論文 한국정밀공학회지 제 12권 제5호 1995. 5
Journal of the Korean Society of Precision Engineering Vol. 12. No 5. May. 1995.

6자유도 병렬형 로봇 메니퓰레이터의 기구학적 해석

류 길 하,* 조 병 관**

A Study on the Kinematic Analysis of a 6-DOF Parallel Robot Manipulator

G. H. Ryu,* B. K. Cho**

ABSTRACT

본 연구의 목적은 6자유도를 가진 병렬형 메니퓰레이터의 기구학적 해석을 하는데 있다. 일반적인 산업용 로봇의 구조는 링크가 직렬로 연결된 형상을 하고 있으며 이러한 형태는 넓은 작업공간의 확보와 유연성이 뛰어난 장점이 있는 반면에 각 링크의 오차가 메니퓰레이터의 끝단에서 누적되어 나타나게 되고 구동력이 증가하게 되는 단점을 지니고 있다. 이러한 단점을 극복하기 위하여 정밀한 위치제어가 필요한 경우에는 병렬형 형태의 링크를 지닌 구조를 사용하고 있다. 병렬형 메니퓰레이터의 역기구학적 해석은 비교적 단순한 데 반하여 정기구학적 해석은 비선형 방정식의 형태로 나타나며 해석적으로 그 해를 구하기가 쉽지 않다. 본 연구에서는 6자유도를 지닌 병렬형 메니퓰레이터의 기구학적 해석을 수행하였으며 예제를 통하여 검정하였다.

Key Words : Parallel robot manipulator, Kinematic analysis, Stewart platform

1. Introduction

Robot manipulators have traditionally been used as a positioning device in the industrial fields. There are two typical types in the structure. One is serial type robot arms which are connected using anthropomorphic open chain mechanism and the other is parallel link mechanism such as a Stewart Platform shown in Figure 1. The serial connected link mechanism is widely used in the industry and has several advantages. This serial type manipulator has large workspace, high dexterity and a good maneuverability. However the cantilevered structure of the open chain mechanism is serious disadvantages because of its low rigidity and low natural frequency. In addition, each successive links from the end effector toward the base have to be large enough to support the weight of the previous links and actuators. This increases the mass and the size of the actuators and affects the

* 한국기술교육대학, 제어기계공학과
** 한국기술교육대학, 동력기계공학과

dynamic performance. Another consequence of the cantilevered structure is that the actuator and geometric errors are accumulated along the open chain mechanism and result in a large position error at the end point of the robot manipulator. Therefore their use in applications that require large load and high accuracy operations is limited.

In the case of a parallel mechanism such as a Stewart Platform, the end effector is attached to a moving plate which is supported in-parallel by a number of actuated links. All actuators can be fixed to the base and the mass of the moving plate is reduced. The truss-like structure of the parallel mechanism can offer high load capability, high stiffness, good positioning accuracy and a good dynamic performance.

Two basic kinematic problems in the study of the robot manipulator are to be known as the forward and inverse kinematic problem. The forward kinematic problem is to compute the position and orientation of the end effector of the robot manipulator provided that a set of actuated joint variables is specified and the inverse kinematic problem is to compute a set of joint variables which could be used to attain the given the position and orientation of the end effector of the robot manipulator.

In the case of a parallel mechanism the inverse kinematic problem is very simple but the forward kinematic problem is very complicated. This is in contrast to the serial connected link manipulator, in which the forward kinematics is relatively easy and the inverse kinematics is relatively difficult. To solve the direct kinematic problem of the parallel manipulators has been studied by many researchers, usually in the configuration known as a Stewart Platform. The most well known parallel manipulator is probably the Stewart Platform, which was originally designed as an aircraft simulator by Stewart in 1965 [1]. Hunt suggested that this mechanism should be used as a manipulator[2]. Many researchers have studied various aspects of the Stewart Platform. Yang and Lee[3] and Fichter[4] studied the kinematic problems of the Stewart Platform and addressed the practical design and considerations. Lee and Shah[5] studied the kinematic analysis of a 3-DOF in-parallel manipulator and other peoples studied to solve this problem. [6-8] But the equations of the forward kinematic problems mentioned above are highly nonlinear and of high degree polynomial equation. The solution of a high degree polynomials is usually very sensitive to numerical precision and requires special cautions in solution procedures. Therefore such scheme is not very useful in real time applications. The aim of this paper is to present a method for the forward kinematic problem of the special 6-DOF parallel manipulator in a closed form in order to achieve a high control performance in the high speed and high accuracy. Closed form kinematic solution is very useful to control the manipulator in a real time.

This paper is organized as follows. Section 2 presents the kinematic analysis for a parallel manipulator. A numerical example and conclusion are described in section 3 and section 4.

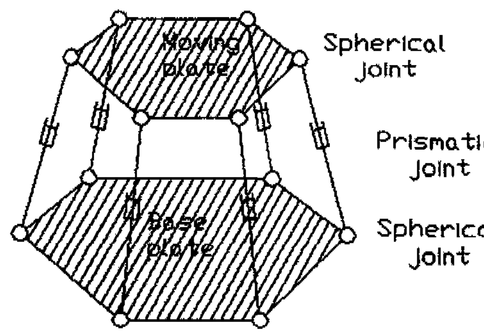

Fig. 1 The General 6-DOF Stewart Platform

2. Kinematic Analysis

The 6-DOF parallel robot manipulator consists of two bodies and connecting six legs which can be varied in length. One body is called the base plate and the other body is called the moving plate. Each of the six legs has one of its end point fixed in the base plate and the other point fixed in the moving plate. The two ends of each legs are fitted with spherical joints.

In this section we deal with the kinematic analysis of the 6-DOF parallel robot manipulator shown in Figure 2. The moving plate has six degrees of freedom. On the base plate the locations of the leg's ends(Bi, i=1, 6) are arbitrary. On the moving plate, the three leg's ends are connected at one of the joint P_1, P_2, P_3, two leg's ends at the second joint P_4, P_5 and one leg end at the third joint P_6. The position of the moving plate is controlled by three prismatic joints with changing the lengths L_1, L_2, L_3 and three other prismatic joints control the orientation of the moving plate in space. Two coordinate systems shown in Figure 2, {Tb} and {Tm}, are attached at the base and moving plate. The reference coordinate system XYZ, with unit vectors i, j and k, respectively, is fixed on the base plate and its origin coincides with the joint B_1. X-axis is chosen to pass through the joint B_2 and Y-axis is set to be in the base plate. Z-axis is upward and perpendicular to the base plate. Similarly an origin of a moving coordinate system xyz, coincides with joint P_1 with the z-axis pointing vertically upward and x-axis passing through joint P_2 and y-axis lying in the moving plate. The relationship between these two coordinate systems is described by the following transformation matrix $[{}^bT_m]$ as;

$$[{}^bT_m] = \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix} \qquad (1)$$

where $P=[x, y, z]^t$ is a 3x1 matrix denoting the position vector of o_m with respect to {Tb}. 0 is a 1x3 zero matrix, and R is a 3x3 rotation matrix representing the orientation of {Tm}, with respect to {Tb}, and can be expressed as;

$$[R] = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix} \qquad (2)$$

The Eq. (1) gives a complete description of the moving plate coordinate system referenced to the base plate coordinate system. Among the nine elements of the directional cosine matrix, there are only three independent ones and the remaining six can be determined by following equations;

$$l_x^2 + l_y^2 + l_z^2 = 1 \qquad (3)$$
$$m_x^2 + m_y^2 + m_z^2 = 1 \qquad (4)$$
$$l_x m_x + l_y m_y + l_z m_z = 0 \qquad (5)$$
$$n_x = l_y m_z - l_z m_y \qquad (6)$$
$$n_y = l_z m_x - l_x m_z \qquad (7)$$
$$n_z = l_x m_y - l_y m_x \qquad (8)$$

Each of six points in the base plate is described by a position vector, B_i, with respect to the base coordinate system {Tb}. Similarly, each of the points in the moving plate is described by a position vector, P_i, with respect to the moving coordinate system {Tm}.

2-1 Inverse Kinematic Analysis

The inverse kinematic problem of the 6-DOF parallel robot manipulator is to compute the lengths of the six legs given the position and orientation of the moving plate. The position vector Pi, referenced to the moving coordinate system {Tm}, can be described with respect to the base coordinate system by transformation matrix, Eq(1).

$$[{}^bP_i] = [{}^bT_m][{}^mP_i] \qquad i = 1,2,\cdots,6$$

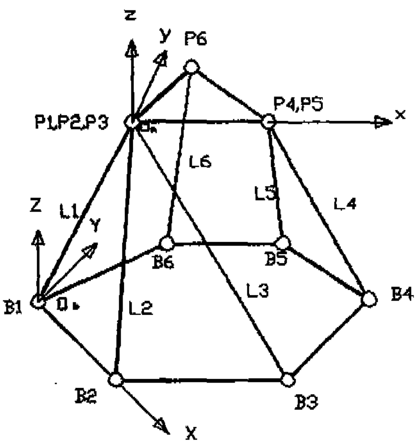

Fig. 2 The kinematic model of a 6-DOF parallel robot manipulator

Because the position of Pi and Bi have been expressed at the same coordinate system {Tb}, the length of each leg is the magnitude of the leg vector Li as follow;

$$L_i = |P_i - B_i| \qquad i = 1,2,\cdots,6 \tag{9}$$

We can rewrite Eq. (9) using Eq. (1) and (8) as;

$$\begin{aligned}&(P_{ix}l_x + P_{iy}m_x + x - B_{ix})^2 \\ &+(P_{ix}l_y + P_{iy}m_y + y - B_{iy})^2 \\ &+(P_{ix}l_z + P_{iy}m_z + z - B_{iz})^2 = L_i^2 \\ &\qquad i = 1,2,\ldots,6\end{aligned} \tag{10}$$

By Eq. (10), we can compute the each leg length given the position and orientation of the moving plate.

2-2 Forward Kinematic Analysis

The forward kinematic problem of the 6-DOF parallel robot manipulator is to compute the position and orientation of the moving plate given the lengths of the six legs. Consider the mechanism shown in Figure 2. The lengths of the legs are expressed as;

$$L_i = |{}^bP_i - B_i| \qquad i = 1,2,3 \tag{11}$$

We will omit the superscript b which means that the position vector is described with respect to a reference coordinate system, {Tb}. Since the position of the P_1, P_2 and P_3 are located at the same point on the moving plate, we have;

$$\begin{aligned}L_1^2 &= (P_{1x} - B_{1x})^2 + (P_{1y} - B_{1y})^2 + (P_{1z} - B_{1z})^2 \\ L_2^2 &= (P_{1x} - B_{2x})^2 + (P_{1y} - B_{2y})^2 + (P_{1z} - B_{2z})^2 \\ L_3^2 &= (P_{1x} - B_{3x})^2 + (P_{1y} - B_{3y})^2 + (P_{1z} - B_{3z})^2\end{aligned} \tag{12}$$

From the above equations we can calculate the position vector, Pi (i=1, 2, 3) as follow;

$$\begin{aligned}P_{ix} &= \frac{L_1^2 - L_2^2 + B_{2x}^2}{2B_{2x}} \\ P_{iy} &= \frac{L_1^2 - L_3^2 + B_{3x}^2 + B_{3y}^2 - 2B_{3x}P_{ix}}{2B_{3y}} \\ P_{iz} &= \pm\sqrt{L_1^2 - P_{ix}^2 - P_{iy}^2}\end{aligned} \tag{13}$$

Next, consider the point, P_4 which coincides the point P_5 on the moving plate shown in Figure 3. The point P_4 will lie on a circle with center located on the line connecting point B_4 and B_5. The center of circle is located P_o and its radius is P_4P_o. The position vector of point Po is given by equation;

$$P_0 = B_4 + t(B_5 - B_4) \tag{14}$$

where t is determined by equation:

$$t = \frac{L_4^2 - L_5^2 - |B_4|^2 + |B_5|^2 - 2B_4 \cdot (B_5 - B_4)}{2|B_5 - B_4|^2}$$

Now the position vector P_4 and P_5 are given by the equation;

$$P_i = P_o + d\cos\phi(\cos\theta\vec{i} + \sin\theta\vec{j}) + d\sin\phi\vec{k} \qquad i = 4,5 \tag{15}$$

where

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{B_{5y} - B_{4y}}{B_{5x} - B_{4x}}\right) + \frac{\pi}{2} \\ d &= L_4^2 - t^2\end{aligned}$$

By the following equation we can determine

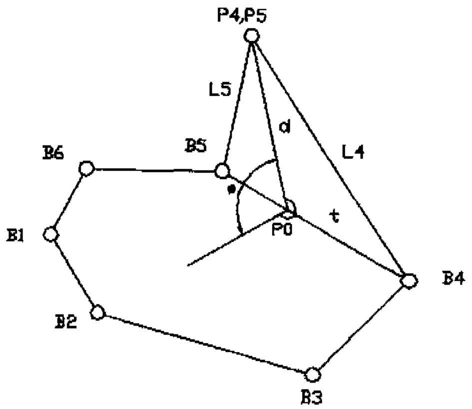

Fig. 3 Calculating the position vector P3

an unknown value ϕ.

$$l_1^2 = |P_4 - P_1|^2 \tag{16}$$

Substitution P_1, P_4 from Eq. (13) and Eq. (15) into above equation and simplification gives;

$$C_1 \cos\phi + C_2 \sin\phi + C_3 = 0 \tag{17}$$

where

$$C_1 = 2d[(P_{0x} - P_{1x})\cos\theta + (P_{0y} - P_{1y})\sin\theta]$$
$$C_2 = -2dP_{1z}$$
$$C_3 = d^2 + |P_0|^2 + |P_1|^2 - 2(P_0 \cdot P_1) - l_1^2$$

From the above equation we can calculate ϕ as;

$$\sin\phi = \frac{-C_2C_3 \pm C_1\sqrt{C_2^2 + C_1^2 - C_3^2}}{C_2^2 + C_1^2} \tag{18}$$

Now the position vector P_4 and P_5 are determined by the Eq. (15).

The position vector P_6 with respect to a reference coordinate system {Tb} can be calculated by Eq. (9). Since the distance between P_6 and B_6 is L_6, we use the following constraint equation to calculate the position vector P_6.

$$(P_{6x}l_x + P_{6y}m_x + x - B_{6x})^2$$
$$+(P_{6x}l_y + P_{6y}m_y + y - B_{6y})^2$$
$$+(P_{6x}l_z + P_{6y}m_z + z - B_{6z})^2 = L_6^2 \tag{19}$$

Simplification of Eq. (18) and the directional cosine properties in Eq. (4) and (5) give as;

$$\begin{aligned} D_1m_x + D_2m_y + D_3m_z + D_4 &= 0 \\ l_xm_x + l_ym_y + l_zm_z &= 0 \\ m_x^2 + m_y^2 + m_z^2 &= 1 \end{aligned} \tag{20}$$

where,

$$D_1 = 2P_{6y}(P_{1x} - B_{6x})$$
$$D_2 = 2P_{6y}(P_{1y} - B_{6y})$$
$$D_3 = 2P_{6y}P_{1z}$$
$$D_4 = |P_6|^2 + |P_1|^2 + |B_6|^2 + 2P_{6x}(l_xP_{1x} + l_yP_{1y} + l_zP_{1z} - l_xB_{6x} - l_yB_{6y}) - 2(B_{6x}P_{1x} + B_{6y}P_{1y}) - L_6^2$$
$$l_x = P_{4x} - P_{1x}/|P_4 - P_1|$$
$$l_y = P_{4y} - P_{1y}/|P_4 - P_1|$$
$$l_z = P_{4z} - P_{1z}/|P_4 - P_1|$$

where P_{6x} and P_{6y} in Eq. (19) and (20) are the position vector components with respect to moving coordinate system {Tm}. Now we consider Eq. (20). First and second equation in Eq. (20) represent planes and third is a sphere. Therefore mx, my and mz are the intersection point between the straight line which is constructed by two planes intersection and the sphere. The intersection straight line can be expressed in a parametric form as;

$$\begin{aligned} m_x &= m_{xo} + ft \\ m_y &= m_{yo} + gt \\ m_z &= m_{zo} + ht \end{aligned} \tag{21}$$

where, m_{xo}, m_{yo} and m_{zo} is the point on the line of intersection nearest to the origin and f, g, and h is directional cosine of the line as;

$$f = D_2l_z - D_3l_y$$
$$g = D_3l_x - D_1l_z$$
$$h = D_1l_y - D_2l_x$$

Substitution Eq. (21) into the third equation in Eq. (20), then we can find m_x, m_y and m_z.

Using Eq. (6)-(8) we can also calculate n_x, n_y and n_z. Since we have calculated all components in Eq. (1), we can calculate the position vector P_6 with respect to a reference coordinate system using Eq. (1) as;

$$\left[{}^{b}P_{6}\right]=\left[{}^{b}T_{m}\right]\left[{}^{m}P_{6}\right] \quad (22)$$

3. Example

To illustrate the kinematic analysis mentioned in section 2, let us consider a numerical example, 6-DOF parallel manipulator, shown in Figure 2. The moving plate has six degrees of freedom. On the base plate the locations of the leg's ends(B_i, i=1,6) are arbitrary. On the moving plate, the three leg's ends are connected at one of the joint P_1, P_2, P_3, two leg's ends at the second joint P_4, P_5 and one leg end at the third joint P_6. The geometrical data and leg lengths are listed in Table 1. The results of the analysis are listed in Table 2 and Table 3.

The transformation matrix, the moving coordinate system which is attached on the moving plate with respect to fixed coordinate system which is attached on the base plate is described as;

$$T_1=\begin{bmatrix} .818 & -.576 & .016 & 39.120 \\ .575 & .816 & -.059 & 41.879 \\ .021 & .058 & .998 & 118.911 \\ .0 & .0 & .0 & 1. \end{bmatrix}$$

$$T_2=\begin{bmatrix} .818 & .292 & -.496 & 39.120 \\ .575 & -.383 & .722 & 41.879 \\ .021 & -.876 & -.482 & 118.911 \\ .0 & .0 & .0 & 1. \end{bmatrix}$$

Table 1. Geometrical Data of the mechanism

Coordinates of the joint on the base platform			
joint No.	x-coord.	y-coord.	z-coord.
1	0.0	0.0	0.0
2	100.0	0.0	0.0
3	150.0	70.0	0.0
4	100.0	140.0	0.0
5	0.0	140.0	0.0
6	-50.0	70.0	0.0

Lateral distance of the moving platform		
50.0	50. 0	50.0

Lengths of the legs					
132.0	140.0	165.0	140.0	160.0	150.0

Table 2. Coordinates of the joints on the moving platform

Coordinates of the joints on the moving platform				
No.	Joint	x-coord.	y-coord.	z-coord.
1	P_1	39.12000	41.87857	118.91094
	P_2	80.00000	70.64878	119.96003
	P_3	34.63762	91.58474	121.94496
2	P_1	39.12000	41.87857	118.91094
	P_2	80.00000	70.64878	119.96003
	P_3	72.21824	39.66071	81.49987
3	P_1	39.12000	41.87857	118.91094
	P_2	80.00000	35.38688	90.86306
	P_3	62.22591	81.43054	98.86702
4	P_1	39.12000	41.87857	118.91094
	P_2	80.00000	35.38688	90.86306
	P_3	44.91784	-.20034	92.53386
5	P_1	39.12000	41.87857	-118.91094
	P_2	80.00000	35.38688	-90.86306
	P_3	44.91784	-.20034	-92.53386
6	P_1	39.12000	41.87857	-118.91094
	P_2	80.00000	35.38688	-90.86306
	P_3	62.22591	81.43054	-98.86702
7	P_1	39.12000	41.87857	-118.91094
	P_2	80.00000	70.64878	-119.96003
	P_3	72.21824	39.66071	-81.49987
8	P_1	39.12000	41.87857	-118.91094
	P_2	80.00000	70.64878	-119.96003
	P_3	34.63762	91.58474	-121.94496

Table 3. Reversely computed lengths of the legs

Reversely computed lengths of the legs						
No.	L1	L2	L3	L4	L5	L6
1	132.000	140.000	165.000	140.000	160.000	150.000
2	132.000	140.000	165.000	140.000	160.000	150.000
3	132.000	140.000	165.000	140.000	160.000	150.000
4	132.000	140.000	165.000	140.000	160.000	150.000
5	132.000	140.000	165.000	140.000	160.000	150.000
6	132.000	140.000	165.000	140.000	160.000	150.000
7	132.000	140.000	165.000	140.000	160.000	150.000
8	132.000	140.000	165.000	140.000	160.000	150.000

$$T_3 = \begin{bmatrix} .818 & .062 & .572 & 39.120 \\ -.130 & .988 & .079 & 41.879 \\ -.561 & -.139 & .816 & 118.911 \\ .0 & .0 & .0 & 1. \end{bmatrix}$$

$$T_4 = \begin{bmatrix} .818 & -.338 & -.466 & 39.120 \\ -.130 & -.897 & .423 & 41.879 \\ -.561 & -.285 & -.777 & 118.911 \\ .0 & .0 & .0 & 1. \end{bmatrix}$$

$$T_5 = \begin{bmatrix} .818 & -.338 & .466 & 39.120 \\ -.130 & -.897 & -.423 & 41.879 \\ .561 & -.285 & -.777 & -118.911 \\ .0 & .0 & .0 & 1. \end{bmatrix}$$

$$T_6 = \begin{bmatrix} .818 & .062 & -.572 & 39.120 \\ -.130 & .988 & -.079 & 41.879 \\ .561 & .139 & .816 & -118.911 \\ .0 & .0 & .0 & 1. \end{bmatrix}$$

$$T_7 = \begin{bmatrix} .818 & .292 & .496 & 39.120 \\ .575 & -.383 & -.722 & 41.879 \\ -.021 & .876 & -.482 & -118.911 \\ .0 & .0 & .0 & 1. \end{bmatrix}$$

$$T_8 = \begin{bmatrix} .818 & -.576 & -.016 & 39.120 \\ .575 & .816 & .059 & 41.879 \\ -.021 & -.058 & .998 & -118.911 \\ .0 & .0 & .0 & 1. \end{bmatrix}$$

4. Conclusion

In this paper, the closed form solution of the forward kinematic analysis of a 6-DOF parallel robot manipulator has been derived. There are eight solutions in the above mentioned mechanism. Although all solutions are not physically meaningful, all solutions satisfy the constraints, the leg lengths. In this paper, we did not consider joint limits on the spherical and prismatic joints of the mechanism as well as interference between links. This work is refered to a further step toward closed form solution of the general 6-DOF parallel robot manipulator.

Reference

1. Stewart D., "A Platform with 6 Degrees of Freedom", Proc. Inst. Mech. Eng., Vol.180, pp.371-386, 1965.
2. K. H. Hunt, "Structural Kinematics of In-Parallel-Actuated Robot Arms.", Trans. ASME. J. Mechanisms, Transmissions, and Automation in Design, Vol.105, pp.705-712, 1983.
3. D.C.H.Yang and T.W.Lee, "Feasibility Study of a Platform Type of Robotic Manipulators from a Kinematic Viewpoint", Trans. ASME. J. Mechanisms, Transmissions, and Automation in Design", Vol.106, pp.191-198, 1984.
4. E.F.Fichter, "A Stewart Platform-Based Manipulator:General Theory and Practical Construction", The International Journal of Robotics Research, Vol.5, NO.2, pp.157-182, Summer, 1986.
5. K.M.Lee and D.K.Shah, "Kinematic Analysis of a Three Degrees of Freedom In-Parallel Actuated Manipulator", IEEE Conference of Robotics and Automation, pp. 345-350, 1987.
6. X.Shi and R.G.Fenton, "Solution to the Forward Instantaneous Kinematics for a General 6-DOF Stewart Platform", Mech-

anism and Machine Theory, Vol. 27, No. 3, pp. 251-259, 1992.
7. Jeans-Pierre Merlet, "Closed Form Resolution of the Direct Kinematicsof Parallel Manipulators using Extra Sensors Data", IEEE Conference of Robotics and Automation, pp. 200-204, 1993.
8. Kevin Cleary and Tatsus Arai, "A Prototype Parallel Manipulator:Kinematics, Construction, Software, Workspace Result, and Singularity Analysis", IEEE Conference of Robotics and Automation, pp. 566-571, 1991.