The Korean Journal of Rheology,
Vol. 7, No. 3, December 1995 pp. 261-266

〈연구논문〉

원관에서 POWER-LAW 유체의 수력학적 입구길이와 입구보정계수에 관한 연구

김선철 · 오광석 · 김윤제* · T.F. Irvine · Jr. · T. Yamasaki****

한국원자력연구소, *성균관대학교, **뉴욕주립대학교 기계공학과
(1995년 11월 23일 접수)

Hydrodynamic Entrance Lengths and Entrance Correction Factors for a POWER-LAW Fluid in a Circular Duct

S.C. Kim, K.S. Oh, Y.J. Kim*, T.F. Irvine, Jr. and T. Yamasaki****

Korea Atomic Energy Research Institute, P.O. Box 105, Yusong, Taejon 305-606, Korea
*Dept. of Mechanical Design, Sung Kyun Kwan Univ., Suwon 440-746, Korea
**Dept. of Mechanical Engineering, SUNY at Stony Brook, Stony Brook, New York 11794, USA
(Received November 23, 1995)

요 약

원관에서 power-law 유체에 대하여 수력학적 입구길이와 입구보정계수를 측정할 수 있는 새로운 방법이 개발되었다. 유변학적 성질을 측정할 수 있는 긴 관과 입구보정계수를 측정할 수 있는 짧은 관을 가진 새로운 모세관 점도계(capillary tube viscometer)를 이용하여 증류수를 실험한 결과 유변학적 성질과 입구 보정계수가 표준값과 비교하여 1%안의 오차를 얻었다. Power-law 유체에 대한 해석 및 실험결과(Carbopol 960 용액)도 이미 보고된 값과 ±6% 이내로 잘 일치하였다.

Abstract - A new technique to measure the hydrodynamic entrance length and entrance correction factor for a power-law fluid in a circular duct has been developed. By using a newly designed capillary tube viscometer which has two tubes i.e. the long tube to measure rheological properties (n and K) and the short tube to measure the entrance correction factor C. Measurements were made with distilled water to verify the present experimental work. As a result, less than 1% difference including rheological properties as compared with the standard values appeared to be satisfactory. For a power-law fluid, present analysis and experiments(for Carbopol 960 solutions) are in good agreement within ±6% with other investigation reported previously.

Keywords: Rheological property, entrance length, entrance correction factor, capillary tube viscometer, power-law fluid.

1. Introduction

In order to properly design piping systems, heat exchangers, etc. for non-Newtonian Fluids, a knowledge of hydrodynamic entrance lengths and associated entrance pressure correction factors is required. For Newtonian fluid systems, such information is readily available, for example, see reference[1]. However, for even the simplest purely viscous non-Newtonian fluid, i.

e. a power-law fluid, conflicting information had been reported in the literature. A power-law fluid is one in which the shear stress at a particular location in a flow field is related to the local velocity gradient by

$$\tau = K\left(\frac{du}{dr}\right)^n = K\dot{\gamma}^n \tag{1}$$

where K and n are rheological properties called the fluid consistency and flow index respectively. The velocity gradient with the symbol $\dot{\gamma}$ is called the shear rate. Another convenient way to write Eq. (1) is to define an apparent viscosity as the shear stress divided by the shear rate giving

$$\tau/\dot{\gamma} = \eta_a = K(\dot{\gamma})^{n-1} \tag{2}$$

Eq. (2) indicates that in Log-Log coordinates the apparent viscosity plotted against the shear rate yields a straight line whose slope is (n-1) and whose η_a intercept when $\dot{\gamma}=1$ is log K. This is the experimental basis for measuring these rheological properties.

There have been several analytical and experimental investigation of the entrance correction factors published previously: measurements by Laniève[2] and Boger and Ramarmurthy[3] and analyses by Collins and Schowalter[4] and Bogue[5]. Considerable differences exist in the results of the several papers especially in the experimental measurements. Accordingly, an investigation was carried out to calculate the entrance length and entrance correction factors for a power-law fluid and to confirm the entrance correction factor calculations by experimental measurements.

2. Analysis

For practical purposes, the hydrodynamic entrance length for a circular duct is usually defined as the length where the center line velocity is within 1% of its fully developed value assuming that the entrance velocity profile is flat. Thus, it is necessary to calculate the changing velocity profiles throughout the entrance region.

The entrance correction factor is most conveniently defined by assuming that the total pressure drop in duct flow is the sum of the pressure drop which would occur if the flow were fully developed throughout the duct length plus an entrance correction factor, C.

$$\frac{p_0 - p(z)}{\rho \bar{u}^2/2} = f\frac{z}{d} + C \tag{3}$$

where $C = C_1 + C_2 + C_3$

C_1=The pressure loss due to the kinetic energy increase as the fluid goes from zero velocity in the reservoir to the average velocity inside the tube.

C_2=The kinetic energy loss caused by the velocity profile change in the entrance region.

C_3=The additional wall shear loss in the entrance section since only the fully developed value is built into the friction factor f.

The value of C_1 is unity because this is simply the conversion of pressure energy to kinetic energy. C_2 can be found by assuming the whole flow to be one large stream tube with average velocity $\bar{u}$ at any arbitrary cross section of the tube. Therefore, C_2 can be calculated from the expression[6].

$$C_2 = \frac{1}{A}\int u^{+3} dA - \frac{1}{A}\int dA \tag{4}$$

Using Eq. (4) C_2 can be calculated from the fully developed velocity profile for a power-law fluid in a circular duct. The result of such a calculation is

$$C_2 = \frac{3(1+3n)^2}{(1+2n)(3+5n)} - 1 \tag{5}$$

Because C_3 can not be calculated directly, it can be obtained by solving the momentum and continuity equations for the total correction factor C and subtracting C_1 and C_2 from it.

The governing equations to be solved in dimensionless form can be written as:

Momentum:

$$u^+\frac{\partial u^+}{\partial z^+}+v^+\frac{\partial u^+}{\partial r^+} = -\frac{dp^+}{dz^+}-\frac{1}{r^+}\frac{\partial}{\partial r^+}[r^+(-\frac{\partial u^+}{\partial r^+})^n] \quad (6)$$

Continuity:

$$\frac{\partial u^+}{\partial z^+}+\frac{1}{r^+}\frac{\partial}{\partial r^+}(v^+r^+)=0 \quad (7)$$

Global Continuity:

$$2\int_0 u^+ r^+ dr^+ = 1 \quad (8)$$

For fully developed flow, the left side of Eq. (6) is zero and the equation can be integrated directly to obtain the velocity profile as

$$u^+ = \frac{1+3n}{1+n}\left(1-r^{+\frac{2+n}{n}}\right) \quad (9)$$

For the developing flow region, since the velocity field is the unknown variable and the local apparent viscosity of the power law fluid is a function of the velocity field, a numerical iterative solution is required. The details of the calculation have been reported in the thesis by Yamasaki[7] and only the results will be presented here.

Since entrance correction factors for Newtonian fluids in a circular duct have been reported extensively, the program was first run for such a system. Table 1 shows the results of the present analysis as compared with previous calculated and experimental values. The calculated value of C=2.249 compares favorably with previously published results.

Next, the values of C and the hydrodynamic entrance lengths were calculated as a function of the power-law flow index, n. The results are shown in Table 2 and 3. A comparison of the entrance correction factors with previously published analytical and experimental results will

Table 1. Comparison of entrance correction factors of Newtonian fluid

Theoretical Investigator	C
Atkinson and Goldstein [9]	2.41
Brogue [5]	2.16
Boussinesq [10]	2.24
Compbel and Slattery [11]	2.18
Collins and Schwalter [4]	2.33
Langhaar [12]	2.28
Siegel(cubic profile) [13]	2.08
Siegel(quadratic profile) [13]	2.24
Sparrow, Lin and Lungren [14]	2.24
Tomita [15]	2.22
Present Analysis	2.249
Experimental Investigator	**C**
Dorsey [16]	2.08, 200
Knibbs [17]	2.27±8%
Nikuradse [18]	2.32
Rieman [19]	2.248±1%
Schiller [20]	2.32±10%
Weltman and Keller [21]	2.2±10%

Table 2. Calcuated correction factor for pseudoplastic fluids

n	C	C_1	C_2	C_3	$C_3/C(\%)$
1.0	2.2490	1	1.000	0.2490	11.07
0.9	2.1806	1	0.9557	0.2249	10.31
0.8	2.1066	1	0.9055	0.2011	9.55
0.7	2.0256	1	0.8481	0.1755	8.76
0.6	1.9358	1	0.7818	0.1540	7.96

Table 3. Calculated entrance lengths

n	$(Le/d)/Re_g'$
1.0	0.0545
0.9	0.0543
0.8	0.0537
0.7	0.0529
0.6	0.0515
0.5	0.0494

be discussed later.

3. Experiment

Values of C were measured using a double tube capillary viscometer as illustrated in Fig. 1. For a single vertical tube, the following equation can be written

$$p_g = \frac{64}{Re_g'}\frac{L}{d}\left(\frac{\rho \bar{u}^2}{2}\right) - \rho g(H+L) + C\frac{\bar{u}^2}{2} \qquad (10)$$

Eq. (10) could be used to determine C if the rheological properties n and K from the power-law equation were known. However, they are best measured in a long tube where the unknown term which includes C is negligible. Under these conditions, however, since the C term is small, it is difficult to determine experimentally. This problem is resolved in the present experiment by using the long tube to measure n and K and the short tube to measure C. Details of the experimental apparatus have

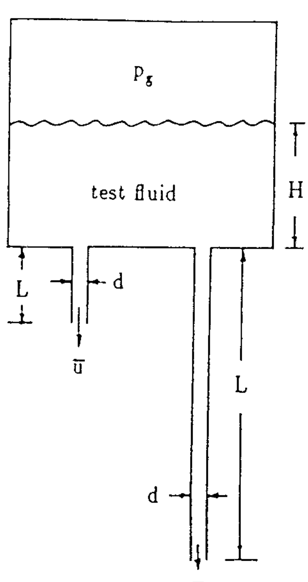

Fig. 1. Schematic digram of capillary tube viscometer to measure the entrance correction factor.

been reported in [8] and again only the results will be presented here.

The first measurements were made with distilled water, a Newtonian fluid, where the flow index, n, is equal to one and the consistency K is equal to the dynamic viscosity μ. The results also shown in Fig. 2, where $n=1.004$, $K=\mu=8.943\times10^{-4}$ N · s/m², C=2.23 (*Temp.*=25.0℃).

The results were that n was different from its value of unity by 0.14%, μ was different from the standard value for water by 0.44% and the experimental value of C was different from its calculated value (See Table 1) by -0.93%. These agreements appeared to be satisfactory and were within the range predicted by a system error analysis.

Entrance correction factors were then measured for Carbopol 960, a pseudoplastic power-law fluid, in concentrations of 600, 800, 1,000, 1,200, 1,400, 1,600, and 2,000 parts per million by weight(wppm). Each run had two sets of data, one for the long tube to determine n and K and the other for the short tube to measure the entrance correction factor C. The results for a typical run are shown in Fig. 3 for a concentration

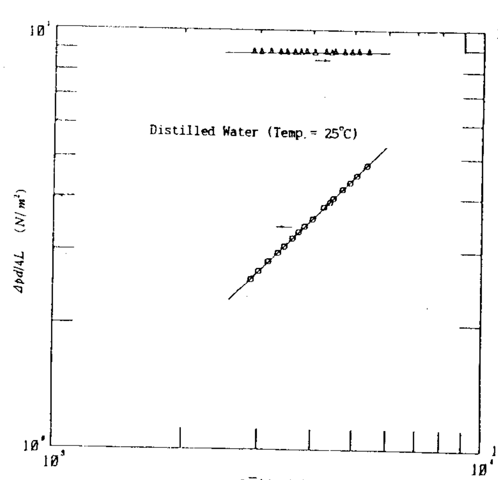

Fig. 2. Flow curve of distilled water.

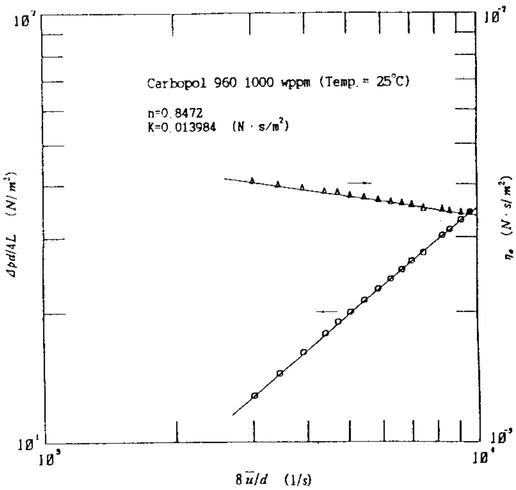

Fig. 3. Flow curve of Carbopol 960 (1,000 wppm).

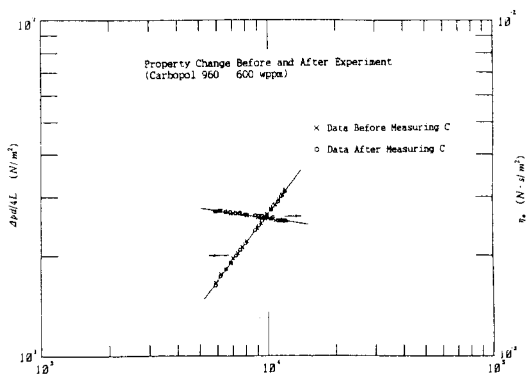

Fig. 4. Property change before and after experiment.

of 1,000 wppm, Fig. 4 shows a special run for 600 wppm to confirm that the rheological properties n and K did not change over the time period in which C was measured.

The final measured values of C for Carbopol 960 are shown in Table 4. They will be discussed and compared with other analysis and experiments in the next section.

4. Discussion of Results

Fig. 5 shows the results of the present analysis and experiments together with several other in-

Table 4. Measured values on n, K, and C for Carbopol 960 solutions (at 25 °C)

wppm	n	K	C
600	0.913	0.0044	2.198
	0.927	0.0039	2.215
800	0.873	0.0088	2.180
	0.869	0.0090	2.180
	0.895	0.0070	2.127
1000	0.847	0.0140	2.127
1200	0.788	0.0302	2.111
	0.775	0.0342	2.124
1400	0.760	0.0471	2.053
1600	0.730	0.0929	2.125
	0.716	0.0889	2.047
2000	0.626	0.3754	1.990

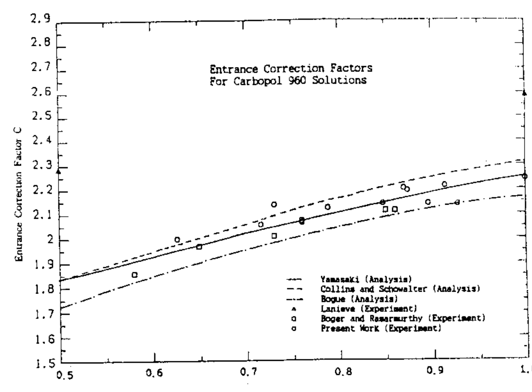

Fig. 5. Entrance correction factor for Carbopol 960 solutions.

vestigation reported previously. It is of interest to note that the present analysis falls roughly between the analysis of Collins and Schwalter[4] and Brogue[5]. Also, the present experiments are in reasonable agreement with the experiments of Brogue and Ramarmurthy[3] while the experiments by Lanieve[2] appears to yield values of C that are much too large. All of the experimental data except for that of Lanieve are in good agreement with the present analysis.

Nomenclature

A : cross sectional area [m^2]
C : entrance correction factor

d or D : diameter of capillary tube [m]

f : darcy friction factor$(2\frac{dp}{dx}d/\rho\bar{u}^2)$

H : height of test fluid [m]

K : power-law fluid consistency $[N \cdot s^n/m^2]$

L : length of capillary tube [m]

L_e : entrance length [m]

n : power-law fluid flow index

Δp : pressure drop across capillary tube $[N/m^2]$

p_g : gauge pressure $[N/m^2]$

p_o : inlet pressure $[N/m^2]$

p^+ : dimensionless pressure drop $(p-p_g-\rho gz/\rho\bar{u}^2)$

r : radial coordinate

r^+ : dimensionless radial coordinate (r/R)

R : duct radius [m]

Re : Newtonian Reynolds number$(\rho\bar{u}d/\mu)$

Re_g : power-law Reynolds number$(\rho\bar{u}^{2-n}d^n/K)$

Re_g' : generalized Reynolds number for a circular tube $(\rho\bar{u}^{2-n}d^n/K(\frac{3n+1}{4n})^n 8^{n-1})$

u : velocity in axial direction [m/s]

$\bar{u}$: duct average velocity [m/s]

u^+ : dimensionless velocity in axial direction $(u/\bar{u})$

v : velocity in radial direction [m/s]

v^+ : dimensionless velocity in radial direction $(Re_g v/2^n\bar{u})$

z : coordinate in axial direction [m]

z^+ : dimensionless coordinate in axial direction $(2^{1+n}z/Re_g D)$

Greek Letters

$\dot{\gamma}$: shear rate [1/s]

η_a : apparent viscosity$(\tau/\dot{\gamma})$ $[N \cdot s/m^2]$

μ : dynamic viscosity $[N \cdot s/m^2]$

ρ : fluid density $[kg/m^3]$

τ : shear stress $[N/m^2]$

References

1. R.K. Shaw and A.L. London, "Advances in Heat Transfer, Supplement 1", Academic Press, New York, 1978.
2. H.L. LaNieve, III, *M.S. thesis*, University of Tennessee, Knoxville, Tenn., 1963.
3. D.V. Boger and A.V. Ramamurthy, *A.I.Ch.E.*, **16**, 1088 (1970).
4. M. Collins and W.R. Schowalter, *A.I.Ch.E.*, **9**, 804 (1963).
5. D.C. Bogue, *Ind. Eng. Chem.*, **51**, 874 (1959).
6. E.R.G. Eckert and T.F. Irvine. Jr., *Fifth Midwestern Conference on Fluid Mechanics* (1957).
7. T. Yamasaki, *M.S. Thesis*, State University of New York at Stony Brook, 1982.
8. S.C. Kim, *M.S. Thesis*, State University of New York at Stony Brook, 1987.
9. S. Goldstein, "Modern Development in Fluid Dynamics", Vol.1, 'Clarendon Press, Oxford, England, 1938.
10. J. Boussinesq, *Compt. Rend.*, *113*, 8, 49, 1891.
11. Compbell, Slattery, and C. Jones, *J. Basic Eng.*, 14 (1963).
12. H.L. Langhaar, *J. Appl. Mech.*, **9**: *Trans. Am. Soc. Mech. Engrs.*, **64**, A-55-58 (1942).
13. R. Siegel, *Sc.D. Thesis*, MIT, Cambridge 1953.
14. E.M. Sparrow, S.H. Lin, and T.S. Lundgren, *Phys. Fluids*, **7**, 338 (1964).
15. B. Tomita, *Soc. Mech. Engrs.*, **4**, No 13, 77 (1961).
16. N.E. Dorsey, *Phys. Rev.*, **28**, 833 (1926).
17. G.H. Knibbs, *Roy. Soc. N.S. Wales*, **29**, 77 (1895).
18. L. Prandtl and O.J. Tietjens, "Applied Hydro and Aerodynamics", McGraw-Hill, New York, 1961.
19. W.J. Rieman, *J. Am. Chem. Soc.*, **50**, 46 (1928).
20. L.Z. Schiller and Z. Angew, *Math. Mech.*, **2**, 96 (1922).