병렬전단벽 구조물의 변환부분의 유한요소해석을 위한 보-변환요소의 개발

Beam Transition Elements for Finite Element Analysis of Transition Regions of Coupled Wall Structures

金 皓 洙*
Kim, Ho-Soo

요 약

본 연구는 竝列剪斷壁 構造物의 變換部分의 효율적인 有限要素解析을 위한 보-變換要素 및 變換部分의 요소를 제시하고자 한다. 먼저 보-變換要素는 보요소와 벽체요소사이의 변형 및 힘의 拘束條件을 근거로하여 보의 기본적인 거동을 동일하게 유지하면서 平面應力要素의 개념으로 대치된 유사보요소로 간주될 수 있으며, 이는 變換部分에서의 보요소와 벽체요소사이의 서로 다른 自由度에 기인한 變形의 不適合性을 합리적으로 해결해준다. 또한 보-變換要素와 직접 연결되는 變換部分의 요소는 보-變換要素의 경우와 동일한 拘束條件이 적용됨으로써 變換部分에 대한 효율적인 要素分割方案을 제시해 준다. 이와 같이 본 연구에서 제시된 요소들은 기본적으로 竝列剪斷壁 構造物 뿐만아니라 보요소와 벽체요소의 相互作用이 고려되는 모든 구조물에 효율적으로 활용될 수 있다.

Abstract

This study presents the formulation of beam transition elements and transition zone elements for the effective finite element analysis of the transition regions of coupled wall structures. Beam transition element can be described as the quasi beam element which is replaced by an equivalent plane stress element, keeping equally, the basic behavior of beam element, based on the kinematic and force constraints between beam and wall element. These beam transition elements solve the incompatibility related to different degrees of freedom between beam and wall element in transition regions. Also, the stiffness matrices of transition zone elements which are directly connected with beam transition elements in transition regions can be derived from the equivalent constraint conditions. These elements provide the reasonable mesh grading schemes for transition regions and can be usefully applied to the transition regions of all structures that the interactions of wall and beam element are considered.

* 청주대학교 건축공학과 조교수

이 논문에 대한 토론을 1995년 12월 31일까지 본 학회에 보내주시면 1996년 6월호에 그 결과를 게재하겠습니다.

1. INTRODUCTION

Coupled wall structures or shear wall-frame structures are frequently used as the structural systems of tall buildings. In analyzing such structures, the transition region in which connecting beam and shear wall are interconnected is normally of considerable interest and is often the weakest area. In general, wall element which is treated as a plane stress element in finite element analysis has two translational degrees of freedom per node, whereas beam element has two translational and one rotational degrees of freedom per node. Incompatibility related to different degrees of freedom between beam, and wall element raises many problems in analyzing the transition region. Therefore, the reasonable connection systems between beam and wall element are required to obtain the reliable analytical results for transition regions.

Many studies have been proposed to solve this incompatibility. Beck and Coull & Choudhury, etc. have presented the continuous medium method in which connecting beams can be replaced by the equivalent laminas.[7],[8] MacLeod and Weaver, etc. have presented the rectangular finite element with additional one or two rotational degrees of freedom per node to satisfy the compatibility with beam element.[9],[13] Also, frame analogy method has been proposed under the concept in which shear walls can be replaced by the equivalent columns and beams.[10]

These conventional methods show many problems such as the phenomenon of stress concentration in the connections, the error in stress redistribution due to the inaccuracy in modeling the structure, and the occurrence of additional moments due to the inconsistency between the center line of beam element and the mesh line of wall element, etc.

This study presents the concept that beam and plane stress element with different degrees of freedom have very close relationship and can be replaced each other. By this concept, the stiffness matrix of beam transition element with the nodal types of plane stress element can be derived from the exact stiffness matrix of Hermitian beam element, based on the kinematic and force conditions. Therefore, this beam transition element can be described as the quasi-beam element which is replaced by an equivalent plane stress element, keeping equally the basic behavior of beam element.

2. STIFFNESS MATRIX FORMULATION OF BEAM TRANSITION ELEMENTS

2.1 Kinematic and force constraint equations

The close relationship between these elements should be recognized though beam and plane stress element have been considered separately in finite element analysis. Two basic assumptions have been considered in beam formulation ; the first is kinematic assumption that plane sections initially normal to the neutral axis remain plane after deformation, and the second is force assumption that stresses normal to the neutral axis are zero.

We can derive the concept that the degrees of freedom of beam element can be replaced by the degrees of freedom of the equivalent plane stress element, namely, beam transition element by considering constraint conditions based on these two assumptions.

In general, a beam element model in plane structures is considered as Fig.1 and the equivalent beam transition elements are given as Fig.2. Here, the beam transition elements are classified as three types(BTRAN 1, 2, 3) ac-

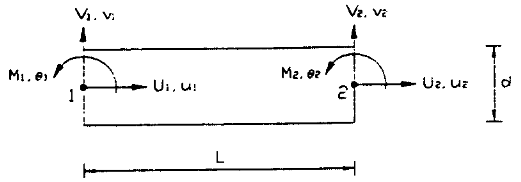
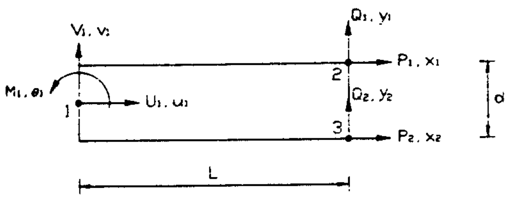
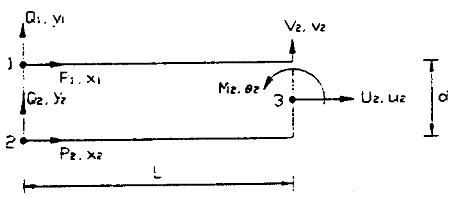


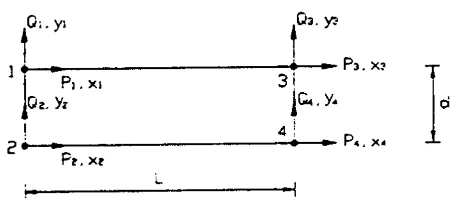
Fig. 1 Beam element



(a) 3-node BTRAN 1



(b) 3-node BTRAN 2



(c) 4-node BTRAN 3

Fig. 2 Beam transition elements

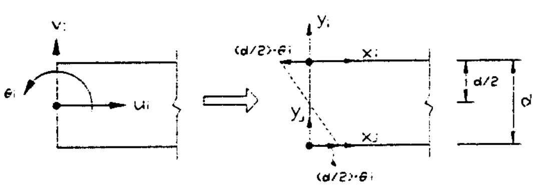


Fig. 3 Kinematic constraint conditions

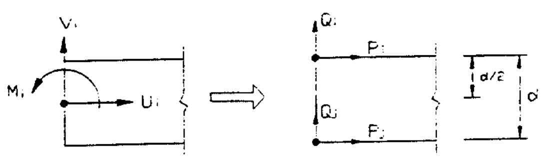


Fig. 4 Force constraint conditions

cording to the location and number of equivalent nodes when the nodes of beam element is replaced by the nodes of beam transition element. Also, Fig.3 and 4 show the comparisons of the basic behaviors according to the kinematic and force constraints between beam element and beam transition element.

In particular, in order to constitute the constraint equations from Fig.3 and 4, the assumptions should be considered that the neutral axis of beam element locates in the center and the upper vertical displacement is equal to the lower vertical displacement in both ends of beam element in case that its depth is not large.

From Fig.3., the kinematic constraint conditions are given that the rotational degree of freedom of beam element corresponds to the couple of horizontal degrees of freedom in the upper and lower ends of beam transition element, and the horizontal degree of freedom of beam element corresponds to the average value of horizontal degrees of freedom in the upper and lower ends of beam transition element. Thus, we can obtain the following kinematic constraint equations.

$$x_i = u_i - (d/2) \times \theta_i \quad (1)$$

$$x_j = u_i + (d/2) \times \theta_i \quad (2)$$

$$y_i = y_j = v_i \quad (3)$$

These equations can be written as

$$\theta_i = (x_j - x_i) / d \quad (4)$$

$$u_i = (x_i + x_j) / 2 \quad (5)$$

$$y_i = y_j = v_i \quad (6)$$

Also, from Fig.4, the force constraint conditions are given that the moment of beam element corresponds to the couple of the horizontal forces in the upper and lower ends of beam transition element, and the horizontal force of beam element corresponds to the sum of the horizontal forces in the upper and lower ends of beam transition element.

Thus, force constraint equations can be written as

$$U_i = P_i - P_j \tag{7}$$

$$V_i = Q_i \tag{8}$$

$$M_i = (P_j - P_i) \times d/2 \tag{9}$$

2.2 Formulation of transformation matrix and beam transition element matrix

Transformation matrices $[T_i]$ for each type of beam transition elements can be derived from the constraint equations. Also, beam transition element stiffness matrix $[K_i]$ can be constituted as eqn(10), considering the transformation matrix $[T_i]$ and the beam element stiffness matrix $[K_B]$. Also, $[R_i]$ and $[D_i]$ represant the force matrix and displacement matrix, respectiochy.

$$\begin{aligned}[R_i] &= [T_i]^T [K_B] [T_i] [D_i] \\ &= [K_i] [D_i]\end{aligned} \tag{10}$$

where

$$[K_B] = \begin{bmatrix} S_1 & 0 & 0 & -S_1 & 0 & 0 \\ & S_2 & S_3 & 0 & -S_2 & S_3 \\ & & S_4 & 0 & -S_3 & S_5 \\ & & & S_1 & 0 & 0 \\ & \text{symm.} & & & S_2 & -S_3 \\ & & & & & S_4 \end{bmatrix}$$

$S_1 = EA/L,\quad S_2 = 12EI/(L^3\beta),\quad S_3 = 6EI/(L^2\beta)$
$S_4 = (4EI/L - \alpha EI/L)/\beta$
$S_5 = (2EI/L - \alpha EI/L)/\beta$
$\alpha = 12EI/(A_s GL^2),\quad \beta = 1 - \alpha$

α : Shear Deformation Factor
A_s : Effective Shear Area
G : Shear Modulus

In particular, a beam element has six degrees of freedom in two-dimensional structures. Here when a beam element is replaced by an equivalent plane stress element, namely, a beam transition element, the number of total degrees of freedom increases according to the additional nodes. Since beam transition element was primarily derived from beam element with six degrees of freedom, as many constraint conditions as correspond to the additional degrees of freedom should be added to the stiffness matrix formulation of beam transition element. Therefore, all matrices of eqn (10) for each beam transition element can be given as follows, considering these constraint conditions.

1) In case of BTRAN 1

It can be assumed that y_2 is dependent on y_1 from eqn(6). Also, from eqn(8), V_2 corresponds to Q_1 in node 2 of Fig.2(a) and Q_2 in node 3 is ignored.

$[K_1]$ = stiffness matrix of BTRAN 1
$[R_1] = [U_1\ V_1\ M_1\ P_1\ Q_1\ P_2]^T$
$[D_1] = [u_1\ v_1\ \theta_1\ x_1\ y_1\ x_2]^T$

$$[T_1] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1/d & 0 & 1/d \end{bmatrix}$$

2) In case of BTRAN 2

It can be assumed that y_2 is dependent on y_1 from eqn(6). Also, from eqn(8), V_1 corresponds to Q_1 in node 1 of Fig.2(b) and Q_2 in node 2 is ignored.

$[K_2]$ = stiffness matrix of BTRAN 2
$[R_2] = [P_1\ Q_1\ P_2\ U_2\ V_2\ M_2]^T$
$[D_2] = [x_1\ y_1\ x_2\ u_2\ v_2\ \theta_2]^T$

$$[T_2] = \begin{bmatrix} 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -1/d & 0 & 1/d & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

3) In case BTRAN 3

Similarly, it can be assumed that y_2 is dependent on y_1 and y_4 is dependent on y_3 respectively. Also, V_1 corresponds to Q_1 in node 1 of Fig.2(c) and V_2 corresponds to Q_3 in node 3 respectively, and also Q_2 in node 2 of Fig.2(c) and Q_4 in node 4 are ignored.

[K_3] = stiffness matrix of BTRAN 3

[R_3] = $[\ P_1\ Q_1\ P_2\ P_3\ Q_3\ P_4\ ]^T$

[D_3] = $[\ x_1\ y_1\ x_2\ x_3\ y_3\ x_4\ ]^T$

$$[\ T_3\ ] = \begin{bmatrix} 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -1/d & 0 & 1/d & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1/d & 0 & 1/d \end{bmatrix}$$

3. FORMULATION OF TRANSITION ZONE ELEMENT

In case that we use the beam transition element stiffness matrix due to constraint conditions in transition region, the equivalent constraint conditions should be considered within the stiffness matrix of wall element, namely, transition zone element which is directly connected with this beam transition element. Therefore, we can reconstitute the stiffness matrix of transition zone element, based on constraint conditions as in chapter 2.

$$[\ K_i\ ] = [\ T_i\ ]^T\ [\ k_i\ ]\ [\ T_i\ ] \qquad (11)$$

where

- $[k_i]$: Stiffness matrices of plane stress elements
- $[T_i]$: Transformation matrices based on kinematic constraints
- $[K_i]$: Constrained stiffness matrices of transition zone elements

In particular, transformation matrices $[T_i]$ in eqn(11) are given as follows, according to the connection conditions with beam transition elements.

3.1 Transition zone elements connected with BTRAN 1

First, since $v_3 = v_2$ as shown in Fig.5(a), it can be assumed that v_3 is dependent on v_2. In this case, transformation matrix $[T_1]$ can be written as

$$\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ u_4 \\ v_4 \end{Bmatrix} = [\ T_1\ ] \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ u_4 \\ v_4 \end{Bmatrix} \qquad (12)$$

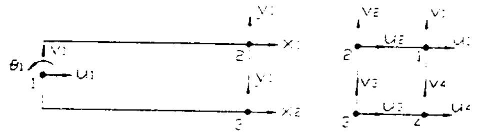
Secondly, since $v_5 = v_2$ as shown in Fig.5(b), it can be assumed that v_5 is dependent on v_2. In this case, transformation matrix $[T_2]$ can be written as

$$\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \end{Bmatrix} = [\ T_2\ ] \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \end{Bmatrix} \qquad (13)$$

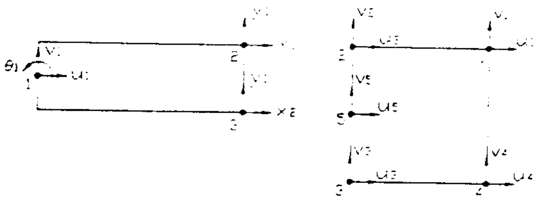
3.2 Transition zone elements connected with BTRAN 2

First, since $v_4 = v_1$ as shown in Fig.6(a), it can be assumed that v_4 is dependent on v_1. In this case, transformation matrix $[T_3]$ can be written as

$$\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \end{Bmatrix} = [\ T_3\ ] \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \end{Bmatrix} \qquad (14)$$



(a) 4-node transition zone element

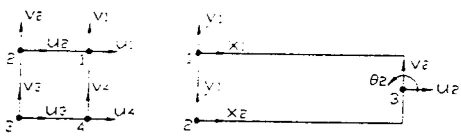


(b) 5-node transition zone element

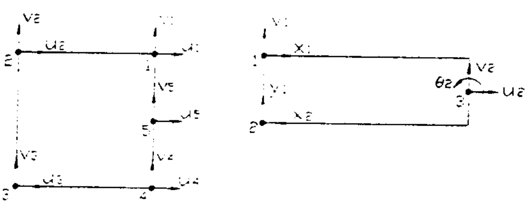
Fig. 5 Transition zone elements connected with BTRAN 1

Secondly, since $v_5=v_1$ as shown in Fig.6(b), it can be assumed that v_5 is dependent on v_1. In this case, transformation matrix $[T_4]$ can be written as

$$\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \end{Bmatrix} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&1 \\ 0&1&0&0&0&0&0&0&0 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \end{Bmatrix} = [T_4] \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \end{Bmatrix} \tag{15}$$



(a) 4-node transition zone element



(b) 5-node transition zone element

Fig. 6 Transition zone elements connected with BTRAN 2

3.3 Transition zone elements connected with BTRAN 3

We can apply the transformation matrices of eqns(14), (15) to the transition zone element which is connected with the left side of beam transition element BTRAN 3 and can apply the transformation matrices of eqns(12), (13) to the transition zone element which is connected with the right side of beam transition element BTRAN 3.

4. NUMERICAL RESULTS AND DISCUSSIONS

A typical coupled wall structure is considered as a numerical example to verify the validity of beam transition elements according to several mesh gradings of transition zone. In order to obtain the reliable numerical results, we introduce the input process of element data from the program ADINA and add the subroutines which represent the beam transition element stiffness matrices.

Fig.7 shows a typical coupled wall structure model and Fig.8 shows the several analytical models and mesh gradings for Fig.7.

CASE 1 model as in Fig.8(a) can be considered as the conventional FEM model, and is entirely subdivided as the mesh of 4-node plane stress elements. Beam transition elements and transition zone elements are applied in CASE 2 and 3 models as shown in Fig.8(b)(c). These transition zone elements provide the efficiency in mesh grading of transition zone, related to the depth size of connecting beam. Also, Fig.9 shows the several modeling types for a connecting beam. A connecting beam can be replaced by two beam transition elements and one beam element as shown in Fig.9(a), or can be directly replaced by only one beam transition element BTRAN 3 as shown in Fig.9(b). The analytical model for frame analogy met-

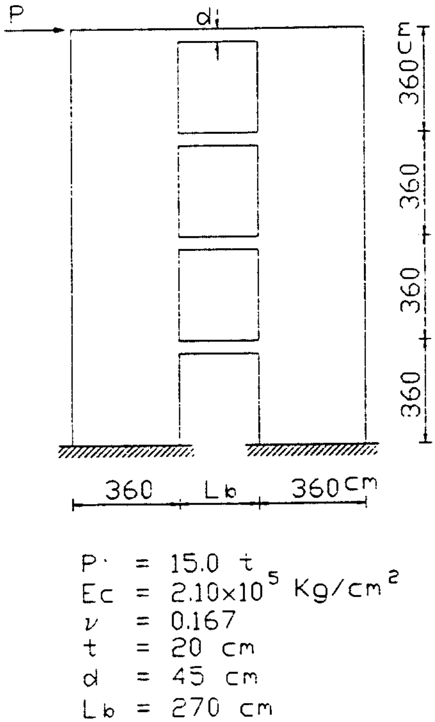
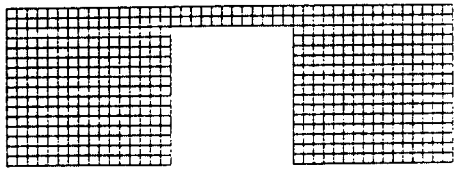
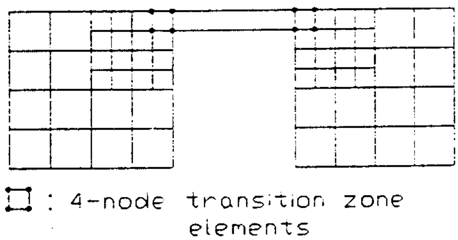


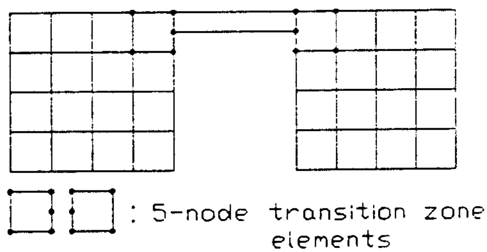
Fig. 7 A typical coupled wall structure



(a) CASE 1 model

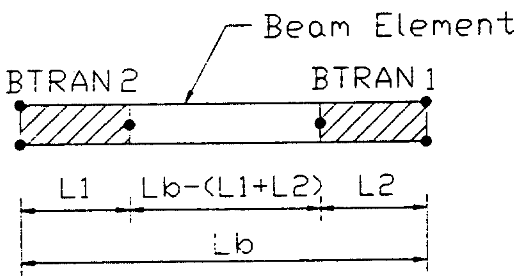


(b) CASE 2 model

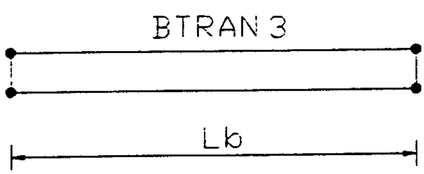


(c) CASE 3 model

Fig. 8 Analytical models and mesh gradings for Fig.7



(a) Beam transition element BTRAN 1, 2 and a beam element



(b) Beam transition element BTRAN 3

Fig. 9 Models for connecting beam

Table 1. Input data for CASE 4 model

Input data	$I_z(\times 10^5 cm^4)$	$A_z(cm^2)$	$A_s(cm^2)$
Equivalent column	777.60	7200.00	6072.48
Equivalent beam	777.60	7200.00	6072.48
Connecting beam	1.51875	900.00	759.05

Table 2. Length effects of beam transition elements

(unit : cm)

Case No.	BTRAN 1 (L_2)	BTRAN 2 (L_1)	Beam element $L_b-(L_1+L_2)$
1	60.0	60.0	150.0
2	60.0	90.0	120.0
3	90.0	60.0	120.0
4	90.0	90.0	90.0
5	135.0	135.0	0.0

hod is considered as CASE 4 model and its input data are given in Table 1. Table 2 shows several cases in order to evaluate the effects of the length variations of beam transition elements for Fig.9(a) when beam length L_b is 270cm.

Fig.10 shows the shapes of lateral displacements according to the height from ground level. Also, Table 3 shows the comparison of end forces based on the behavior of connecting beam.

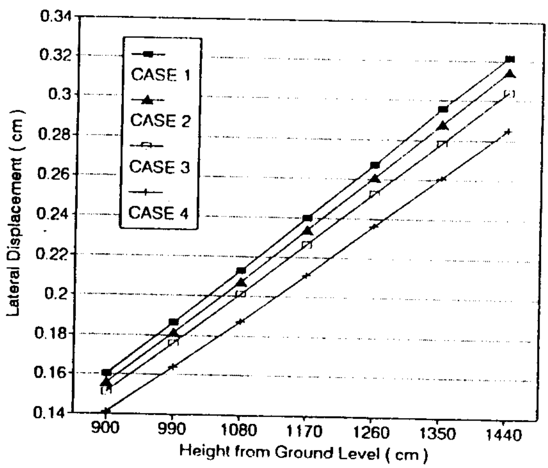


Fig. 10 Lateral displacements according to the height

Table 3. The comparison of end forces of connecting beams

Mesh model	Story	M_{ab} (t·cm)	M_{ba} (t·cm)	P (t)	V (t)
CASE 1	4	304.58	318.87	6.36	2.31
	3	381.60	375.23	2.02	2.80
	2	308.57	307.60	−0.25	2.28
	1	183.17	183.65	−0.30	1.36
CASE 2	4	306.18	317.96	6.32	2.31
	3	366.63	360.27	2.08	2.69
	2	296.18	295.41	−0.27	2.19
	1	175.89	176.33	−0.32	1.30
CASE 3	4	327.12	339.16	6.32	2.47
	3	368.98	363.75	2.09	2.71
	2	299.26	298.39	−0.27	2.21
	1	178.53	178.91	−0.31	1.32
CASE 4	4	410.10	403.30	6.85	3.01
	3	393.30	389.50	1.28	2.90
	2	321.30	320.40	−0.24	2.38
	1	188.90	189.00	−0.21	1.40

The characteristics and validity of beam transition elements can be evaluated from these numerical results. The results in case that a connecting beam is replaced by two beam transition elements BTRAN 1, 2 and one beam element as in Fig.9(a) are equal to the results in case that a connecting beam is replaced by only one beam transition element BTRAN 3 as in Fig.9(b). This shows that we can obtain the same results for the behavior of the connecting beam irrespective of the length variations of beam transition elements BTRAN 1, 2 as in Table 2. Therefore, we can obtain the efficiency in mesh grading and the simplicity in the input process of data in that a connecting beam can be replaced by only one beam transition element BTRAN 3.

In particular, CASE 2 and 3 models using beam transition elements show much closer results to CASE 1 model, compared with CASE 4 model. Moreover, the convergence of beam transition elements can be verified in that CASE 2 model which is more refined than CASE 3 model in the mesh of transition zone gives the improved results.

5. CONCLUSIONS

It has been shown that beam transition element method presents the improved results than frame analogy method, and is superior to the conventional FEM in saving the computer time and in simplifying the input procedure, based on the numerical investigation. In particular, the same results have been obtained irrespective of the length variations of beam transition elements BTRAN 1, 2 in modeling the connecting beam. This shows that beam transition element represents exactly the behavior of the connecting beam and a connecting beam can be replaced by an equivalent plane stress elements, namely, a beam transition element BTRAN 3. Thus, this beam transition element BTRAN 3 provides a great efficiency in the analysis of the transition regions of coupled wall structures and also can be used as a basic finite element in the substructuring process. Also, the transition zone elements, related to these beam transition elements, provide the great flexibility in modeling the connections of coupled wall structures.

In conclusion, beam transition elements provide the effective and reliable results in the analysis of the connections of coupled wall structures, the accuracy in modeling connecting beam, and the simplicity in input process of data. Also, these beam transition elements and transition zone elements can be usefully applied to the transition regions of all structures that wall and beam element are interconnected.

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