학술기사

우리 학회에서는 국제교류의 일환으로 해외과학자 초청세미나를 개최하고 있다. 지난 94년 11월 15일에는 University of California, Irvine의 A. H.-S. Ang 교수와 University of Missouri-Rolla의 F. K. Cheng 교수가 구조물의 내진설계에 대한 연구를 발표하는 기회를 가졌다. 여기서는 Ang 교수의 발표내용은 발표자가 준비한 원고를 게재하고, Cheng 교수의 발표내용은 발표자의 자료에 황재승 회원(서울대 건축학과)과 민경원 회원(인천대 건축공학과 전임강사) 이 해설을 추가하여 게재하도록 한다.

Development of Optimal Seismic Safety and Performance Criteria for Design of R/C Buildings

A. H-S. Ang and D. de Leon*

1. Introduction

In earthquake resistant design, the determination of the proper level of safety or performance requires consideration of acceptable risk. Proposed here is a systematic approach for determining the acceptable risks for life safety and damage control on the basis of minimizing the respective expected life-cycle costs, from which the corresponding criteria for design may be derived.

The current generation of seismic building codes are concened almost exclusively with life safety. This concern for life safety, of course, is proper and indeed deserves priority consideration in aseismic design. However, of equal importance is the development of criteria for design to limit or control structural damage. Whereas life safety requires prevention of collapse, damage control requires prevention of excessive damage ; the latter requires the definition of a tolerable or allowable damage level.

Cost is invariably important in engineering design ; in particular, the total life-cycle cost of the system is of special concern. Therefore, in the formulation or development of criteria for design, a balance between performance and life-cycle cost is an important objective. Traditionally, this balance is "achieved" or attempted through judgement and intuition. A systematic and quantitative approach for this purpose is suggested, which requires the following :

(i) A systematic method for the quantitative damage assessment of a structure subjected to a specified or anticipated earthquake.

(ii) Development of realistic cost functions that are related to or vary with the level of safety(or probability of failure), and the formulation of expected life-cycle costs.

(iii) Consideration of the seismic hazard at the site of a structure ; e.g. as defined through a hazard curve.

(iv) Trade-off between safety (or performance) and life-cycle cost to determine the optimal acceptable risks.

The theoretical and analytical tools for the above purposes are now available, as described briefly below.

---

* Professor, University of California, Irvine.

2. Damage Assessment

Clearly, in design for damage control, a tolerable level of structural damage must be defined. For this purpose, a predictive model is needed in order to calculate the damage expected during a given or anticipated earthquake. The model, however, must be related to or consistent with observed seismic damages. This can be accomplished by calibrating the calculated damages of a selected group of structures with the corresponding observed damages of the same structures from past earthquakes. On the basis of this calibration, and the subsequent disposition of the damaged structures, a reparable damage level may be defined.

2.1 Damage Model for Reinforced Concrete Elements

A seismic damage model for reinforced concrete has been developed by Park and Ang (1985) Damage is assumed to be caused by the maximum structural deformation and the dissipated hysteretic energy, and the total damage is expressed in terms of a damage index

$$D=\frac{x_m}{\delta_u}+\frac{\beta}{Q_y\delta_u}\cdot E_h \qquad (1)$$

where : x_m=maximum deformation ;

E_h=dissipated hysteretic energy ;

δ_u=deformation capacity ;

Q_y=static yield strength ;

β=constant.

The above damage index has been normalized, such that at collapse the value of D is 1.00. Based on extensive analyses of available test data for beam and beam column elements, the damage index at collapse has been determined(Park and Ang, 1985) to have a lognormal distribution with a median of 1.0 and a c.o.v. of around 50%.

2.2 Damage of Structure

The above model refers only to the damage of structural elements or components(e.g. beams and columns). The global damage of a complete structure is, of course, a function of the damages of its constituent components. The determination of the global damage may be based on the following :

$$(D_t>d)=\underset{i}{U}\,(D_i>d) \qquad (2)$$

where, D_i=the damage of component i ;

D_t=the globasl damage of the structure ; and

U=the union of events.

Eq. (2) means that the global damage will exceed d when one or more of its components have damages exceeding d. It follows that the probability of the global damage can be assessed as,

$$P(D_t>d)=P[\underset{i}{U}\,(D_t>d)] \qquad (3)$$

For a specified value of d, representing the tolerable damage level, the pertinent damage exceedance probability, Eq. (3), can be assessed, e.g. by fast Monte Carlo simulations(Wang and Ang, 1993).

In the above damage assessment the response statistics(namely, the maximum deformation and dissipated hysteretic energy) of the components are required. This involves the nonlinear response analysis of the structure under a given earthquake. The model for this analysis is the following.

The equation of motion may be expressed in matrix form as,

$$M\ddot{x}+C\dot{x}+Kx+(1-\alpha)Kz=-M\ddot{x}_g \qquad (4)$$

where, $\ddot{x}$, $\dot{x}$ and x=relative acceleration, velocity and displacement vectors ;

M, C and K=mass, viscous damping, and initial stiffness matrices ;

α=ratio of post-to pre-yielding stiffnesses ;

$\ddot{x}_g$=ground acceleration.

And z is governed by

$$\dot{z}=\frac{1}{\eta}[Ax-\nu(\beta|\dot{x}||z|z+\gamma\dot{x}|z|^2)] \qquad (5)$$

where A, ν, β, γ and η are parameters that govern the shape and amplitude of the hysteresis loop.

3. Formulation of Cost Functions

The optimal risk or safety level may be determined on the basis of achieving the minimum expected life-cycle cost. Therefore, the cost items that are pertinent are those that would vary with the acceptable risk ; any cost item that is independent of the risk will not affect the determination of the optimal risk and may be neglected.

For the purpose of formulating the expected life-cycle cost and determining the optimal acceptable risks for design, the cost items may be classified into three categories as follows :

1. Those that vary with risk ; i.e. the cost will increase or decrease with the risk underlying the design of a structures, e.g. initial cost of a structure ; damage /repair cost.
2. Those that are independent of risk, e.g. insurance premium ; cost of site preparation.
3. Those that are consequences of damage or collapse of a structure ; e.g. economic losses (business interruption, loss of revenue), cost of injury.

Ideally, insurance premium ought to vary with the acceptable risk underlying the design of a structure. Realistically, however, it is more likely to be independent of the underlying risk.

Cost items of the first category are explicitly functions of the underlying acceptable risk, whereas in the second category the costs are constants and thus will not influence the determination of the optimal risk. For the third category, the expected cost of each consequence is the actual cost multiplied by the risk.

3.1 Initial Cost Function

The initial cost of a structure will vary with the safety or acceptable risk underlying its design ; in particular, this cost will decrease with risk(or increase with safety level), as shown in Fig. 1. To develop this initial cost function for a structure or a class of structures, the structure must be designed repeatedly for varying levels of safety or risk ;e.g. using several values of the seismic base coefficients. The cost for each of the designs can then be estimated which may consist of material cost, construction cost, design cost, etc. Under the earthquake for a designated site(e.g. defined by a spectrum shape) with a given intensity, the probability of damage exceedance for each of the designs can be assessed through Eq. (3). These calculated probabilities may then be related to the respective costs for the different designs, thus obtaining the initial cost function for the given intensity. By varying the intensity of the same earthquake, a family of initial cost functions is developed for the structure as illustrated in Fig. 1.

3.2 Damage /Repair Cost Function

In the case of damage and subsequent repair, the cost would increase with the acceptable risk(i.e. decrease with safety level) ; a structure with low safety level can be expected to sustain more severe damage than if it were designed with higher safety. Therefore, the damage/repair cost will increase with risk as shown in Fig. 2.

The repair costs of specific structures that have been damaged from past earthquakes are needed and should be obtained. The global damages(in terms of the damage index) of the same structures can be calculated, from which a regression relation may be obtained between the damage/repair cost and the median global damage, as illustrated in Fig. 4 for the data from Mexico City.

Subjected to a given intensity of the earthquake for a designated site, the median damages of the structures designed earlier can be assessed, from which the damage/repair costs for the different designs are obtained from the pertinent cost relationship(e.g. Fig. 4). The damage exceedance probabilities for the same designs can be calculated, and thus the damage/repair cost as a function of the damage probability is developed for a given intensity of the earthquake. By varying the intensity, a family of similar damage/repair cost functions are generated for the structure, as illustrated in Fig. 2.

3.3 Other Cost Items

The other cost items that also constitute the total life-cycle cost of a structure would include the insurance premium, the economic loss caused by structural damage or collapse, and in the case of severe damage and collapse the cost of injury and life loss. The latter costs are direct or indirect consequences of structural damage or collapse.

There is increasing interests in the study of potential economic and other losses from seismic damage and collapse of structures(e.g. ATC-13, 1985 ; NRC, 1989 ; NRC, 1992). Data on building stocks or inventories that may be subject to risk are also being collected and examined(e.g. Jones, et al, 1987 ; Jones and Chang, 1993). This and similar information will be pertinent also to the development of improved criteria for design, particularly criteria for damage control in which the potential economic loss may be of principal importance.

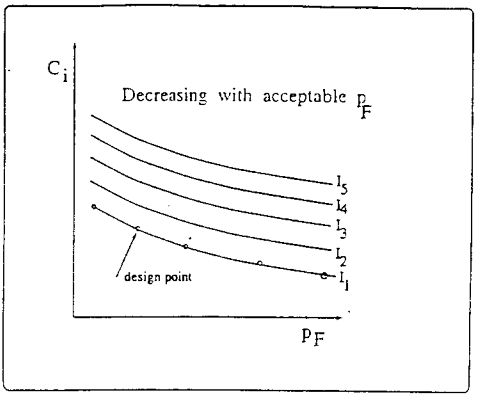


Fig. 1. Initital Cost Functions

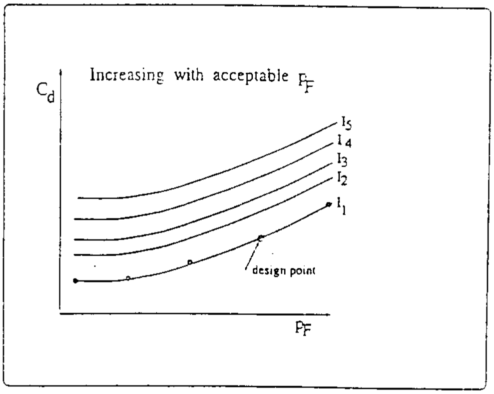


Fig. 2. Damage/ Repair Cost Functions

3.4 Expected Life-Cycle Costs

For damage control or prevention, the total expected life-cycle cost function of a structure, therefore, would be

$$C_t = C_i + p_f C_d + p_f C_c + p_f C_e + p_f C_{nd} \qquad (6)$$

where, C_i=the initial cost function ;

C_d=the damage /repair cost function ;

C_c=loss of contents(partial) ;

C_e=the economic loss caused by a structural damage ;

C_{nd}=the cost of non-disabling injury caused by a structural damage ;

p_f=the probability of excessive damage.

Observe that C_c, C_e, and C_{nd} are, respectively, the loss of contents, economic loss and cost of injuries which are all consequences of a structural damage ; therefore, the respective expected costs are obtained by multiplying the actual cost by p_f

In the case of life safety, the total expected life-cycle cost would be,

$$C_t=C_i+p_fC_r+p_fC_c+p_fC_e+p_fC_{in}+p_fC_f \qquad (7)$$

where, C_i=the initial cost function

C_r=the replacement cost of the collapsed or demolished structure ;

C_c=loss of contents(total) ;

C_e=the economic loss caused by a structural collapse ;

C_{in}=cost of all injuries(disabling and non-disabling) ;

C_f=cost of human fatality ;

p_f=probability of structural collapse.

Eq. (7) is similar to Eq. (6), except that the damage /repair cost is replaced by the replacement cost C_r, and the cost of human fatality C_f is added ; moreover, the cost of injuries includes both disabling and non-disabling injuries, and C_c represents the total loss of the contents.

4. Determination of Optimal Risks

The optimal risks for damage control and life safety may be determined on the basis of respective minimum expected life-cycle costs ; i.e. minimizing C_t of Eq. (6) or (7) to obtain the corresponding p_f.

4.1 Damage Control

For a given intensity of earthquake, the expected life-cycle cost function can be determined according to Eq. (7) ; by varying the intensity, a family of expected life-cycle cost functions are generated as shown in Fig. 3. Observe that for a given intensity, there is an optimal probability of damage exceedance(optimal risk) corresponding to the minimum life-cycle cost ; this optimal risk will very with the intensity as indicated in Fig. 3.

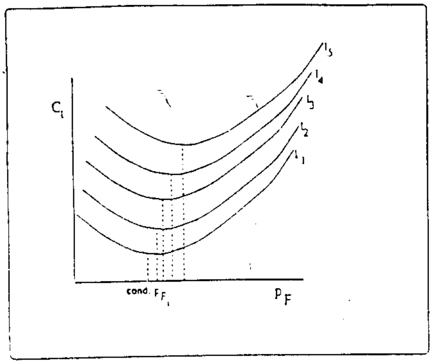


Fig. 3. Total Life-Cycle Cost Functions

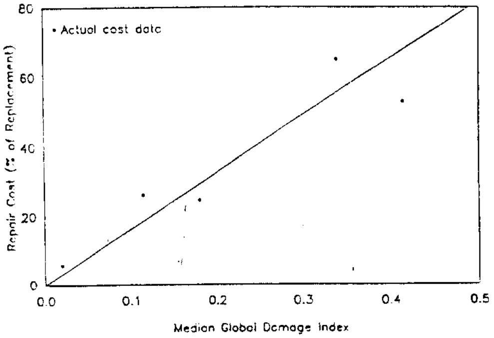


Fig. 4. Repair Cost vs. Damage Relationship for Mexico City

The seismicity or seismic hazard at the site of a structure will naturally affect the safety and performance of the structure. Properly, therefore, the seismic hazard over the life of a structure should be taken into account. To include this factor, the optimal risks determined above for the different intensities(which are conditional risks) should be weighted by the respective probabilities of occurrence of the different intensities, resulting in the expected acceptable risk for damage control,

$$E(p_f)=\sum_{\text{all } i}(p_f/a_i)\cdot P(a_i) \qquad (8)$$

where, $(p_f|a_i)$=conditional optimal risk at intensity a_i ;

$P(a_i)$=probability that the lifetime intensity is a_i.

4.2 Life Safety

Again, for different seismic intensities at the site, a family of expected life-cycle cost functions for life safety can be generated based on Eq. (7) ; the resulting cost functions would be similar to those shown in Fig. 3, except that the cost will be a function of the probability of collapse $P(D_t>1.0)$.

Observe that for a given intensity, there will also be a conditional optimal probability of collapse corresponding to the minimum life-cycle cost ; this optimal risk will also vary with the intensity, and thus by weighing the conditional optimal risks for the various intensities by the corresponding probabilities of occurrence of the respective intensities, the expected optimal risk for life safety is obtained through Eq. (8), where $(p_f|a_i)$ is the optimal probability of collapse for given intensity a_i.

5. Illustrative Application

The approach is applied to the development of criteria for reinforced concrete buildings in Mexico City. Damage and cost data are based on reports available for the 1985 Mexico City earthquake. In particular, a class of framed R/C apartment buildings of five to ten stories is considered in this study.

5.1 Analyses of Damaged Buildings

A number of R/C buildings that were damaged to different degree of severity during the 1985 Mexico City earthquake were analyzed using the damage model described earlier. The respective median global damage indices were obtained, the calculated indices were then calibrated with the observed damages of the respective buildings. Also, based on the subsequent disposition of each of the buildings(some were repaired whereas others were demolished), the reparable damage level was determined to be at $D_t=0.5$.

For a limited number of buildings(five), the actual repair costs were reported(Guerrero, 1990). From these repair cost data and te calculated median global damage indices, D_m, for the corresponding buildings, the relationship between repair cost and the median global damage index is obtained as shown in Fig. 4.

5.2 Development of Cost Functions

For the initial cost function, a seven-story R/C building is designed in accordance with the 1987 Mexican Code(DDF, 1987). The resulting structure is intended to represent a class of R/C frame buildings of 5-10 stories. The building plan and elevation are shown in Fig. 5. The same building is designed repeatedly using several seismic base coefficients as enumerated in Table 1.

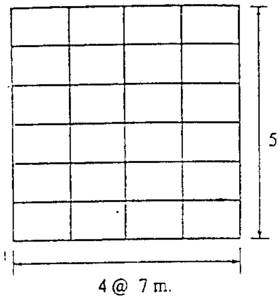
Table 1. Seismic base coefficients used in design

0.10	0.15	0.20	0.25	0.30	0.35	0.40
0.45	0.50	0.55	0.60	0.65	0.70	0.75

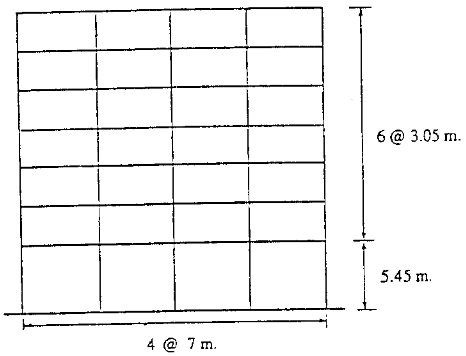
Based on the resulting design for each building, the cost for material is estimated, whereas the cost for construction is assumed to be the same as the material cost. The cost for design will most likely not change with the base coefficient, and therefore is constant. The cost associated with the loss of contents is estimated as 0.5 C_i(for collapse) and D_mC (for intermediate damage).

Each of the buildings is then subjected to the 1985 Mexico City earthquake(defined by the SCT spectrum) as shown in Fig. 6; the intensity(in terms of the peak ground acceleration, PGA) is assessed, which can be related to the respective initial costs; thus yielding the initial cost function for the particular intensity, as indicated in Fig. 7. By varying the intensity, similar initial cost functions are obtained for other intensities, as illustrated in Figs. 7 and 8 for PGA of 0.15g and 0.25g, respectively.

Under a given ground motion intensity, the median global damage indices of the various



(a) PLAN VIEW



(b) ELEVATION

Fig. 5. Plan and Elevation of 7-Story R/ C Building

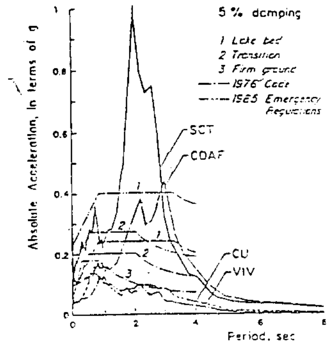


Fig. 6. Accleration Response Spectra for Mexico City

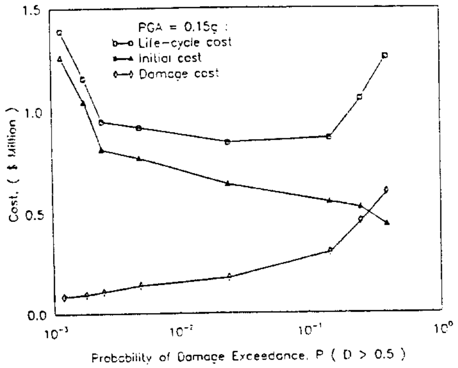


Fig. 7. Cost Functions for Damage Control(PGA=0.15g)

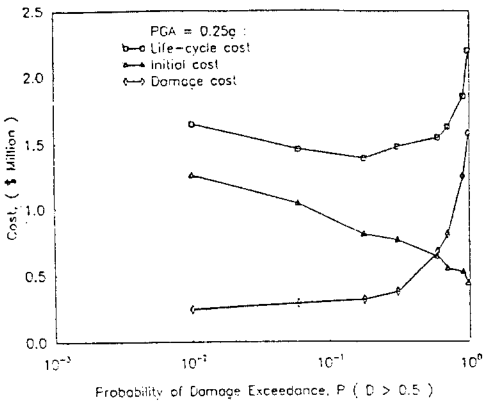


Fig. 8. Cost Functions for Damage control(PGA=0.25)

designs are also calculated as part of the above damage assessments. Then from the repair cost vs. damage relationship of Fig. 4, the required damage/repair costs for the respective designs are obtained. These costs are then related to the corresponding calculated damage exceedance probabilities, thus generating the required damage/repair cost function for the particular intensity. For other intensities, similar cost functions are obtained as illustrated also in Figs. 7 and 8 for two intensities.

The economic loss caused by structural damage(business interruption, loss of revenue, etc) clearly will depend on the usage of the structure and the severity of the damage. This loss may be assumed to be a quadratic function of the median global damage, with the maximum loss at $D_t=0.5$. In general, the economic consequences of structural damage or collapse are difficult to determine(NRC, 1992), and available data are scarce; any loss estimation is subject to high degree of uncertainty. In the case of a residential apartment building, the maximum economic loss amy be estimated as the loss of rentals during the period of reconstruction. For this case, the maximum loss for Mexico City may be estimated based on the following assumptions:

(i) the reconstruction period is 2 years;

(ii) the average monthly rental is $20 per square meter of floor area.

Thus, the maximum possible economic loss caused by structural damage for apartment buildings is,

$$C_e = 20 \times 12\left(\frac{A}{0.5}\right) = 480A \qquad (9)$$

Therefore, for a median global damage of D_m, the economic loss would be

$$C_e = 480A\left(\frac{D_m}{0.5}\right)^2 \qquad (10)$$

5.3 Injury and Fatality

The cost associated with human injury and fatality may be estimated as follows.

For Mexico City, the cost of injuries may be estimated based on the following assumptions:

(i) the cost of each disabling injury is $167,000, whereas for each non-disabling injury the cost is $1667;

(ii) the average number of injuries per unit area of collapsed buildings is 0.0168(estimated from data reported in Tokyo Mertropolitan Goverment, 1986; UNAM Inst. of Engineering, 1985);

(iii) 2/3 of all injuries are non-disabling, and 1/3 are disabling.

With these assumptions, the cost of non-disabling injury is,

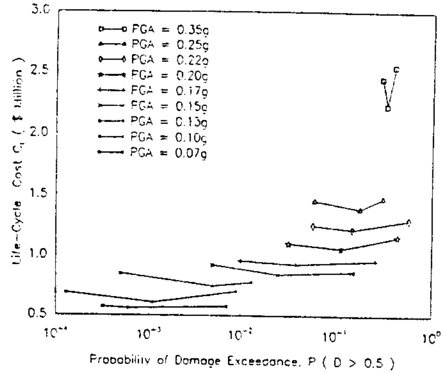


Fig. 9. Life-Cycle Cost Functions with Respective Conditional Optimal Risks for Damage Control

$$C_{nd}=0.0168\times\frac{1}{3}A\times1667\left(\frac{D_m}{0.5}\right)=37.33AD_m \qquad (11)$$

where A=the floor area of building; whereas for total injuries, the cost would be,

$$C_{in}=0.0168 \left(\frac{1}{3}\times 167.000+\frac{2}{3}\times 1667\right) A=653A \quad (12)$$

Finally, the cost of a human life may be estimated as the economic loss to society caused by the loss of a life; e.g., this may be estimated as the present value of the average contribution to the gross national product during the remaining productive life of an individual(Rosenblueth, 1976). For Mexico the average per capita income is $4680(Europa Publ., 1993). The average number of fatality per unit area of a collapsed building is estimated to be 0.0122(Tokyo Metropolitan Goverment, 1986; UNAM Inst. of Engineering, 1985). On these bases, and assuming an average remaining porductive life of 30 years, the cost associated with loss of life caused by structural collapse is,

$$C_f=0.0112A\times 4680\times 30=1572A \quad (13)$$

where A=floor area of a building.

The insurance premium and legal cost are assumed to be independent of the acceptable risk, and thus will not effect the determination of the optimal risk.

5.4 Determination of Optimal Risks

Damage Control-- For damage control, the life-cycle cost functions were developed in accordance with Eq.(6) for different potential intensities in Mexico City; the resulting cost functions are displayed in Fig.9 in which the optimal risk for each intensity is indicated at the corresponding minimum cost.

The seismic hazard curve for Mexico City is shown in Fig. 10, presented for periods of one year and 50 years. This is generated following the model suggested by Esteva and Ruiz(1989) for the seismic hazard of Mexico City. By weighing or convolving the optimal risks for the different intensities with the hazard curve for a life of 50 years, the expected optimal risk for damage control is obtained as,

$$E\left[P(D_t>0.5)\right] = 0.105 \quad (14)$$

By reviewing the family of life-cycle cost functions(see Fig. 9), and observing the design that corresponds to this expected optimal risk, the corresponding seismic base coefficient is determined to be 0.30. These results are for a tolerable damage of d=0.5 which is the limit of reparable damage. Obviously, other(lower) tolerable damage levels may be specified for which the corresponding expected optimal risks and base coefficients may similarly be obtained.

Life Safety--For life safety, the corresponding life-cycle cost functions were developed following Eq.(7) and using the cost components of Eqs.(9) through(13), again for various potential earthquake intensities in Mexico City; the resulting family of cost functions is shown in Fig. 11, in which the conditional optimal risks for the various intensities are indicated at the respective minimum costs. By convolving these optimal risks with the 50-year hazard curve for Mexico City, the expected optimal risk for life safety is obtained as

$$E\left[P(D_t>1.0)\right] = 1.6\times 10^{-3}$$

The associated seismic base coefficient is determined to be 0.45. Observe that in the new seismic code of Mexico City(revised in

Table 2. Summary of Results

Limit State	Expected Optimal Risk*	Seismic Base Coefficient
d =0.5	0.105	0.3
d = 1.0	1.6×10^{-3}	0.45

1987), the base coefficient is 0.40 which is close to the optimal value of 0.45.

6. Conclusions

Structural design criteria for damage control and life safety consistent with the respective acceptable risks can be developed systematically on the basis of achieving minimum life-cycle costs.

Monetary cost is an important and essential factor in the formulation of structural design criteria. In this consideration, the total expected life-cycle cost is most pertinent, which should include the potential damage /repair cost and possible replacement cost over the life of a structure.

The technical tools and theoretical models are now available to integrate the technical aspects of engineering with the essential economic factors in the systematic determination of optimal acceptable risks from which appropriate conventional requirements for codified design can be derived.

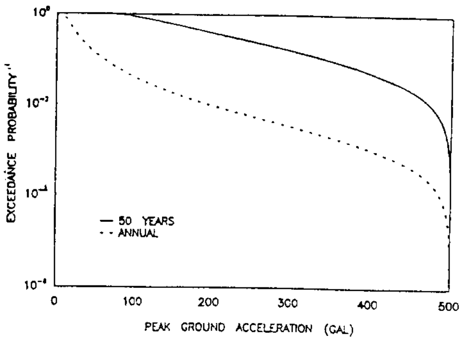


Fig. 10. Seismic Hazard Curve for Mexico City

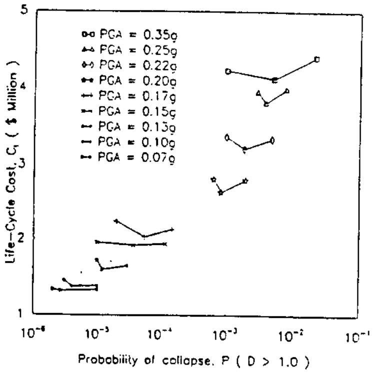


Fig. 11. Life-Cycle Cost Functions with Respective Conditional Optimal Risks for Life Safety

The approach can be applied for a specific structure or a class of structures; in each case, the appropriate cost functions must be developed.

7. Acknowledgements

This study was conducted under the partial support of the National Science Foundation under Grant CMS-9201812, and the fellowship support from the National Council for Research and Technology of Mexico. These supports are gratefully acknowledged.

REFERENCES

1. Applied Technology Council (ATC-13), "Earthquake Damage Evaluation Data for California", 1985.
2. Departamento del D.F., "Reglamento de construcciones para el Distrito Federal. Normas Tecnicas" (in Spanish), Mexico, D. F., 1987.
3. Esteva, L. and Ruiz S., "Seismic Failure Rates of Multistory Frames," *J. of Structural Engineering*, ASCE, Vol. 115, No. 2, 1989, pp.268-284.
4. Guerrero, J.J., "Vulnerabilidad Sismica de Edificios de Concreto Reforzado en la Delegacion Cuauhtemoc", *Master thesis*(in Spanish), DEPFI, UNAM, Mexico, D.F., 1990.
5. Institute of Engineering, UNAM, "Efectos de los sismos de septiembre de 1985 en las construcciones de la Ciudad de Mexico. Aspectos estructurales," *Report*(in Spanish), November, 1985.

6. Jones, B.G and Chang, S.E., "Indirect Methods for Estimating the Built Physical Environment for Risk and Damage Assessment and Relief, Recovery, and Reconstruction Planning," *Proc. National Earthquake Conference,* Memphis, 1993, pp.593-602.
7. Jones, B.G., Manson, D.M., Hotchkiss, C.M. and Savonis, M.J., "Determination of Building Stocks for Urban Vulnerability Analysis and Earthquake Loss Estimation," *Working Paper,* Cornell University, 1987.
8. National Research Council, "Estimatiog Losses from Future Earthquakes," *Panel Report,* National Academy Press, 1989.
9. National Research Council, "The Economic Consequences of a Catastrophic Earthquake," *Proc. of a Forum,* National Academy Press, 1992.
10. Park, Y.J. and Ang, A.H-S., "Mechanistic Seismic Damage Mdeol for Reinforced Concrete," *J. of Structural Engineering,* ASCE, Vol. 111, No. 4, 1985, pp.722-739.
11. Rosenblueth, E., "Towards Optimum Design Through Building Codes," *J. of Structural Division,* ASCE, Vol. 102, No. 3, March 1976, pp.591-607.
12. Europa Publications Limited, "South America, Central America and the Caribbean, 1993", 4th Edition, England, 1993.
13. Tokyo Metropolitan Government, "Report on the Investigation of the Earthquake in Mexico(September 19, 1985)", June 1986.
14. Wang, G.S. and Ang, A.H-S., "Adaptive Kernel Method for Evaluating Structural System Reliability," *Proc. ICOSSAR '93,* Innsbruck, 1993, pp.1495-1500.