論文 95-6-3-7

레일레이 페이딩채널에서 선택성 결합방식의 공간다이버시티를 사용한 QAM의 심벌오율

(Symbol Error Rates of QAM with Selection Combining Space Diversity in Rayleigh Fading Channels)

김창주*, 김영수*, 정구영*, 문재경*, 이혁재**

(Chang-Joo Kim*, Young-Su Kim*, Goo-Young Jeong*, Jae-Kyoung Mun*, Hyuck-Jae Lee**)

Abstract

This paper derives the symbol error rate (SER) for quadrature amplitude modulation (QAM) with L-fold selection combining (SC) space diversity in Rayleigh fading channel. No analysis has been reported yet for theoretical SER performance of QAM with SC space diversity in Rayleigh fading channels. The formula is obtained by averaging the symbol error probability of M-ary QAM in an additive white Gaussian noise channel over the distribution of the maximum signal-to-noise ratio among all of the diversity channels. By giving the order of diversity, L, and the number of signal points, M, we have been able to obtain the SER performance of QAM with general SC space diversity. Analytical results show that the probability of error decreases with the order of diversity. We can also see that the incremental diversity gain per additional diversity decreases as the number of branches becomes larger.

I. Introduction

Digital cellular systems have been widely developed to provide mobile communication service. With the increasing demands of the service, an important topic is to use a spectrally efficient modulation technique to raise the spectrum efficiency in the limited frequency bandwidth. Quadrature amplitude modulation (QAM) is an effective technique to achieve high spectral effciency in additive white Gaussian noise (AWGN) channel. Also it is a good candidate for high spectral efficiency in a Rayleigh fading channel with channel state information (CSI).

In order to achieve high spectral efficiency in the land mobile communication system, Sampei et al. [1,2,3] have introduced M-ary QAM with two-branch maximal ratio combining (MRC) space diversity and they have employed a pilot symbol assisted method (PSAM) to obtain the CSI. Space diversity is a well-known technique to

* 전파신호처리연구실, 전파공학연구부, 이동통신기술연구단, 한국전자통신연구소
(Radio Signal Processing Section, Radio Science Department Mobile Communicaton Division, ETRI)
** 전파공학연구부, 이동통신기술연구단, 한국전자통신연구소
(Radio Science Department Mobile Communication Division, ETRI)

combat multipath fading in mobile radio communications. As a result of computer simulation and laboratory experiments, they have obtained a desirable performance of QAM with MRC space diversity based on the PSAM channel sounding technique. By the way, their theoretical analysis has been limited only to the QAM with two-branch MRC space diversity. In [4], Kim et al. have extended the order of diversity to the general MRC space diversity. However, no analysis has been reported yet for theoretical symbol error rate (SER) of QAM with selection combining (SC) space diversity in Rayleigh fading channels.

In this paper, we derive the expression of the symbol error probability for QAM with general SC space diversity in Rayleigh fading channels. The formula derived is obtained by averaging the symbol error probability of QAM in AWGN channel over the distribution of the maximum signal-to-noise ratio among all of the diversity channels. The main advantages of SC diversity, in which only one signal at a time coming from two or more spatially separated antennas is connected to a detector, are its relative simplicity and lower cost, since it needs only one receiver, regardless of the number of antennas employed. On the other hand, MRC requires the use of the same number of receivers as antennas and is more complex.

This paper is organized as follows. Following Introduction, the probability of symbol error of QAM for SC space diversity is derived in Section Ⅱ. Some results are discussed in Section Ⅲ and Conclusions are made in Section Ⅳ.

Ⅱ. Probability of Symbol Error

Let us consider only the rectangular signal sets. For such signal structure, the $M=2^k$ signal points result in a symmetrical form of QAM when k is even. In this case, QAM can be viewed as two separate pulse amplitude modulation signals impressed on phase-quadrature carriers. The probability of symbol error for M-ary QAM signals in the AWGN channel can be expressed as [5]

$$P_{bnon}=2\left(1-\frac{1}{\sqrt{M}}\right)erfc\left[\sqrt{\frac{3k}{2(M-1)}\frac{E_b}{N_o}}\right]\times\left[1-\frac{1}{2}\left(1-\frac{1}{\sqrt{M}}\right)erfc\left\{\sqrt{\frac{3k}{2(M-1)}\frac{E_b}{N_o}}\right\}\right] \quad (1)$$

where

$$erfc(x)=\frac{2}{\sqrt{\pi}}\int_x^\infty exp(-t^2)dt.$$

The frequency nonselective channel results in multiplicative distortion of the transmitted signal. The condition that the channel fades slowly implies that the multiplicative process may be regarded as a constant during at least on signaling interval. For a fixed attenuation α, (1) can be repr- esented as

$$P_{bnon}(\gamma_b)=2\left(1-\frac{1}{\sqrt{M}}\right)erfc\left\{\sqrt{\frac{3k}{2(M-1)}\gamma_b}\right\}\times\left[1-\frac{1}{2}\left(1-\frac{1}{\sqrt{M}}\right)erfc\left\{\sqrt{\frac{3k}{2(M-1)}\gamma_b}\right\}\right] \quad (2)$$

where

$$\gamma_b=\alpha^2\frac{E_b}{N_o}$$

We view (2) as a conditional symbol error probability, where the condition is that α is fixed. To obtain the error probability when α is random, we average P_{bnon} over the probability density function (pdf) of γ_b. That is,

$$P_{bfad} = \int_0^\infty P_{bnon}(\gamma_b) P(\gamma_b) d\gamma_b \quad (3)$$

where $P(\gamma_b)$ is the probability density function of γ_b when α is random.

In SC [6,7], This scheme is to select signal with the largest instantaneous power from the L branch signals coming from the different antennas. It is assumed that the received signal has a Rayleigh fading envelope. This is a reasonable assumption for the case in which the transmitted signal is reflected by a multitude of scatters surrounding the received antennas. It is also assumed that the signals coming from the L different antennas are statistically independent and identically distributed (iid) random processes. This assumption will be true if the spatial separation between any two antennas is greater than half a wavelength of the carrier singal.

Now we evaluate the symbol error probability in a traditional way by averaging the result for a time invariant channel over the distribution of the maximum SNR among of the diversity receptions. The probability density function of the maximum selection SNR over L iid diversity paths is well-known. Let z be the maximum SNR. The mean of each branch signal is assumed to be equal. Under these conditions, the pdf is given by [6]

$$\begin{aligned} P_z(z) &= \frac{L}{\bar{\gamma}_b}(1-e^{-z/\bar{\gamma}_b})^{L-1} e^{-z/\bar{\gamma}_b} \\ &= \frac{L}{\bar{\gamma}_b} \sum_{k=0}^{L-1} (-1)^k \binom{L-1}{k} e^{-(k+1)z/\bar{\gamma}_b} \end{aligned} \quad (4)$$

Let us transform (2) into another form.

$$P_{bnon} = \frac{\sqrt{M}-1}{M} \times \{(\sqrt{M}+1) - 2\, erf(\sqrt{\beta\gamma_b}) - (\sqrt{M}-1) erf^2(\sqrt{\beta\gamma_b})\} \quad (5)$$

where

$$\beta = \frac{3k}{2(M-1)}$$

Then, (3) is obtained by

$$P_{bfad} = \frac{\sqrt{M}-1}{M} \cdot \{(\sqrt{M}+1) - 2I_1 - (\sqrt{M}-1) I_2\} \quad (6)$$

where

$$\begin{aligned} I_1 &= \frac{L}{\bar{\gamma}_b} \sum_{k=0}^{L-1} (-1)^k \binom{L-1}{k} \int_0^\infty erf(\sqrt{\beta\gamma_b}) e^{-(k+1)\gamma_b/\bar{\gamma}_b} d\gamma_b \quad (7) \\ &= L \sum_{k=0}^{L-1} (-1)^k \frac{1}{k+1} \binom{L-1}{k} \sqrt{\frac{3k\bar{\gamma}_b}{2(M-1)(k+1)+3k\bar{\gamma}_b}} \end{aligned}$$

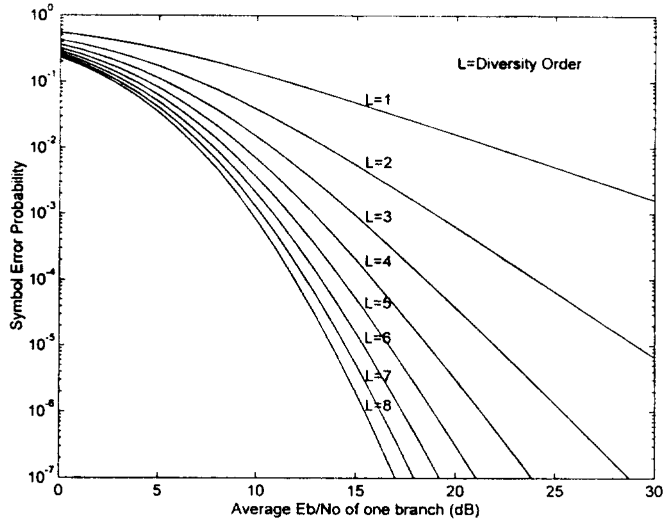
and

$$\begin{aligned} I_2 &= \frac{L}{\gamma_b} \sum_{k=0}^{L-1} (-1)^k \binom{L-1}{k} \int_0^\infty erf^2(\sqrt{\beta\gamma_b}) e^{-(k+1)\gamma_b/\bar{\gamma}_b} d\gamma_b \\ &= \frac{4L}{\pi} \sum_{k=0}^{L-1} (-1)^k \frac{1}{k+1} \binom{L-1}{k} \frac{1}{\sqrt{1+\frac{\bar{\gamma}_b}{(k+1)\beta}}} \tan^{-1} \left(\frac{1}{\sqrt{1+\frac{\bar{\gamma}_b}{(k+1)\beta}}} \right) \end{aligned} \quad (8)$$

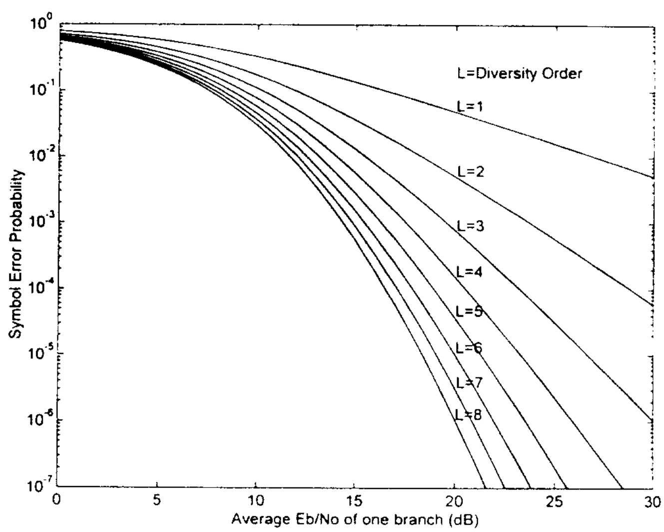
In deriving (8), we make use of the mathematical formula in [8].

Ⅲ. Results and Discussion

In this Section, the SER performance of QAM with SC space diversity is obtained from (6). [Fig. 1] shows the SER performance of 16QAM with SC space diversity in Rayleigh fading environments for L=1~8. [Fig. 2] shows the SER performance of 64QAM with SC space diversity in Rayleigh fading environments for L=1~8. From figures 1 and 2, we can see that the probability of error decreases with the order



〔Fig. 1〕 Performance of 16 QAM with SC space diversity in Rayleigh fading channels.



〔Fig. 2〕 Performance of 64 QAM with SC space diversity in Rayleigh fading channels.

of diversity. The results illustrate the advantage of diversity as a means for combating the fading phenomena. We can also see that the incremental diversity gain per additional diversity decrease as the number of branches becomes larger.

Ⅳ. Conclusions

In this paper, we have analyzed the symbol error probability for QAM with general SC space diversity in Rayleigh fading channels. By choosing the order of diversity, and the number of signal points from the derived formula, we can obtain the SER performance of the QAM with

SC space diversity in Rayleigh fading channels. Analytical results show that the probability of error decreases with the order of diversity. We can also see that the incremental diversity gain per additional diversity decreases as the number of branches becomes larger. By giving the order of diversity, L, and the number of signal points, M, we have been able to obtain the SER performance of QAM with general space diversity. We can also obtain the SER performance of quadrature phase shift keying with SC space diversity in Rayleigh fading channels from the derived formula by setting M=4.

References

[1] S. Sampei and T. Sunaga, "Rayleigh Fading Compensation Method for 16 QAM in Digital Land Mobile Channels," *39th IEEE veh. Technol, Conf.*, pp.640-646, May 1989.

[2] S. Sampei and T. Sunaga, "Rayleigh Fading Compensation for QAM in Land Mobile Radio Communications," *IEEE Trans. Veh. Technol.*, vol.VT-42, no.2, pp.137-147, May 1993.

[3] T. Sunaga and S. Sampei, "Performance of Multi-Level QAM with Post-Detection Maximal Ratio Combining Space Diversity for Digital Land-Mobile Radio Communications," *IEEE Trans. Veh. Technol.*, vol.VT-42, no.3, pp. 294-301, Aug. 1993.

[4] C.J. Kim, Y.S. Kim, G.Y. Jung, and H.J. Lee, "BER Analysis of QAM with MRC Space Diversity in Rayleigh Fading Channels," *6th IEEE PIMRC Conf.*, pp.482-485, Sept. 1995.

[5] J.G. Proakis, *Digital Communications*. New York: McGraw-Hill, 1989.

[6] G.T. Chyi, J.G. Proakis, and C.M. Keller, "On the Symbol Error Probability of Maximum-Selection Diversity Reception Schemes Over a Rayleigh Fading," *IEEE Trans. Commun.*, vol.COM-37, no.1, pp.79-83, Jan. 1989.

[7] C. Leung, "Optimized Selection Diversity for Rayleigh Fading Channels," *IEEE Trans. Commun.*, vol.COM-30, no.3, pp.554-557, Jan. 1982.

[8] D.B. Owen, "A Table of Normal Integrals," *Commun. Stat. Simul. Comput.*, vol.B9(4), pp. 389-419, 1980.