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MODEL ON THE DYNAMIC BEHAVIOR OF CONDUCTIVE FERRO-MAGNETIC MATERIAL WITH NEGLIGIBLE COERCIVITY

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Differential equations governing dynamic behavior of toroid-shaped ferro-magnetic material having a small gap of uniform width were derived incorporating Maxwell equations of electromagnetic induction relevent to the system and Newtonian equation of motion. Once the external uniform magnetic field was applied within the material through dc-circuit around the toroid, gap begin to change which lead to the abrupt variation of field in the material and gap according to the differential equations already derived. Characteristics of current and electromotive force with respect to time in the circuit consisting of inductance and resistance in series could be predicted from numerical solutions of these equations. As current in the circuit increases, magnetic field in the material increases, thus, the gap starts to shrink due to increased attractive force between the gap surfaces. As current decreases the gap become to expand due to reduced attraction between gap and elastic restoring force in the material. With an appropriate selection of elastic constant of toroidal ferromagnetic material and design of gap structure it is possible to obtain the specified in both linear and nonlinear magnetic characteristics , such as current dependent and independent inductance.

I. INTRODUCTION

Usually ferromagnetic material of high permeability reaches the saturation magnetization at very weak field strength applied which is very efficient in application where large magnetic flux is required for very small field signal represented as electric,magnetic etc. However this kind of substance cannot be used for high field application due to very small threshold field strength above which any change in magnetization is not observed. Therefore for higher fields applications it be required that the substances should have relatively large permeability and methods to extend the range where μ can be effective. It includes giving a small gap in the toroid substances. Even a slight change in gap width causes an enormous variation in magnetic field within both the material and the gap according to Eq. [1][1],[2]

$$B = \frac{\mu H_{ext}}{1+\mu d/\ell} \qquad [1]$$

A large value of inductance can be obtained due to $\mu \gg 1$,when there is no gap i.e. $d/\ell = 0$[1]. Given a gap effective permeability $\frac{\mu}{1+\mu d/\ell}$ can be reduced significantly due to presence of gap. Once gap starts to be closer by attraction between poles, induced at each inner surfaces, it is possible to increase effective permeability, which lead to the increased inductance. As long as magnetic field in substance is varying it could be worked as power transmitting device such as inductor. In this work transient and steady state behavior of magnetic field and the relevent motion in the gap structures were predicted according to the series of governing differential equations derived, based on Maxwell equations Newton's equations of motion.

II. THEORETICAL

Suppose a ferromagnetic ring(toroid) of permeability μ, circumferential length, ℓ cross section A is uniformly wound with n-turns of wire carrying current i. Because μ of air is so small compared to that of ferro-magnetic material the presence of gap of variable length, d, greatly influence the magnetic field within the substance and gap according to Eq.[1]. The distribution of magnetic field across the radial position, B(r), is derived as follows;

$$\nabla \times \nabla \times B = g \nabla \times E + \varepsilon \frac{\partial}{\partial t}(\nabla \times E) \qquad [2]$$

where g and ε are conductivity and dielectric constant respectively. If we assume the conductive media, i.e. $\varepsilon/g\mu \ll 1$, where we can neglect the second of RHS in Eq. [2]. Based on vector theorem and mathematical manipulations[1] equations describing B across the radius are Eqs. [3] and [4], assuming B is harmonically varying with respect to time[3],[4];

$$\frac{\partial^2 B}{\partial t} + \frac{1}{r}\frac{\partial B}{\partial t} - g\mu \frac{\partial B}{\partial t} = 0 \qquad [3]$$

$$B = B(r)\exp(j\omega t) \qquad [4]$$

Substituting Eq.[4] into [3] results in radial distribution equation for B as follows;

$$r^2 \frac{\partial^2 B}{\partial t} + r\frac{\partial B}{\partial t} - jg\omega r^2 \mu B = 0 \qquad [5]$$

Solution of Eq. [5] is Eq. [6] with the boundary value B_o at the surface

$$B(r,t) = B_0 \frac{\left\{ber^2\left(\frac{\sqrt{2}}{\delta}r\right) + bei^2\left(\frac{\sqrt{2}}{\delta}r\right)\right\}^{\frac{1}{2}} e^{j\omega t}}{\left\{ber^2\left(\frac{\sqrt{2}}{\delta}R\right) + bei^2\left(\frac{\sqrt{2}}{\delta}R\right)\right\}^{\frac{1}{2}}} \qquad [6]$$

$$B_0 = \frac{\mu H_{ext}}{1+\mu d/\ell} = \frac{\mu n i_o}{1+\mu d/\ell} \qquad [7]$$

where $\delta = \sqrt{\frac{\rho}{\pi f \mu}}$ and B_0 is the magnetic field at surface of conductor, induced by external field H_{ext} which is proportional to maximum external current i_o. If the ratio $R/\delta \ll 1$ i.e., low frequency, low permeability, small radius, we can assume there is no radial variation in magnetic field as shown in Figs. 1 and 2. Therefore time independent spatial part of B(r,t) and subsequent magnetic flux can be written as Eq. [8][5];

$$B = \frac{\mu n i}{1+\mu d/\ell}; \quad \Phi = nBA = \frac{\mu n^2 A i/\ell}{1+\mu d/\ell} \qquad [8]$$

And the inductance L can be written as Eq. [9]

$$L = \frac{\mu A n^2/\ell}{1+\mu d/\ell} \qquad [9]$$

Emf ε is induced from the rate of magnetic flux according to Eq. [10]

$$\varepsilon = \frac{d\Phi}{dt} = \frac{d}{dt}\left(\frac{\mu/\ell A n^2 i}{1+\mu d/l}\right)$$

$$= \frac{A n^2 \mu}{l}\frac{d}{dt}\frac{i}{1+\mu d/l} \qquad [10]$$

The attraction force between pole surfaces can be derived from energy, W, stored in the gap volume.

$$W = \int B^2 dv \qquad [11]$$

$$F = \frac{\partial W}{\partial d} = B^2 A = \frac{\mu^2 n^2 A\, i^2}{\ell^2(1+\mu d/\ell)^2} \qquad [12]$$

Considering substance as an elastic medium having elastic constant k mass m and initial length of gap d_o, the difference between pole-pole attraction and restoring force in the elastic medium can contribute to the motion of gap, as expressed in Eq.[13];

$$m\frac{\partial d}{\partial t} = k \times (d_o - d) - \frac{\mu^2 n^2 A\, i^2/\ell^2}{(1+\mu d/\ell)^2} \qquad [13]$$

Current can be determined at each instant from following Eq. [14];

$$\frac{d\Phi}{dt} = \frac{A n^2 \mu}{\ell}\left[\frac{di}{dt}\frac{1}{1+\mu d/\ell} - \frac{i\mu\frac{\partial d}{\partial t}}{(1+\mu d/\ell)^2}\right]$$

In circuit consisting of inductance and resistance in series Eq. [15] holds;

$$\frac{d\Phi}{dt} + R\, i = \varepsilon \qquad [15]$$

If we substitute Eq [13], [14] for Eq [15] following equation for current in the circuit at any moment t can be obtained:

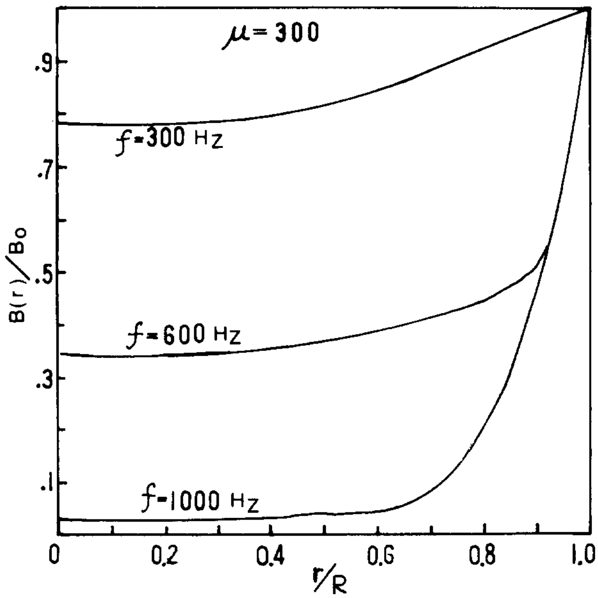

Fig. 1 Radial distribution of magnetic field intensity.

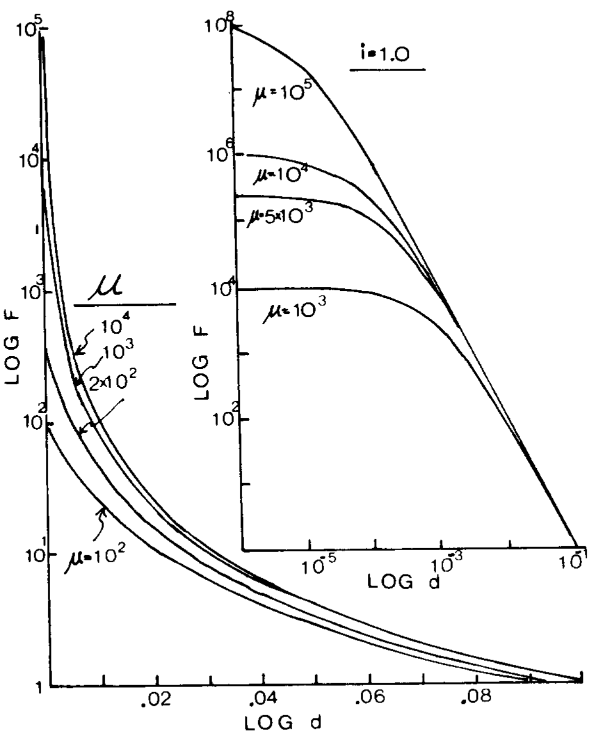

Fig. 2 Semi-log plot of attraction between poles versus gap length under the various permeabilities.

$$\frac{di}{dt} = -\frac{\mu}{\ell}\frac{dx}{dt}(1+\mu x/\ell)\, i - \frac{(1+\mu x/\ell)R}{\mu/\ell\, An^2}\, i + \frac{(1+\mu x/\ell)}{\mu/\ell\, An^2}\,\varepsilon \qquad [16]$$

where x and ε represent length of gap at time t and emf applied to the circuit as the form of such as ε(dc), $\varepsilon_0 \sin\omega t$ respectively. When gap start to vibrate current, length of gap and velocity of gap are subject to vary according to the differential equations [13]-[16]. Parameters describing the system can be determined from numerical integration of the equations derived. Setting and substituting the initial conditions, t=0, $d = x = d_o$, and $i = 0$, for Eq.[16], the current at t= t + Δt can be determined. Width of gap at the next very moment also can be given from insertion of newly obtained i into Eq.[13]. From successive iteration of this process(Runge-kutta 4th) all the parameters describing the system as i, d, dx/dt, ε etc at any instant t would be predected.

If we consider the gas in gap-volume as compressible media equation of motion must be modified due to the damping nature of the fluid in the volume. Suppose the compression/expansion cycle caused by motion of gap be adiabatic process. Relation between pressure and volume of gap can be written as Eq. [17]

$$PV^{\gamma} = P_0 V_0^{\gamma}\ ; \quad AP = AP_0\left(\frac{d_o}{d}\right)^{\gamma} \qquad [17]$$

where P_0, V_0, γ are pressure, volume and constant dependent on the type of gas ,respectively, at time $t=0$. Finally the equation of motion of the gap must be transformed to Eq. [18];

$$m\frac{d^2x}{dt^2} = k\,(d_o - x) + P_0 A\left[\left(\frac{d_o}{x}\right)^{\gamma} - 1\right] - \frac{n^2 A\mu^2 i^2}{(1+\mu x/\ell)^2} \qquad [18]$$

Ⅲ.RESULTS AND DISCUSSION

As the width of gap decreases, attractive force between the poles increases according to Eq. [12] as shown in Fig. 3, where the rate of increase is pronounced in region of very narraow gap. Figs. 4, 5, and 6 illustrate how gap and current are varying w.r.t. time when dc-emf is applied to the circuit. From the figures it is seen that d and i are oscillating toward the fixed value with decreasing amplitude and the rate of variation is getting faster as time progresses. Fig. 6 shows the oscillation of current w.r.t. time when the magnitude of dc-emf applied is 10 times greater than that of Fig. 5 where amplitude and rate of oscillation becomes significantly increased. Fig. 6 represents variation of current w.r.t. time when the external emf in the form of $\varepsilon_o \sin(\omega t)$ is supplied to the circuit varying elastic constants. As seen in the figure with elastic constant varying, the longer time interval is required for the suppression of current at initial stage. When the gap is held fixed it reaches the maximum current of scale 9 at time 0.003 compared to that of less than 4 when the gap were set in motion. It is observed that there is certain period of time during which current remains to be zero. This can be explained from the discrepancies of phases between attraction and restoring force at the poles of gap. Fig. 7 represents variation of gap distance with the progress of time under the different values of elastic constants and resistances. It is seen that the range of variation is much greater when the R is larger. This can be explained from the fact that large current can cause the increased attraction which lead to the wide-opening of gap. Fig.8 shows how current responds with time when alternating emf is applied to the circuit where current-voltage follows the relation $i=[\exp(V^2)-1]$ typical characteristics of gas-discharge process in which even a slight increments in voltage yields a tremendous current boost. Because the gap is in motion current does not exceed the range -4 to 5.5

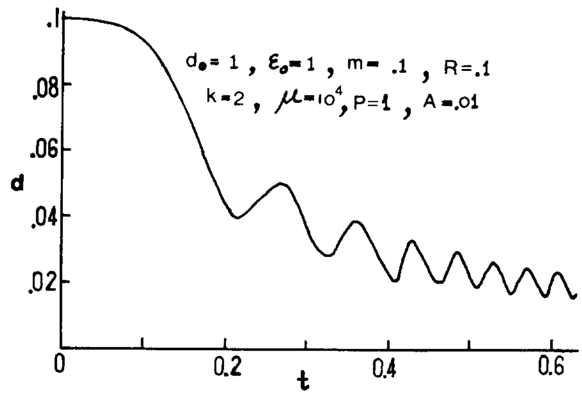

Fig. 3 Variaton of gap width w.r.t. time at dc emf ε_o=1 (dc).

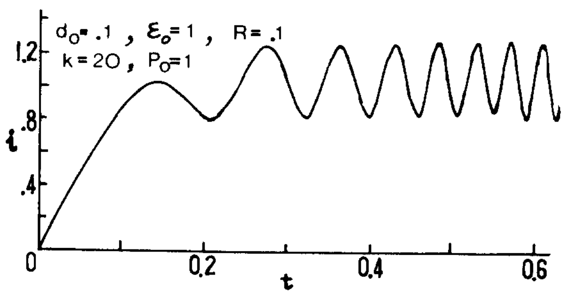

Fig.4 Variation of current w.r.t. time at dc emf ε_o=1 (dc).

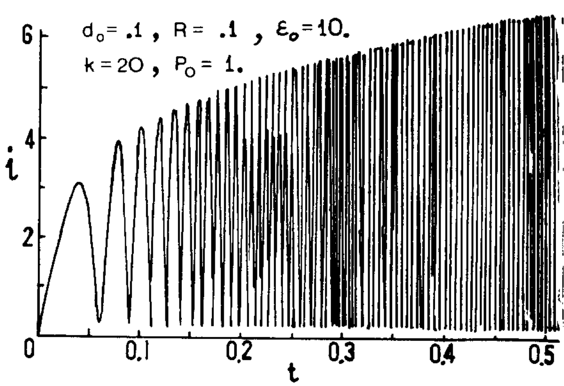

Fig. 5 Variation of current w.r.t. time at ε_o= 10 (dc).

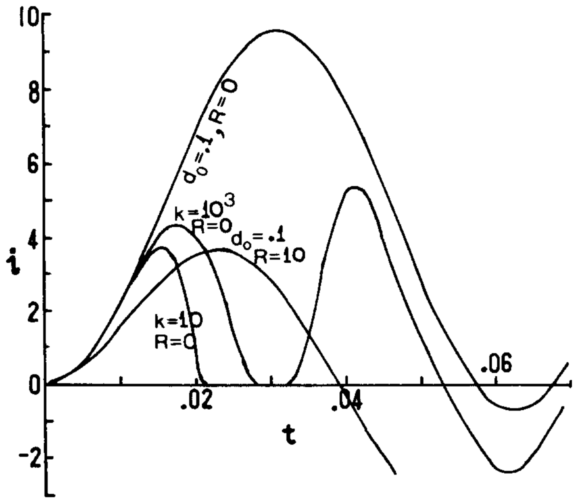

Fig.6 Variation of current with time at various k, d and R

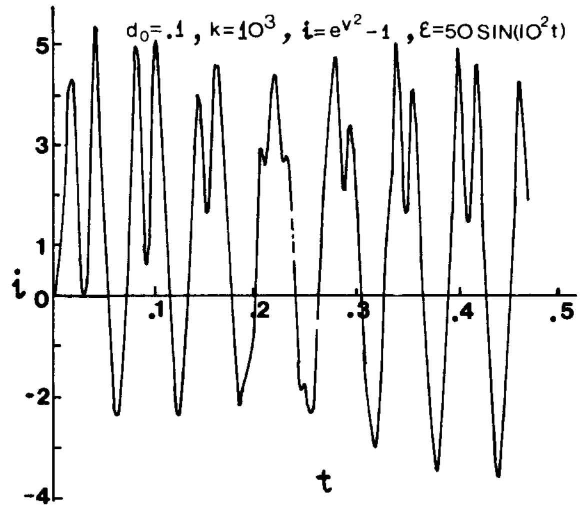

Fig.8 Variation of current w.r.t. time at various k and R.

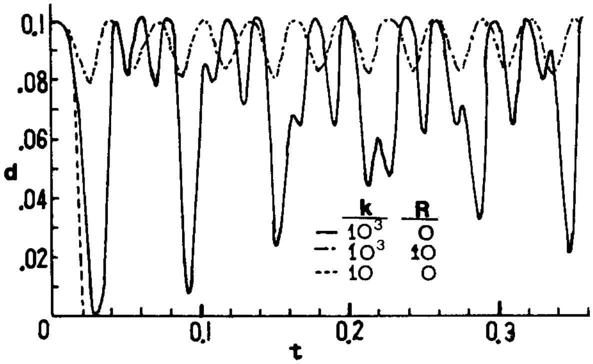

Fig.7 Variation of gap width w.r.t. time when the sinusoidal emf is applied in the circuit.

which means that current can be well suppressed.

IV. CONCLUSIONS

A model for predicting the characteristics of elastic ferro-magnetic materials having a moving gap was presented. Based on the model parameters concerning behavior of material, such as the instantaneous field intensity, attractive force between the poles, length of gap, and induced current/ emf in the circuit can be determined from numerical calculation of the governing equations. One of the applications might be the inductor with movable gap and adjustable elastic constant, which can be effective in the suppression of large surge current as in the gas discharge process.

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