF Structures," *Proceedings of ICASP*, Mexico City, Mexico, 1991.
- [ 5 ] Inoue, T., *Seismic Hazard Analysis of MDOF Structures*, Engineering Degree Dissertation, Department of Civil Engineering, Stanford University, Standford, California, 1990.
- [ 6 ] International Conference of Building Officials (ICBO), *Uniform Building Code*, 1991.
- [ 7 ] Nassar, A. A., and Krawinkler, H., *Seismic Demands for SDOF and MDOF Systems*, Report No. 95, Department of Civil Engineering, Stanford University, Stanford, California, 1991.
- [ 8 ] Osteraas, J. D., and Krawikler, H., *Strength and Ductility Consideration in Seismic Design*, Report No. 90, Department of Civil Engineering, Stanford University, California, 1990.
- [ 9 ] Riddel, R., Newmark, N. M., and Hall, W. J., *Statistical Analysis of the Response of Nonlinear Systems Subjected to Earthquakes*, Structural Research Series No. 486, Department of Civil Engineering, University of Illinois, Urbana, Illinois, 1979.
- [10] Wakabayshi, M., *Design of Earthquake Resistant Buildings*, McGRAW Hill, 1986.

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