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# 탄소섬유/에폭시 복합재료의 압축파괴 거동에 부하 스트레스 상태가 미치는 영향

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# Effect of State of Stress on Compressive Failure in Carbon-Fiber/Epoxy Composites

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## ABSTRACT

여러가지 두꺼운 복합재료 구조물은 3차원 압축 부하 상태에 노출되는 경우가 발생한다. 이런 경우에 있어서의 복합재료 압축 강도는 압축 평균 응력을 이용하면 예측이 가능할지도 모른다. 이번 연구에서는 압축 평균 응력을 이용하여 탄소 섬유 강화 복합재료들의 압축 강도를 예측하는 모델을 개발하고자 한다.

이 모델은 압축강도에 영향을 주는 요소, 초기 misalignment를 고려하였고, 탄소섬유와 수지사이에 접합강도가 임계값을 초과할때 복합재료의 파괴가 일어난다고 가정한다. 또 여러가지 문헌들을 통하여 유압이 접합강도에 미치는 점들을 보여준다.

본 모델을 이용한 예측값들은 가해지는 유압에 따라 증가되며, 실험값들과 비교 분석될 것이다.

**Key Words** : Compressive Strength, Carbon-Fiber/Epoxy, Composite, Pressure Effect, Compressive Failure, Bond Strength

## 1. Introduction

Composite materials are being developed for structural applications ranging from submersible structures to primary structures for commercial transport aircraft. Since composites are frequently used in strength-critical applications and the compressive strength of fiber composites is in many cases lower than the tensile strength, knowledge of the compressive strength properties of composites can often be essential for rational design. However, the factors affecting the compressive strength are not completely understood.

Composites are being considered for applications involving thick laminates loaded in compression, such as in a submersible structures. The special features associated with thick laminates often involve the possibility of defects in the fiber place-

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ment(wavy fibers).(1) In addition, however thick composite laminates will tend to have three-dimensional states of stress. There is some experimental evidence that three-dimensional compressive stress states can actually improve the apparent compressive strength of the laminates.(2) This apparent strength increase is likely associated with a compressive mean stress that acts as a superposed pressure. There is evidence in the literature that superposed hydrostatic pressure acts to increase the apparent compressive strength of fiber composites.(3,4) A further question is thus to explain the mechanisms behind this beneficial pressure effect. The present paper will attempt to show that this strength increase can be explained in terms of known effects of pressure on the polymer matrix, and the interaction of the matrix with compressive strength of fiber composites.

Although many researchers have studied compression failure mechanisms of composite structures, there is not at present a well established theoretical and/or experimental basis for the prediction of compressive strength. A number of models for compressive strength of unidirectional composites have been suggested. The problem is complex, not only because of the mechanics of analysis, but also because of the variable failure mode of different composite materials. In fact, even when the failure mode is agreed upon, there is controversy as to the correct way to model the failure. Nevertheless, the various micromechanical models available in the literature are valuable in gaining an understanding of the controlling damage mechanisms and in directing future efforts at increasing compressive strength. The failure of carbon fiber composite materials has been studied by a number of investigators, including Rosen,(5) Chaplin,(6) Evans and Adler,(7) Budiansky,(8) DeTeresa et al.,(9) Hahn and Williams,(10) Swanson,(11) Argon,(12) Wronski and Howard,(13) Piggott,(14) DeFerran and Harris,(15) Greszczuk,(16) and Hayashi,(17) Budiansky,(8) and many others. Guynn et al.(18) give a number of additional references.

In the following, theoretical models of compressive failure of fiber composites will be reviewed. Modifications of the models to incorporate pressure effects on polymeric matrices will be shown. The results of compression tests carried out on carbon-fiber/epoxy laminates under hydrostatic pressure will be presented and compared with the model predictions.

## 2. Theoretical models

There have been a large number of theoretical models put forth to attempt to explain the mechanisms in compressive failure of fiber composites. A number of these models consider compressive failure to be governed by bifurcations bucking of perfectly straight fibers that are stabilized by the matrix. The matrix is typically represented by the modulus only, although a tangent modulus can be incorporated.(5 10 19) The fibers are typically assumed to act cooperatively, as in the classical shearing and extension models of Rosen.(5) A second approach considers that the models are initially imperfect, by assuming that the fibers have an initial periodic waviness. Examples are given in the work of Herrman et al.,(20) Lanir and Fung,(21) and Hahn and Williams.(10) This category of models does not exhibit bifurcation bucking, but rather fails by excessive deformation of the fibers, leading to either excessive fiber or matrix stresses or strains.

The authors, and many others, believe that models involving initially wavy fibers are more realistic for carbon-fiber/epoxy composites. As noted above, models with initially straight fibers predict a compressive strength that depends on the matrix stiffness, while the initially wavy fiber models include not only the stiffness but either the fiber or the matrix strength. In previous results reported by our laboratory, tests were carried out on fiber composites in which the fiber-matrix bond strength had been artificially reduced with a release agent, but leaving the matrix stiffness unchanged. The apparent laminate compressive strength was reduced by up to a factor of four, indicating the importance of the matrix bond strength and thus supporting the use of imperfect fiber models.(21) Systematic changes in compressive strength with fiber-matrix adhesion strength were also reported by Madhukar and Drzal.(22)

The basic ideas of the model(11,22) will be illustrated in the following. The critical axial plies are assumed to have an initial fiber waviness given by

$$v_0 = f_0 \sin \lambda x \tag{1}$$

where $f_0$ : amplitude of initial fiber waviness
$\lambda$ : wavelength of fiber

The subsequent lateral deformation is taken as

$$v - v_0 = (f - f_0)\sin \lambda x \tag{2}$$

where $v$ : fiber waviness
$f$ : amplitude of fiber waviness

The deformation under load can than be solved for fiber lateral displacement by using the minimum potential energy theorem, with the parameter f governing the amplitude of the bending deformation of the fibers. The task is to formulate the strain energy of the axial fibers and matrix, and then to minimize the potential energy under applied axial compression displacement.

The strain energy terms are given as follows:

(1) In-plane shear in axial plies:

$$\Gamma_{xy} = \frac{\partial(v - v_0)}{\partial x} = (f - f_0)\lambda(\cos \lambda x) \tag{3}$$

$$U_1 = \int_0^1\int_0^1\int_0^t \frac{1}{2} G_m \gamma_{xy}^2 \, dzdydx = G_m t \lambda^2 (f - f_0)^2 / 4 \tag{4}$$

(2) Axial compression of axial plies

$$U_2 = \int_0^1\int_0^1\int_0^t \frac{1}{2} E_{11} \varepsilon_A^2 \, dzdydx \tag{5}$$

$$\varepsilon_A = \varepsilon_{11} - \varepsilon_b \tag{6}$$

$$\varepsilon_b = \int_0^1 \frac{v'^2}{2} dx - \int_0^1 \frac{v_0'^2}{2} dx = \lambda^2 (f^2 - f_0^2)/4 \tag{7}$$

where $\varepsilon_{11}$ is the applied displacement per unit length in the axial direction, $\varepsilon_A$ is the axial strain in the fibers, and $\varepsilon_b$ is the axial displacement per unit length due to the bending undulation of the fibers. The bending energy in the fibers could also be included; but as discussed previously(10,11) this term has little effect. Substituting these expressions for strain energy into the potential energy, and minimizing with respect to $f$, the fiber bending displacement parameter, under an applied axial displacement per unit length of $\varepsilon_{11}$, gives

$$G_m(f - f_0) + E_{11}\lambda^2(f^2 - f_0^2)f/4 - E_{11} f \varepsilon_{11} = 0 \tag{8}$$

The limiting condition is assumed to be established by the shear stress in the matrix or the fiber matrix bond. This is calculated by multiplying the shear strain in the matrix, given by Eqn 3 above, by the matrix shear modulus, to get

$$\tau = G_m(f - f_0)\lambda \tag{9}$$

It can also be observed from Eqn 1 that $\lambda_f$ is the maximum value of the fiber misalignment angle, so that Eqn 9 can be written as

$$\phi = \frac{\tau}{G_m} + \phi_0 \tag{10}$$

where $\phi$ and $\phi_0$ are the current and initial values of fiber of misalignment angle. The maximum value of the fiber misalignment angle is thus determined by the ultimate allowable matrix shear stress $\tau_u$, giving

$$\phi_u = \frac{\tau_u}{G_m} + \phi_0 \tag{11}$$

Substituting this value for $\phi_u$ into Eqn 8 gives an expression for the axial compressive stress as

$$\sigma_{11} = -\{E_{11}(\phi_u^2 - \phi_0^2) + G_m(\phi_u - \phi_0)/\phi_u\} \tag{12}$$

Thus the model predicts a compressive strength related to the initial and ultimate fiber misalignment angle, and the ultimate fiber misalignment angle is determined by the allowable matrix shear stress.

It is known that the stiffness and strength of polymeric materials increases with pressure.(24 25) Thus the pressure effect can be directly incorporated into the above model of compressive strength. The effect of pressure on the fiber-matrix bond strength was taken from data of Shin and Pae(26) given in the nondimensional form of

$$\tau/\tau_0 = 1 + \alpha_1 (p/\tau_0) \tag{13}$$

with $\alpha_1 = 0.159$, and where $\tau$ and $\tau_0$ are the matrix bond strength with and without pressure. The effect of pressure on the modulus is much lower than the effect on matrix bond strength, and did not change the calculation significantly. This expression for the increase of matrix bond strength with pressure is quite similar to that given by Groves et al.(27) at lower pressures.

A generalization from compression under superposed hydrostatic pressure to general three dimensional states of stress is required. One possibility is to replace the pressure with the mean stress. However, in view of the directional properties of the material and the presumed failure mechanism of failure of the fiber-matrix bond, the stress component normal to the fibers may be more appropriate. In particular, it is suggested here that in general the pressure term be replaced by the least compressive value of either the in-plane normal stress $\sigma_2$ or the through-the-thickness normal stress $\sigma_3$. However, this point must necessarily be established by experiments under general states of stress.

## 3. Experimental

Experimental were also carried out to measure the effect of pressure on compressive strength. These tests utilized a multiaxial apparatus with a tubular specimen. In this case a uniform pressure was applied to the inside and outside of the specimen, and the specimen was then loaded in axial compression. The specimen is nominally 50.8 mm (2.0″) inside diameter and is lined with a thin rubber bladder. The specimens reported here were made of AS4/3501-6 or IM7/8551-7 carbon-fiber/epoxy laminate in 3 layups consisting of a quasi-isotropic, $([(0/90/\pm45)_2]_s$ or $[(90/\pm45/0)_2]_s$, an axial biased layup of $([0_2/\pm45)_2]_s$, and a primarily axial layup of $[90/0_6/90/0_6/90)]_t$. Each specimen was tested in an MTS 1000 KN (225 Kip) servo-hydraulic test machine in the Structure Integrity Laboratory at the University of Utah. The test machine was run in stroke control. The crosshead displacement rate used was 1.3 mm/min. All data were measured with a computer-controlled digital data acquisition system based on a Zenith 286 PC-compatible computer that had been developed for characterizing the mechanical response of composite materials subjected to a variety of loading arrangements. The system utilizes a Data Translation DT2821 card that provides 16 channels of analog to digital conversion. The software was written in the C programming language and allowed the user to access standard test configurations, standard or customized data reduction routines, and port data to an Apple Macintosh. After each specimen reached a testing pressure by the hydrostatic fluid pumping tool, the specimen was monotonically loaded in uniaxial compression at a constant displacement rate of 1.3 mm/min until failure. Strains in all tests were measured by means of the resistance strain gages and all load data were obtained from a load cell contained on the MTS servohydraulic test machine. Fig. 1 shows the configuration of two-inch (50.8 mm) diameter multiaxial tube speciumen. More detail on the experiments is given in Jee and Swanson.(28)

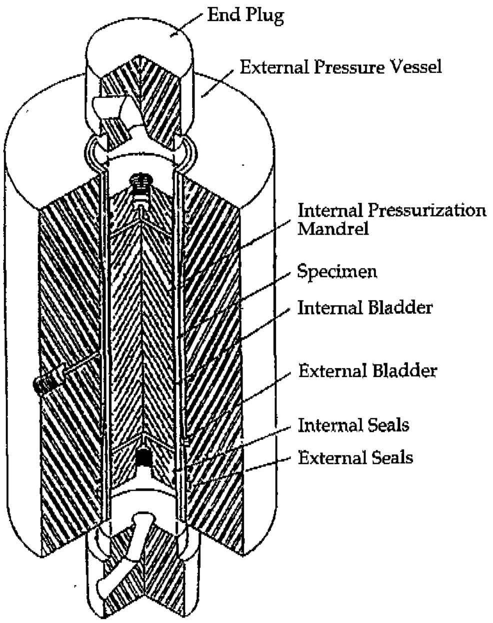

Fig. 1 Configuration of two-inch (50.8 mm) diameter multiaxial tube specimen.

## 4. Results

A comparison of average axial strain-axial stress for the specimens without and with hydrostatic pressure in $[(90/\pm45/0)_2]_s$ layup of the AS4/33501-6 system is shown by Fig 1. The AS4/3501-6 system in the quasi-isotropic $[(90/\pm45/0)_2]_s$ layup showed the increase with pressure. The predicted increase of compressive strength with pressure based on Eqns 11-13 is shown in Fig 2, indicating the general effect pressure to be expected. To make this computation, a value of matrix allowable strength equal to the measured interlaminar shear strength of 95.9 MPa for AS4/3501-6 was used, along with an initial fiber misalignment angle of 3.8°. These values are consistent with the measured compressive strength without pressure. The initial misalignment angle is also consistent with a value of 3° reported by Jelf and Fleck.(29)

The relative increase of compressive strength with pressure can then be predicted by using the increases in matrix and/or fiber matrix bond strength given in Eqn 13, taken from Shin and Pae.(27) The results are compared with the reported data Weaver and Williams(30) and Parry and Wronski(4) in Fig. 3, and with the experiments for AS4/3501-6 and IM7/8551-7 carbon-fiber/epoxy in Fig. 4.

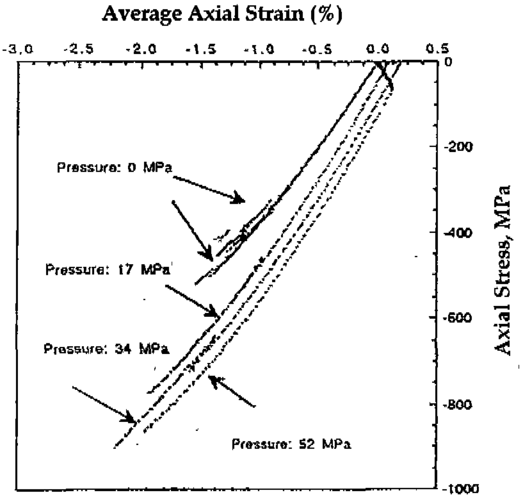

Fig. 2 Effect of pressure on average axial strain-stress response for AS4/3501-6 in a $[(90|\pm45|0)_2]_s$ layup.

It can be seen that the data of Weaver and Williams(30) and Parry and Wronski(4) show somewhat different trends with pressure, particularly at pressures below about 200 MPa. In general the

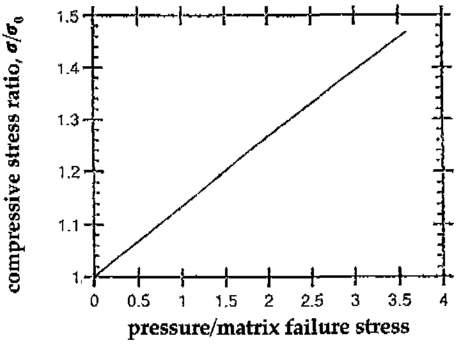

Fig. 3 Effect of pressure on the compressive strength of carbon-fiber/epoxy composites as predicted by the present model.

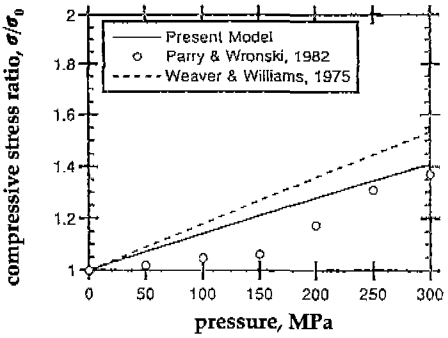

Fig. 4 Comparison of model prediction for pressure effect with data of Parry & Wronski's, 1982, (60 % $V_f$ Type III carbon-fiber/epoxy) and Weaver & Williams, 1975, (36 % $V_f$ Modmur Type II/ Epikote 828 epoxy).

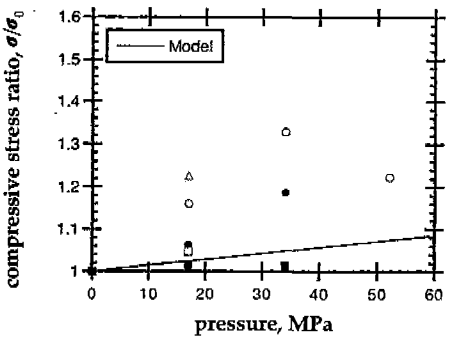

Fig. 5 Comparison of model prediction for pressure effect with measured compressive strength of carbon-fiber/epoxy laminates. Open symbols are 3 layups of AS4/3501-6, closed symbols are 3 layups of IM7/8551-7.

model predicts an effect of pressure that is between that reported in the Weaver and Williams(30) and Parry and Wronski(4) papers.

It can be seen that the model predicts a somewhat lower effect than seen in the experiments shown in Fig. 5, although the data are quite varied. It is possible that additional mechanisms may be involved in these data. It was reported by Jee and Swanson(28) that the pressure also tended to suppress delamination in these specimens, a mechanism that may be specific to the particular layups and use of tubular specimens.

## 5. Discussion

The major point of the the present model is to establish a plausible mechanism for the effect of hydrostatic pressure on compression failure in carbon-fiber/epoxy laminates. As mentioned in the introduction, it is believed that this effect should be applicable to thick laminates, in which 3-d stress-states can be important. The first basic premise of the model is that fiber strength or fiber-matrix bond strength is important in establishing compressive strength of carbon-fiber/epoxy composites. This idea is certainly not universally acknowledged, as the usual models of bifurcation buckling of fibers depend on the matrix stiffness, and not the matrix strength. However it has been shown by many investigators that incorporating initial imperfect fibers into the failure model provides a straightforward way to establish a matrix bond strength dependence. The experiments reported by Swanson and Colvin(22) in which an artificially lowered fiber-matrix bond reduced the laminate compressive strength by up to a factor of four also lends experimental support to this idea, as well as the results of Madhukar and Drzal.(23) The second basic premise of the model is that hydrostatic pressure tends to increase the matrix and bond strength. As discussed in the Introduction, it is well established in the literature that pressure can enhance the strength of polymers. The data of Shin and Pae(27) provide a quantitative assessment of the increase of epoxy bond strength with pressure, and was used here in the present model. Additionally, in a qualitative sense the effect of compressive normal stress on epoxy bond shear allowables is readily apparent and was used both by Groves et al.(27) and in our laboratory(31) to design end grips for tubular test specimens. It thus seems that the basic ingredients of the model are quite plausible. It should be immediately apparent, however, that the specific implementation of these ideas into a model involves a high degree of idealization.

The comparison with the experimental data are generally in support of the model, but do not really establish or disprove the quantitative validity of the model, as the experimental results for the effect of pressure on the compressive strength of carbon-fiber/epoxy composites are too varied. Clearly it would be helpful to have more experimental data available.

## 6. Conclusions

The compressive failure model that demonstrates a dependence of compressive strength of carbon-fiber/epoxy composites on hydrostatic pressure is presented. The model is based on an initial misalignment of axial fibers, and on failure of the fiber-matrix bond. Literature values for the effect of pressure on the bond strength are employed. The model predicts an increase of composite compressive strength with hydrostatic pressure. Comparisons with experimental data support the idea of an increase in compression strength with pressure, but the data are too varied to permit a quantitative assessment of the model. The results are believed to be applicable to states of stress in thick composites that can include three-dimensional compression.

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