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Analysis of Stress Distribution in the Hoop Test using Finite Element Method

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유한요소법을 이용한 Hoop Test에서의 응력분포 해석

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요 약 암석의 인장강도 실험법의 하나로써 새롭게 개발된 Hoop Test에 대한 이론적 뒷받침을 하고자 유한요소법을 사용하여 응력분포를 해석하였다. 간단하며 사용하기 편리한 실험장치이지만 시료가공시 생길 수 있는 내부공의 지름에 대한 오차, 내부공벽면과 하중장치외벽과의 마찰 등이 있어 이들로부터 발생할 오차를 평가해 보았다. 또한 시료의 안지름과 바깥지름의 비율이 응력의 크기에 끼치는 영향을 조사하였다. 응력해석결과, 일반적인 시료준비에 필요한 주의만 기울이면 실험치에 별다른 영향을 주지않는 것으로 제안되었고 이는 그동안의 실험결과의 신뢰성을 이론적으로 보충해 주었다.

1. INTRODUCTION

The hoop tension test devised by Xu *et al.* (1988) was suggested as a much simpler test to perform and a more reliable technique for studying failure in extension (Fig. 1). Initial studies of the test were conducted with some experimental work on sandstone and marble, which was theoretically supported by simple finite element analysis (John *et al.*, 1991).

The previous finite element analysis (John et al., 1991) was conducted under the following simple assumptions: (i) the loading platen is so rigid as to apply evenly distributed unidirectional load at the boundary between platen and rock, (ii) the rock behaves linearly elastically and there is no consideration of plasticity. Under the above assumptions, a two dimensional plane stress element was used for the modelling of the hoop such as that shown in Fig. 2a. Their results showed that the maximum tensile stress occurs in a plane parallel to the line of platen opening (Fig. 2b) and its magnitude was much higher (i.e. four times higher) than the maximum shear stress, which is high enough to fail a hoop under tension (compare Fig. 2c and d).

After their first work, no further analysis has been conducted. However, there are many other interesting points to be studied such as the effect of sample geometry, and the effect of boundary conditions at the contact between the loading platen and the hoop. Therefore this study is devoted to find the effects of contact conditions at the boundary and the effect of geometry on the hoop stress distribution.

2. GEOMETRY EFFECT

Using a similar model to the first one (see Fig. 2a), an attempt was made to analyse the effect of hoop size, viz., outer radius (r_o) and inner radius (r_i) of hoop, on the hoop stress distribution. A commercial PC-based software, i.e. COSMOS (version 1.61) was used. Material properties used

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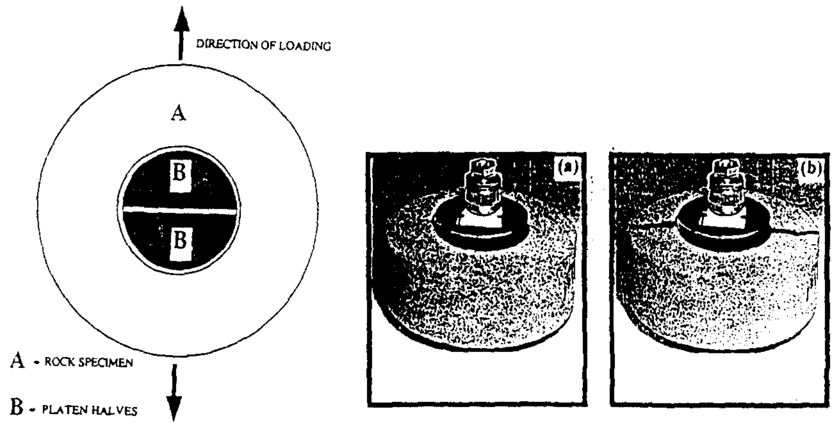

Fig. 1. Hoop tension test: (a) before and (b) after failure.

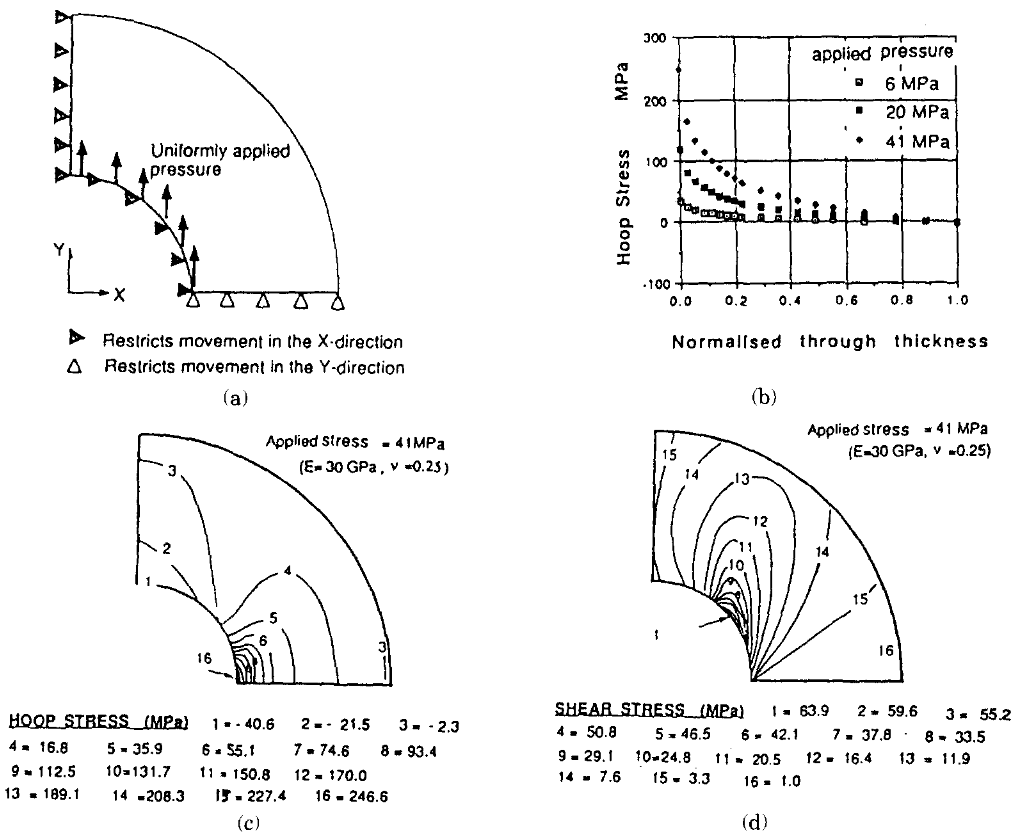

Fig. 2. Previous FEM modelling of hoop test (from John *et al.*, 1991). (a) Boundary condition. (b) Distribution of normalised maximum hoop stress. (c) Contour of hoop stress. (d) Contour of shear stress.

are 100 GPa for Youngs modulus and 0.2 for Poisson's ratio. Boundary conditions are the same as those used in the previous FEM model except that a different type of element, i.e. an 8-node isoparametric quadrilateral element, was adopted in this study because an 8-node quadrilateral

Table 1. Results of the geometry effect on the hoop stress using FEM analysis (hoop stresses are normalised through the applied stress, P).

Group	Inner diameter (d_i, cm)	Outer diameter (d_o, cm)	Ratio m (d_o/d_i)	Max. hoop stress ($\sigma_{T,max}/P$)
SL	5.9	9.5	1.6102	7.4607
SS	4.3	9.5	2.2093	5.8630
LL	5.9	15.07	2.5542	5.3346
LS	4.3	15.07	3.5047	4.4267
BR1	6.3	10.2	1.6190	7.4240
BR2	5.4	10.2	1.8889	6.5537
BR3	3.8	10.2	2.6842	5.1729

element is more suitable for the modelling the curvilinear or circular shape of the platen-hoop interface than 4-node elements which was used in the previous study.

The total number of elements used was 144 which is not much different from that in the previous model (i.e. 112) but there is a significant increase in the total numbers of nodes, i.e. 493 in this study and 135 in previous study.

Four different sizes of hoop were selected from the previous experimental work undertaken by Al-Samahiji (1992). They are combinations from two different inner bore sizes (i.e. diameters of 4.3 cm and 5.9 cm) and two different outer bore sizes (i.e. diameters of 9.5 cm and 15.07 cm) (Table 1). Other sizes of hoop were additionally selected using combinations of some popular sizes of bores, i.e. 3.8 cm, 5.4 cm, and 6.3 cm diameter for the inner bore of the hoop and 10.2 cm diameter for the outer bore of the hoop. Results are shown in Table 1.

Depending on the geometry, (i.e. the ratio between outer diameter and inner diameter), the hoop stress pattern changes. The smaller the ratio (m), the larger the maximum hoop stress is (see also Fig. 3) and the rate of change of hoop stress increases as the ratio (m) decreases.

The larger the ratio (m), the larger the area under compression is (Fig. 4). There is no compression zone in a SL hoop (d_o/d_i=1.6102), BR1 hoop (d_o/d_i=1.6190), and BR2 hoop (d_o/d_i=1.8889).

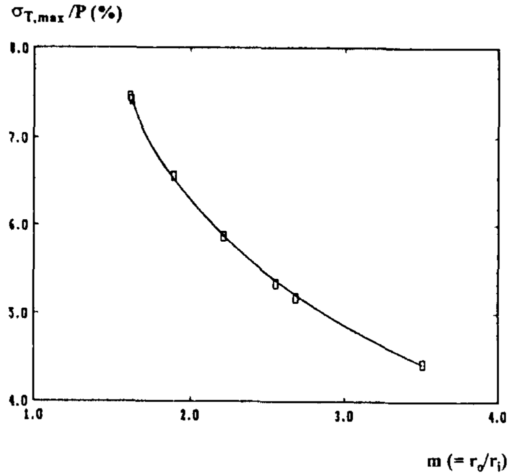

Fig. 3. The effect of geometry on the hoop stress: r_o and r_i are outer and inner radius of hoop, respectively.

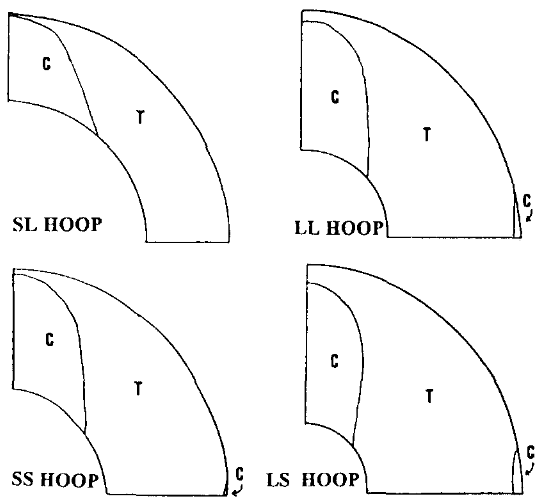

Fig. 4. The effect of geometry on the areas under tension (T) and compression (C).

Such samples can be recommended as being of good size for determining hoop tensile strength because there is no potential influence of a compression zone on the failure of the hoop.

3. ECCENTRICITY EFFECT

During the sample preparation, two coring phases are required, and they should be coaxial,

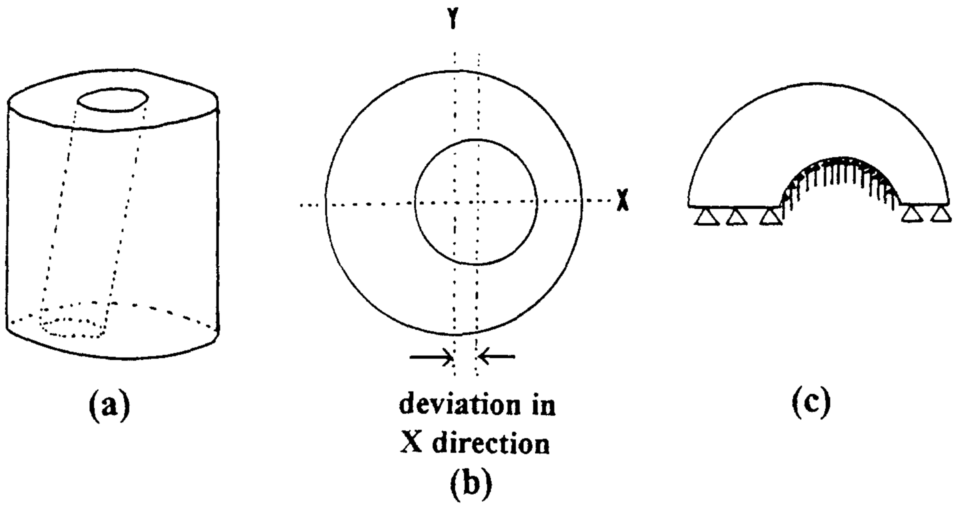

Fig. 5. Eccentricity problems in the hoop test and its analytical model.

Table 2. Results of the analysis of the effect of eccentricity on the hoop stresses.

Name	δ (mm)	$\frac{\sigma_{max.L}}{P}$	$\frac{\sigma_{max.R}}{P}$	$\frac{\sigma_{max.R}}{\sigma_{max.L}}$ (%)	$\frac{\sigma_{max.R}}{(\sigma_{max.L})_{\delta=0}}$ (%)	$\frac{\tau_{max.L}}{P}$	$\frac{\tau_{max.R}}{P}$	$\frac{\tau_{max.R}}{(\sigma_{max.L})_{\delta=0}}$ (%)
EC 0	0.00	7.4609	7.4609	100	100.00	2.2434	2.2434	100.00
EC 5	0.09	7.3501	7.5854	103	101.67	2.2195	2.2718	102.35
EC10	0.18	7.2510	7.7247	107	103.54	2.2001	2.3054	104.79
EC15	0.27	7.1637	7.8806	110	105.63	2.1849	2.3447	107.31
EC20	0.36	7.0872	8.0571	114	107.99	2.1737	2.3903	109.96
EC25	0.45	7.0196	8.2581	118	110.69	2.1666	2.4430	112.76

but there is always a certain deviation from such a perfect shape. It was expected that this might cause local stress concentration in a thinner annular part of hoop.

Such an eccentricity can occur differently in three dimensional (Fig. 5a) but it was decided that only one direction of eccentricity should be considered in a two dimensional plane stress condition because it is much simpler than the three dimensional analysis and is informative enough for an initial study. Therefore deviation in the X-direction only was considered in this modelling (Fig. 5b) and the mesh was generated in the upper half of hoop (Fig. 5c). The SL size hoop was selected for analysis and the deviation used had six different values from 0% to 25% of the annulus (= r_o-r_i=1.8 cm in this case). Interest was given to the values of maximum tensile stresses and maximum shear stresses at the points in the left and right part of inner bore wall.

Normalised stress through the applied pressure are summarised in Table 2 and the normalised stresses through the maximum hoop stress are plotted in Fig. 6. Stress contours are also plotted in Fig.7.

From Fig. 6, it can be seen that when deviation increases the hoop stresses at the right part of the inner bore of the hoop becomes larger than the maximum hoop stress in a hoop with no deviation (Fig. 6a). This means that when deviated coring is involved during sample preparation, the maximum hoop stress can be a bit larger than that expected with a perfect coaxial hoop: for the case of SL size hoop, there will be a 5.63%

increase caused by a deviated coring of 15% relative to the annulus (see Table 2).

The absolute value of tolerable deviation under a certain tolerance of stress is decided by the absolute size of the annulus. For example, if it is said that 15% of deviation can be allowed with a 5.63% tolerance of hoop stress value in all cases of hoop then it should be noted that 2.7 mm is such a tolerance for an SL size hoop.

4. FRICTION EFFECT

To investigate the effect of friction on the hoop stress distribution, a modified model was introduced using the result of theoretical analysis as a boundary load (Park and de Freitas, 1995) (Fig. 8), under the assumption of perfect contact condition (i.e. no gap and no platens are considered). The normalised results through the value at friction = 1.0 (Table 3), shows the linear proportional relationship between coefficient of friction and the maximum hoop stress.

The effect of friction was so significant in $\sigma_{T,max}$ and hence, the changes of the area of compression zone were investigated. The movement of neutral point (i.e. the boundary between areas under compression and under tension) and the changes of the area are shown in Fig.9.

All these results show the significant effect of friction on the hoop stress, but in practice, much evidence from measurements of displacement using photoelasticity and laser holography (Al-Samahiji, 1992) showed that the value for friction is often close to 1.0 during the main part of loading. The change in operative friction (from any initial friction value less than 1.0 to 1.0) occurs in a relatively short period, i.e. during the initial stage of loading when the platens are mating with the sample, i.e. at a low level of stress; these low values of frictional resistance therefore do not affect the hoop tensile strength value. The initial friction value has little contribution to the behaviour of a hoop sample during the main part of loading once the gap between the platens and sample has been closed.

5. GAP EFFECT

Although the above analyses showed a good qualitative and quantitative analysis of the hoop behaviour, it has a strong drawback: perfect contact between rock and platen was assumed, which cannot be met in the real performance of a hoop test. Therefore, it was decided to create a new model which can consider the loading platen as well as the rock.

(1) Basic assumptions for the new modelling

Hoop test has a contact problem between rock and loading platen. When contact exists, especially on a curvilinear surface, there arise two important points: (i) friction between the two materials (i.e. rock and metal platen) may affect the boundary load at contact and (ii) real contact area can differ in each test due to the particular differences in the size of inner bore.

To consider such a contact problem, some assumptions were made in this study:

(i) Rock is isotropic and behaves linearly elastically.

(ii) Metal (of which platen was made) is isotropic and behaves linearly elastically.

(iii) Youngs modulus of rock is same under compression and under tension.

(iv) Youngs modulus of metal is same under compression and under tension.

(v) At the contact surface, the load is transferred from platen to rock and the degree of transfer (i.e. the distribution of boundary load) is dependent on the friction and contact surface. Sliding of one material relative to the other material can occur. No restriction on the movement is made on the inner bore wall, (cf) restriction of movement in the X-direction in

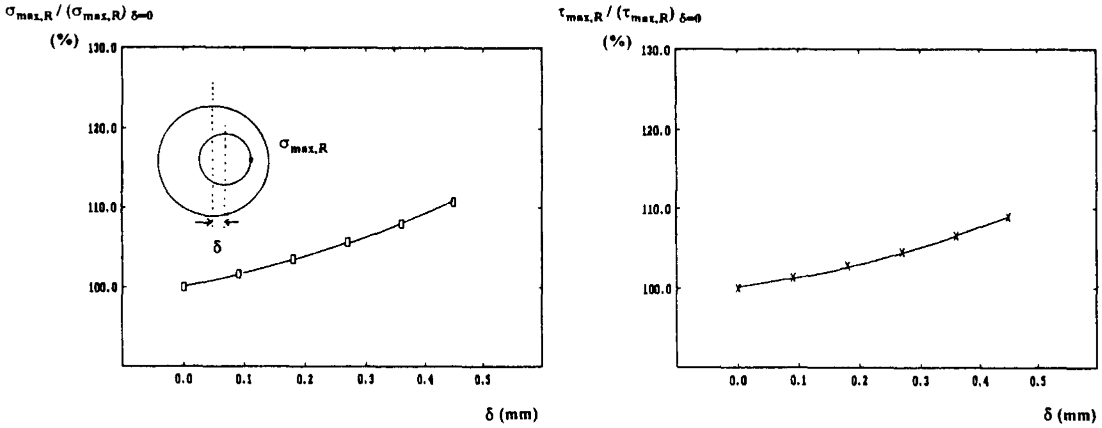

Fig. 6. Graphic results of the effect of eccentricity on the hoop stress (a) and shear stress distribution (b): $(\sigma_{max,R})_{\delta=0}$ is maximum hoop stress on righthand side when deviation to right (δ) is zero.

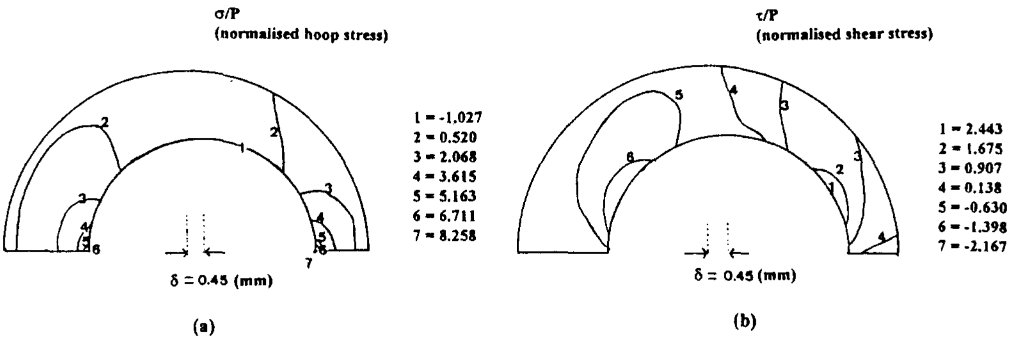

Fig. 7. Contour plots of the variation of the hoop stress distributions caused by the eccentricity: (a) hoop stress and (b) shear stress.

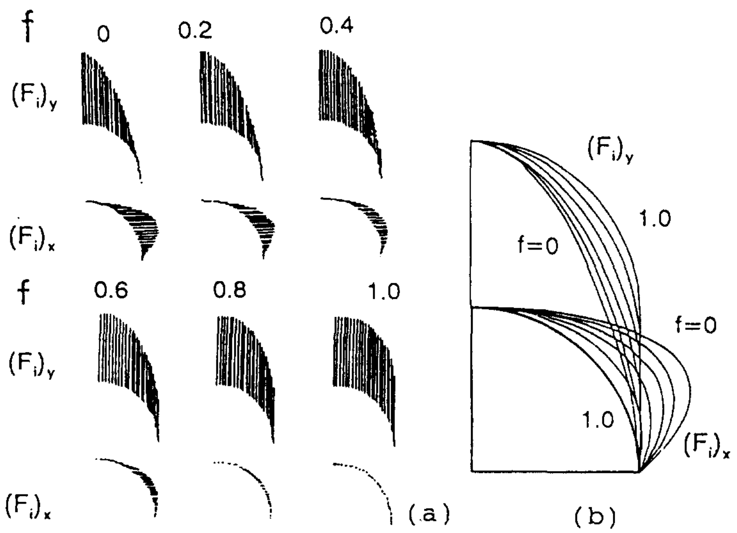

Fig. 8. The relationship between friction (f) and the magnitude of the load line force components, $(F_i)_x$ and $(F_i)_y$, at the platen-sample contact.

Table 3. The effect of friction on the hoop stress.

Name	friction coeff.(f)	$\sigma_{T,max}/(\sigma_{max})_{f=1}$
TN1	0.0	63.6
TN2	0.1	67.3
TN3	0.2	70.9
TN4	0.3	74.5
TN5	0.4	78.2
TN6	0.5	81.8
TN7	0.6	85.5
TN8	0.8	92.7
TN9	1.0	100

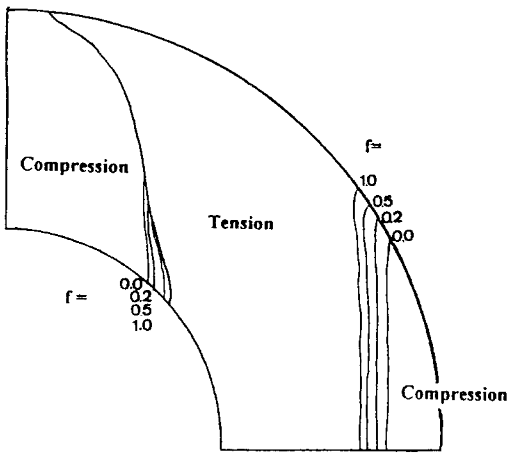

Fig. 9. The changes of stress distribution by the variation of friction (f).

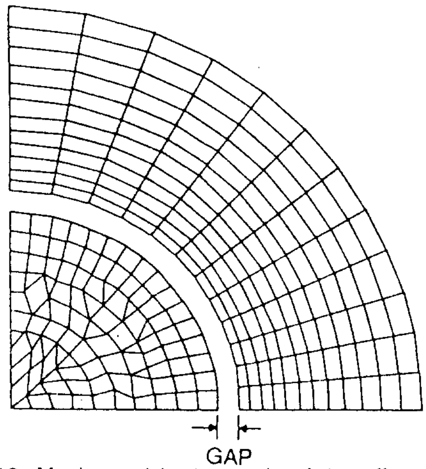

Fig. 10. Mesh used in the study of the effect of eccentricity.

previous model (see Fig. 2a).

(vi) Two dimensional plane stress condition was adopted.

Material properties are selected based on the real values: (i) the metal platen has a Youngs modulus of 300 GPa and a Poisson's ratio of 0.28 which are the exact value of normal steel and (ii) the rock has a Youngs modulus of 100GPa and a Poisson's ratio of 0.2, which are some normal value of rock.

However it should be noted that the Youngs modulus of rock in tension is lower than that in compression, which is beyond the capability of this software.

(2) New model on the behaviour of loading platen and hoop

Due to the symmetry of hoop specimen, only a quarter was modelled for the analysis. To represent both rock and platen, an 8-node isoparametric plane stress element was employed. To represent the interface, node-to-curved line type of contact was adopted: a gap element was employed at each node at the contact surface of platen and each gap surface, defined by three nodes, was employed at the contact surface of the inner bore of the hoop (Fig. 10).

The mesh for the platen was created with a 3-node triangular element and was upgraded to 8-node quadrilateral elements using an auto-meshing facility of the COSMOS software. The mesh for rock was created with the spacing ratio of 2 both in the radial direction and in the circumferential direction, due to the large gradient of hoop stress at the inner bore in the plane parallel to the line of platen opening.

Almost the finest meshing allowable in this software was employed in this new model: the total numbers are as follows: 277 elements, 798 nodes, and 1523 equations. The created meshing was tested under the self-checking facility in the software and no errors were found. Boundary

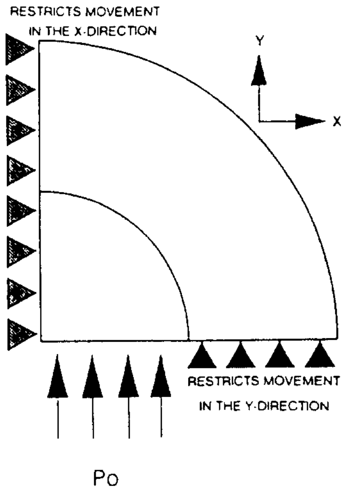

Fig. 11. Boundary condition in a new model.

Table 4. The effect of gap on the hoop stress.

Name	Gap (cm)	$\frac{\sigma_{T,max}}{P}$	$\frac{\sigma_{T,max}}{\sigma_{T,max,norm}}$ (%)	contact angle (°)
CF21	0.01	5.647	100.0	45
GF31	0.05	5.667	100.4	22.5
GF41	0.10	5.968	105.7	15
GF51	0.20	6.268	111.0	<15
GF61	0.30	6.593	116.6	-
GF71	0.40	6.958	123.2	-

conditions are schematically shown in Fig. 11.

(3) Analysis of the effect of gap on the hoop stress distribution

In reality, when the sample dimension (r_i) is not equal to the radius of the platen, a gap will exist between the platens and the sample. This gap could occur due to poor preparation of the inner diameter of hoop during coring whereas the outer diameter of the platen (which is machined) is likely to be more constant. Therefore when different gaps are adopted, they are made by changing the size of the inner radius of hoop.

To investigate the effect of gap on the hoop stress distribution, six different values of gap were used. The effect of gaps were studied, with that of 0.01 cm being taken as good normal

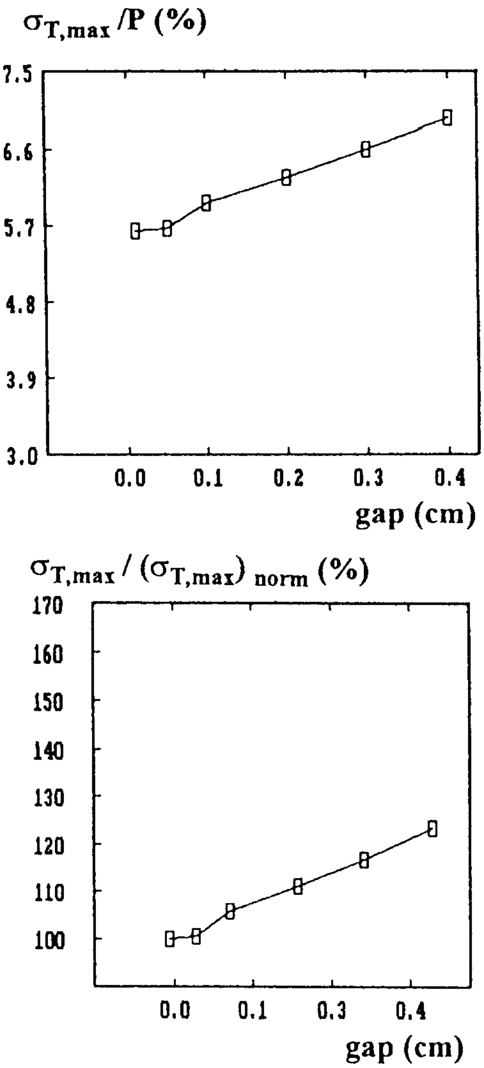

Fig. 12. The effect of gap on the maximum hoop stress: $(\sigma_{T,max})_{norm}$ is the maximum hoop stress at the plane parallel to the platen opening when gap is equal to 0.01 cm.

practice based on the previous work using the hoop test for defining the tensile strength of sandstone, marble, amphibolite and schist (Xu *et al.*, 1988, Al-Samahiji, 1992; Butenuth *et al.*, 1993 & 1994). These works indicate that a gap of 0.01 cm or less is likely to provide a value of tensile strength which correlates well with that defined by tests of direct tension, with a correlation coefficient of 0.9 or better. It is therefore taken as the norm against which the effects of larger gaps can be measured.

The results (Table 4, Fig. 12) show that the maximum hoop stress increases when the gap increases. The increase of hoop stress may be caused by the high stress concentration on the local area. This can be seen from the changes of the approximate contact angle which was presumed from the number of closed gaps after loading. It is also seen that the normalised hoop stress varies within 5% with gap range of 0.01-0.1 cm: this tolerance can easily be achieved by rock coring with normal care, e.g. Gentier et al. (1991), Park (1995).

6. CONCLUSION

Several analyses using either previous models or new models showed useful results and they can be summarised and some suggestions made as follows.

(1) The maximum hoop stress increases when the ratio (m) of outer radius (r_o) to inner radius (r_i) decreases, i.e. thinner sample (i.e. having a low m) fails under lower pressure than that required for the failure of thicker sample (i.e. having a high m) of the same material.

(2) The effect of friction is significant but previous experimental results showed that the friction seemed to be 1.0 at the main part of loading. Therefore the contribution of relatively low initial friction between the rock and platen can be negligible to the failure of a hoop.

(3) The effect of a gap can be controlled to within 5% variation of the maximum hoop stress by keeping the tolerance between 0.01 cm and 0.1 cm, which is easily achieved by coring with normal care. However, when a weaker plane lies on the plane parallel to the direction of loading, failure could occur on that plane due to the relatively high stress in that plane.

(4) For a better picture of real conditions, a new analysis using BEM (Boundary Element Method) is recommended because fracture propagation can be easily analysed by BEM.

7. ACKNOWLEDGEMENTS

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