

Pier Scour Prediction in Pressure Flow

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ABSTRACT : In this experimental paper, the maximum scour depth at pier was studied. The model of the pier of San Gye bridge in the Bocheong stream was set for the experimental studies. Several model verification processes were conducted through the roughness comparisons between model and prototype, pursuing scour depth variations with time depending upon channel bed variation, the comparison of the ratios between falling velocities and shear velocities in the model and prototype, and the comparison of pier scour depths between experimental data and field measuring data. The experiments were conducted in the free flow conditions and pressure flow conditions. The maximum scour depth at piers in the pressure flow conditions is almost twice as much as compared to the free flow conditions. Also, the maximum scour depth variations are indicated in the figures based on the Froude numbers, opening ratios, water depths and approaching angles in the free surface flow conditions.

1. Introduction

The formation of vortexes is the basic mechanism causing local scour at bridge piers. The formation of these vortexes results from the pile up of water on the upstream face and subsequent acceleration of the flow around the nose of the pier. The action of the vortex removes fine materials and/or sediment away from the existing base bed. If the transport rate of sediment removing away from the local region is greater than the transport rate entering into the region, a scour hole develops. As the scour depth is increased, the strength of the vortexes is reduced, thus reducing the transport rate. As equilibrium is reestablished, scouring ceases and the scour hole will not be enlarged further. For piers, there is also an additional vertical vortex downstream of the pier, which is denoted as the

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wake vortex. Both vortexes remove the fine materials and/or sediment from around the pier. In many cases the material which is removed by these vortexes is redeposited immediately downstream of the pier. The literature review about the local scour at pier shows that most researches about the pier scour were conducted using the models in the experimental lab, because the measurement of the flow characteristics which cause the pier scour is relatively easy. However, in the field, the decision of the shear stress which is the main parameter for calculating the pier scour in the field, is so difficult. The first experiment for the scour was done by Durand-Claye in 1873. He conducted the experiments using three types of pier models which are triangular, rectangular, and circular type models, and compared the maximum scour depth. He concluded the maximum scour depth occurs around the rectangular pier model and the minimum scour depth occurs around the triangular pier model (Cornelism, 1982). After Durand-Claye, Ishihara (1945), Laursen and Toch (1953), Tanaka and Yano (1967), Shen et al. (1969) and so on established the theoretical background for the local scour and begin to use the concept of the horseshoe vortex which is the main mechanism of the pier scour. Also, they analyzed the characteristics of the local scour and suggested the scour equations which can compute the maximum scour depth by the pier Reynolds numbers. Laursen (1962) conducted the theoretical and experimental researches related to the contraction scour including the critical shear stress. Altinbilek (1973) conducted the research for the channel bed scour by jet flow. Jain and Fisher (1980) conducted the research related the relationship between Froude number and the maximum scour depth. Hughes (1980) conducted the research for the scour at piers using the several types of bed material, and Raudkivi and Ettema (1977), Jain (1981), Rajaratnam and Newachukwu (1983), Melville (1984) and so on conducted the research related to the pier scour.

On the other hand, just a few researches related to the scour were conducted in Korea. Lee (1984) conducted the research about the local scour at bridge piers and concluded the pier Reynolds number and turbulent strength are the main parameters for forecasting the maximum scour depth. Kim (1985a) conducted the experiments about the maximum scour depth including the vortex mechanism at circular pier. Kim (1985b) compared the maximum scour depths calculated using the existing scour equations with his experiment data which were obtained from the model tests using the diatomite as the channel bed material. Lee (1992) conducted the experimental research related to the scour protection at bridge piers.

However, those researches are conducted on the basis of free surface flow. In the recent days, it is indicated that the scour in the pressure flow condition is more dangerous compared to the scour in the free flow condition. The pressure flow can be indicated when the water flows without the free surface through hydraulic structures such as bridges, tunnels or boxes in the channel. In the pressure flow, the turbulence and vortices, which can affect the scour around the structures, are increased. Before considering the pressure scour, just a few researches are conducted in the pressure flow condition. Wang (1970) conducted the research about the induced pressure which is indicated when the bridge deck is submerged. He found that the pressure head under the submerged bridge deck is twice as much as hydrostatic pressure. Also, Wiggert (1972) analyzed the transient phenom-

ena from the free flow to pressure flow using the numerical analysis. The researches for the scour under the pressure flow condition are initiated in the recent years. The serious research about the pressure scour was initiated by Abed in 1991 and continued by Richardson (1993), Jones (1993) and so on. Abed (1991) concluded the maximum scour depth in the pressure flow is 2 to 3 times in $Fr > 0.5$, 2 to 4 times in $0.35 < Fr < 0.5$, over 10 times $Fr < 0.35$ compared to the maximum scour depth in the free flow through the very limited experiments. She found out that the maximum scour depth is increased by decreasing the opening ratio under the same velocity and water depth. Richardson et al. (1993) conducted the experiments about the scour widths under the free and pressure flow conditions. Jones (1993) concluded that the scour depth at bridge pier under the pressure flow condition is almost the same as the sum of the local and contraction scour depths. However, most researches were conducted within the very restricted boundary conditions. Therefore, the more researches are necessary for proper application in the natural channels or rivers. In this paper, the model was built based on the pier of San Gye bridge which is constructed in the Bocheong stream in the Geum river watershed and the researches for the maximum scour depth at semi-circular pier were conducted under the free flow condition and pressure flow condition.

2. Experimental Set-Up and Channel Verification

2.1 Applied Similarity Laws.

The major external forces, which act in the fluid in the channel, are gravity, viscosity, pressure, surface tension and elastic forces. For the perfect dynamic similarity the scaled ratios between the prototype and the model should be the same for the external forces. The dimensionless numbers between these external forces and the inertial force is indicated in Eq. (1).

$$f\left[\frac{V}{\sqrt{gL}}, \frac{\rho VL}{\mu}, \frac{V}{\sqrt{\nabla p/\rho}}, \frac{\rho V^2 L}{\sigma}, \frac{\rho V^2}{E}\right] = 0 \quad (1)$$

In Eq. (1), five dimensionless numbers are called as Froude number (Fr), Reynolds number (Re), Euler number (Er), Weber number (Wr) and Cauchy number (Cr), respectively. These numbers indicate the influence by the gravity, viscosity, pressure, surface tension and elastic forces, respectively. Also, the similarity laws are called as Froude's, Reynolds's, Euler's, Weber's and Cauchy's similarity laws, respectively.

Because it is impossible to keep five dimensionless values between the prototype and the model constant, it is common to determine the scale by considering the components of major force according to the experimental purpose. In the open channel flow, the influence by the gravity force is larger than that by the others. Therefore, Froude's similarity law is chosen as the basic similarity law for experimental set-up for this paper.

2.2 Roughness Coefficient in the Prototype Stream.

Before starting experiments it is important to grasp the characteristics of the prototype stream. Especially, the roughness coefficient, which influences the velocity, should be determined. In this paper, the roughness coefficient of the stream bed around the San-Gye bridge in Bocheong stream, which is a tributary in the Keum river basin, is obtained using the water level and discharge data. Those data had been measured by International Hydrologic Program (IHP) during several years. The computed roughness coefficient variation is shown in the Fig. 1. As shown in the figure, although the roughness coefficient is a little different by the water level, the range of the roughness coefficients is from 0.027 to 0.048.

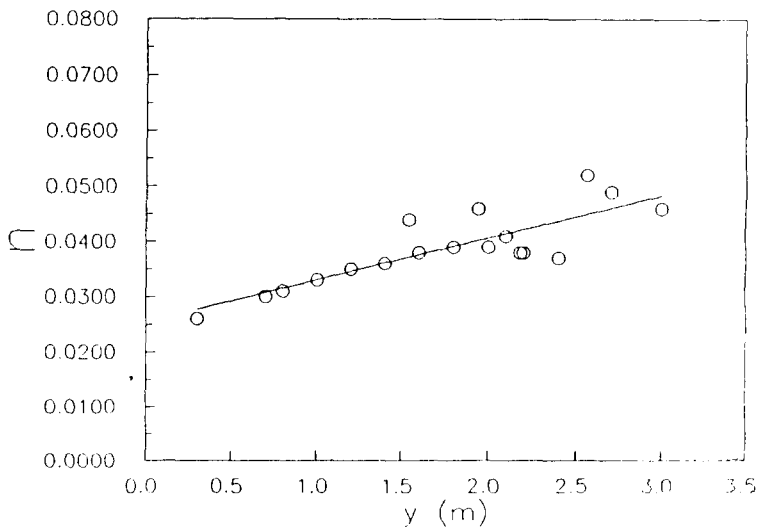


Fig. 1. Manning's Roughness Coefficient Variation by Observed Data

2.3 Model Set-Up and Determination of the Roughness Coefficient

The pier model was installed in the open channel, which is 12m in length, 40cm in width, and 40cm in height. The open channel consists of glass and it is possible to observe the flow. Also, a pump, which supply water to the channel and recirculate water from the ground water tank to the upper water tank, is installed. The channel bed slope in the open channel flume can be change from 0.0% to 4.0%. It is possible to control the discharge by the valve, to measure the discharge by the triangular weir in the upper tank. The flow depth can be controlled using the vertical gate in downstream end. The velocity can be measured using the small magnetic velocity meter, and the scour depth can be measured using the point gage.

As shown in Fig. 2, the bed material is put in the bottom of the channel with the depth of 10cm, and the length of 6.0m. The pier model was built using the cement-mortar with the scale of 1/40, which is a model of the pier of San-Gye bridge in Bocheong stream.

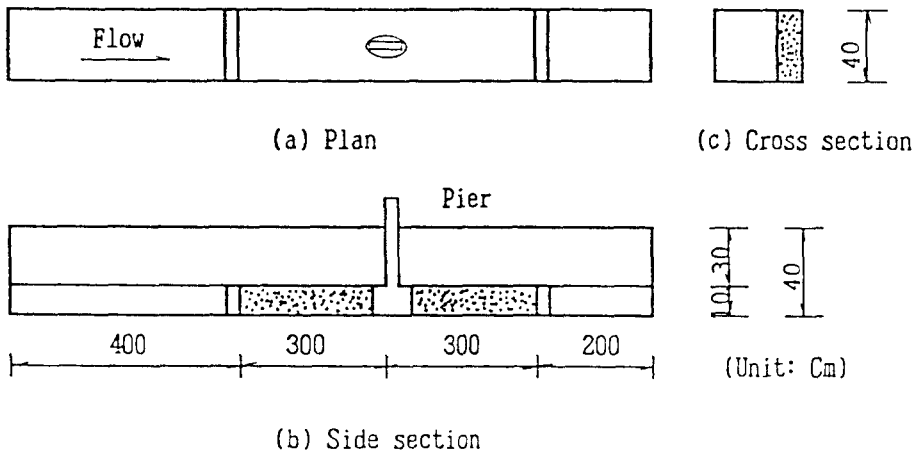


Fig. 2. Open Channel Apparatus for the Scour Experiment

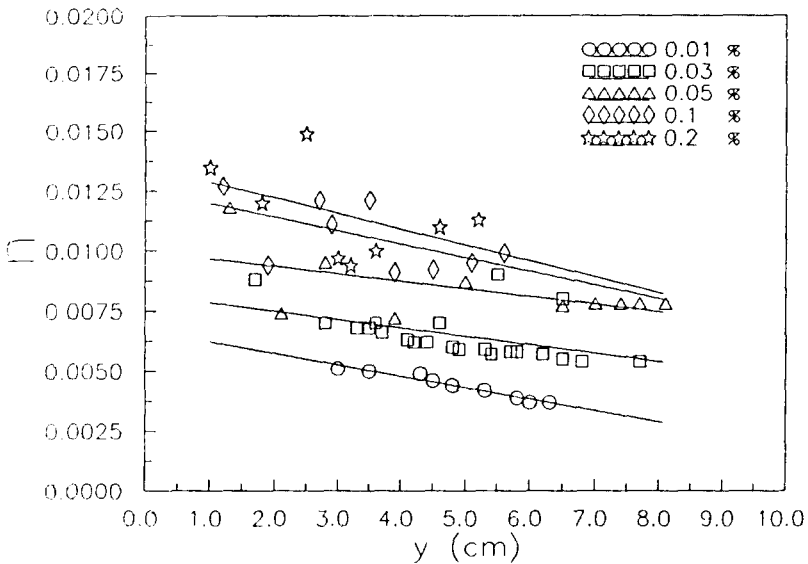


Fig. 3. Manning's Roughness Coefficient Variation by Water Depth in the Model

In the hydraulic model test having movable bed, it is difficult to keep constant similarity between the prototype and the model. Therefore, before starting the model test, it is necessary to have several model verification processes. Usually, in the movable bed two types of bed material are used. The first type is to use the same material with the material in the prototype stream and the second type is to use the properly reduced material, which is the material having different characteristics compared to the material selected in the prototype stream. Even though the same type of material having different sizes is used, the material, which size is smaller than the mean diameter of 0.8mm,

may not have similar hydraulic characteristics with the material in the prototype. The bed material around the San-Gye bridge in Bocheong stream, which is the prototype stream, is composed of sand having gravels. The mean diameter of 50mm was obtained through the sieve analysis. However, in the observation of the scour in the field, it is seen that the material moved by scour is only sand (the average diameter is 0.8mm in sand only). Therefore, the bed material of mean diameter of 0.8mm was utilized in the experiment and the channel bed slopes are changed to fit the hydraulic properties with the prototype stream. In order to determine the channel bed slope, the roughness coefficient is computed by the change of water stage in the each channel bed slope as shown in Fig. 3.

2.4 Decision of the Ranges of the Channel Bed Slope

In general, the flow in the natural stream or river is turbulent, and the resistance by flow occurred by the form drag rather than the friction drag. Therefore, the resistance equation such as Manning's roughness equation can be applied. The Manning's roughness equation can be written as the following equation using the similarity laws.

$$V_r = \frac{I}{n_r} R_r^{2/3} S_r^{1/2} \quad (2)$$

In the open channel flow, the velocity ratio, V_r , and the time ratio, T_r , can be expressed as the following equations by applying Froude's similarity.

$$V_r = L_r^{1/2} \quad (3)$$

$$T_r = \frac{L_r}{V_r^{1/2}} \quad (4)$$

Eq. (3) can be combined with Eq. (4).

$$L_r^{1/2} = \frac{I}{n_r} = L_r^{2/3} S_r^{1/2} \quad (5)$$

For determining the channel bed slopes, the roughness coefficients in the prototype and model channel are utilized. That is, when the roughness coefficient ratio (n_r) between the prototype and the model is determined, the channel bed slope ratio (S_r) can be determined using the following equation.

$$S_r = \frac{n_r^2 L_r}{L_r^{4/3}} = \frac{n_r^2}{L_r^{1/3}} \quad (6)$$

The channel bed slope in the prototype stream is obtained as 0.25% through the field measurement. Through the above processes the proper channel bed slopes in the model flume were selected as the ranges of 0.013% ~ 0.065%.

2.5 Verification of Experimental Set-up

2.5.1 Movement and Falling velocity of the Suspended Sediment

The local scour around the pier is initiated by moving the sand around the pier by the suspended load or the bed load. The suspended load or bed load is determined by the flow characteristics such as flow velocity, flow depth and so on. In this paper, the experiment was planned to be conducted with the consideration of suspended load. In the several experiments the similarity was obtained by considering the ratio between falling velocity w of the suspended load and friction velocity u_* in the model is the same as the ratio in the prototype. In this paper, it is determined to be satisfied similarity using above rule. In the uniform flow in the open channel having the channel bed slope of θ , the force equation can be written as the following equation by applying the conservation of force between the friction force applied to the perimeter and water weight W with the cross sectional area of A , the channel length of L , and the wetted perimeter of P .

$$\tau_o L P = W \sin \theta = \rho g L \sin \theta = w L A \sin \theta \quad (7)$$

where, let $\sin \theta = i$, and $A/P = R$:

$$\tau_o = w R i = w h i \quad (8)$$

where, τ_o is the friction force per unit area and is called as the tractive force. The friction velocity can be written as $u_* = \sqrt{\tau_o / \rho}$ and the following equation can be obtained.

$$u_* = \sqrt{g R i} \quad (9)$$

The fall velocity indicates the terminal velocity, which is occurred when the solid particle is fallen into the still water. It is used as an important component of the characteristics which affect sediment transport and sedimentation. The fall velocity of the solid particle is obtained with the condition that the particle weight in the water is equal to the drag force which resists the particle to be fallen.

$$W_s = \frac{1}{6} \pi d_s^3 (\gamma_s - \gamma) \quad (10)$$

$$F_D = \frac{1}{2} \rho C_D w^2 \frac{\pi d_s^2}{4} \tag{11}$$

$$w^2 = \frac{4 g d_s \gamma_s \tau}{3 C_D \gamma} \tag{12}$$

where d_s is the diameter of the particle, γ_s is the unit weight of the soil particle and γ is the unit weight of water. ρ is the density of water and C_D is the drag coefficient.

Through applying the above equation, the channel bed slope of 0.03% should be maintained to satisfy the same ratios of w/u_c between the prototype and the model having the mean diameter of 0.8mm.

2.5.2. Measurement of Scour Depth in the Prototype Stream

The measurement of scour depth was conducted during summer flood period in 1993. Fig. 4 indi-

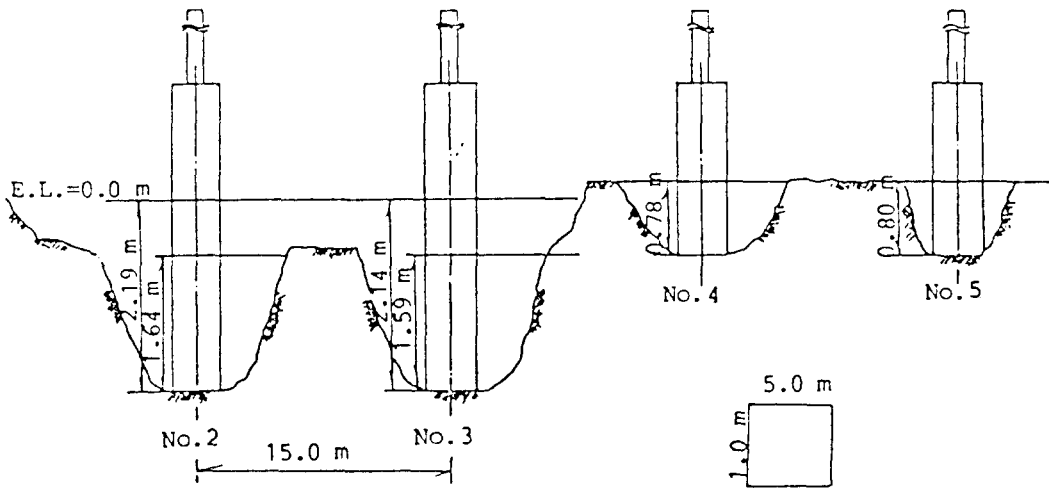


Fig. 4 The Maximum Scour Depth Variation in the Field

Table 1. The Measured Maximum Scour Depth for Each Bridge Pier

(unit : m)

No. of Pier	1st		2nd		3rd		4th	
	water depth	scour depth	water depth	scour depth	water depth	scour depth	water depth	scour depth
No.2	3.55	1.64	-	-	-	-	-	-
No.3	3.40	1.59	-	-	-	-	-	-
No.4	1.96	0.76	0.48	0.17	1.88	0.75	1.83	0.70
No.5	1.96	0.80	-	-	1.88	0.75	-	-

icates the cross sectional area having the maximum scour depth. In the figure, the maximum scour depths of No. 4 and No. 5 piers were obtained through measurement in the field immediate after a flood in the channel bed with refilling holes scoured in the previous flood. The measurement results are indicated in the Table 1.

2.5.3. Maximum Scour Depth Comparison Between Model and Prototype

To compare the scour depth between the model and prototype, the experiments for the maximum scour depth were conducted based on the water depths and channel bed slopes. The experimental results are shown in Fig. 5 and Table 2. As shown in Table 2, it is indicated that the maximum scour depth(d_s/b) in the prototype is almost same with the maximum scour depth(d_s/b) in the model with the channel bed slope of 0.03%.

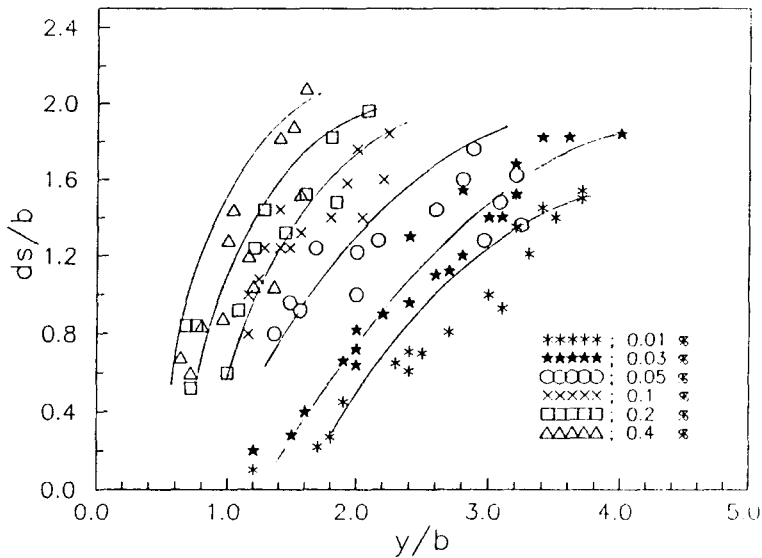


Fig. 5. The Maximum Scour Depth in the Model Flume

Table 2. The Comparison of Maximum Scour Depths between Model and Prototype

depth (prototype)		Maximum scour depth(proto.)		Maximum scour depth of each bed slope for model channel, (d_s/b)					
y(m)	y/b	ds(m)	d_s/b	0.01%	0.03%	0.05%	0.1%	0.2%	0.4%
0.48	0.48	0.17	0.17	—	—	—	—	—	—
1.88	1.88	0.75	0.75	0.30	0.70	1.20	1.45	1.80	2.10
1.96	1.96	0.78	0.78	0.39	0.80	1.25	1.60	1.90	2.15
1.96	1.96	0.80	0.80	0.39	0.80	1.25	1.60	1.90	2.15
3.40	3.40	1.59	1.59	1.40	1.60	1.80	1.90	—	—
3.55	3.55	1.64	1.64	1.50	1.70	1.80	—	—	—

2.5.4 Model Verification

To determine the final model verification the following steps were utilized. First, Froude's similarity law was selected as the basic similarity law. Second, the range of the applicable channel bed slopes range was determined through the measurement of the roughness coefficient in the prototype and model. Third, the channel bed slope of the model was adjusted by verifying through the ratio between fall velocity and friction velocity. Finally, the channel bed slope, which was adjusted in the third step, was verified comparing the maximum scour depth between the model and prototype.

3. Maximum Scour Depth in the Free Flow

Water depth, Froude number, the opening ratio and the angle of attack are selected as the sensitive parameter about the scour depth at pier through the literature review and sensitivity tests in the free flow. Using the selected parameters the maximum scour depth in the free flow were compared.

3.1 Maximum Scour Depth Variation by the Flow Depth.

The experiments were conducted based on the flow depths in the fixed opening ratio. The water depths were adjusted by changing the channel bed slopes. The experimental results were analyzed using the dimensionless number which is obtained by the relationship between the maximum scour depth (d_s) and the pier width (b). The maximum scour depth variation depending upon the flow depth is shown in the Fig. 6. As shown in Fig. 6, the maximum scour depth is increased by increas-

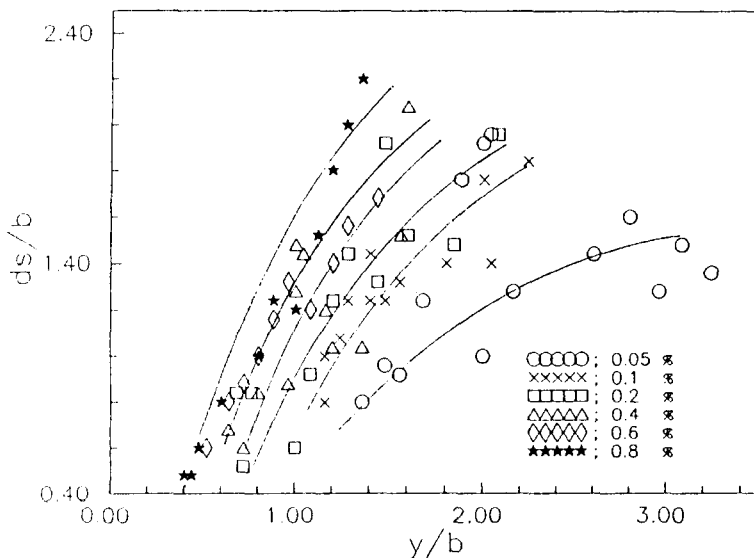


Fig. 6. Maximum Scour Depth Variation with Water Depth Increment

ing the water depth. The increasing rate of the maximum scour depth is reduced by increasing the channel bed slope.

3.2 Maximum Scour Depth Variation by Froude Number.

Froude number(=Fr), which is the dimensionless number, indicates the influence of the gravity force to the inertia force. Also, Froude number is changed by the flow depth and the channel bed slope. The experimental results are shown in the Fig. 7. As shown in the figure, the increment rate of the maximum scour depth in the higher Fr, which is obtained in the higher channel bed slope, is reduced. Also, in the same channel bed slope the maximum scour depth is increased by increasing Fr.

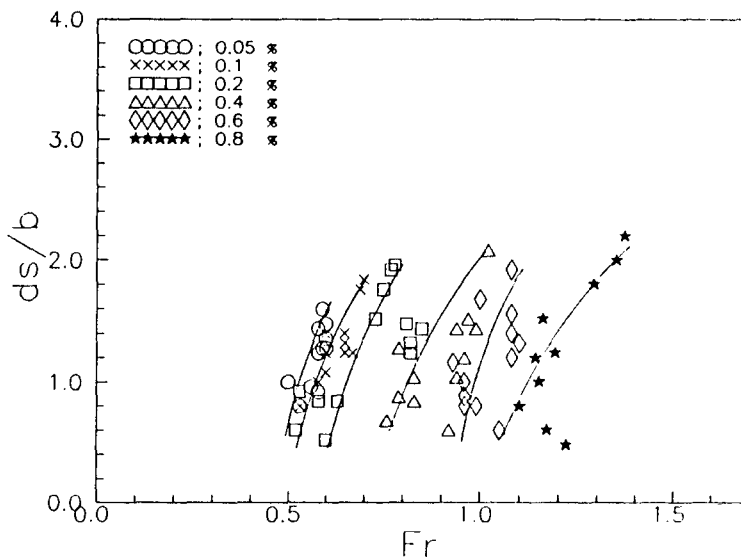


Fig. 7. Maximum Scour Depth Variation with Froude Number Increment

3.3 Maximum Scour Depth Variation by the Opening Ratio.

The maximum scour depth variation by increasing the opening ratios were compared based on the Froude number as shown in Fig. 8. As shown in the figure, the maximum scour depth in the each opening ratio is increased by increasing Froude number. However, the increasing rate of the maximum scour is decreased by increasing Froude number. The maximum scour depth is increased by increasing the open ratios in the same Froude number.

3.4 Maximum Scour Depth Variation by the Angle of Attack

The maximum scour depths by changing the angles of attack were compared based on the channel bed slopes of 0.03%, 0.1% and 0.2%. The comparison is shown in Fig. 9. As shown in the figure, the maximum scour depth is increased by increasing the angle of attack. However, in the angle of attack over 60 degree the maximum scour depth seems not to be changed even though the angle of attack is increased. Also, the

maximum scour depth is increased by increasing the water depth.

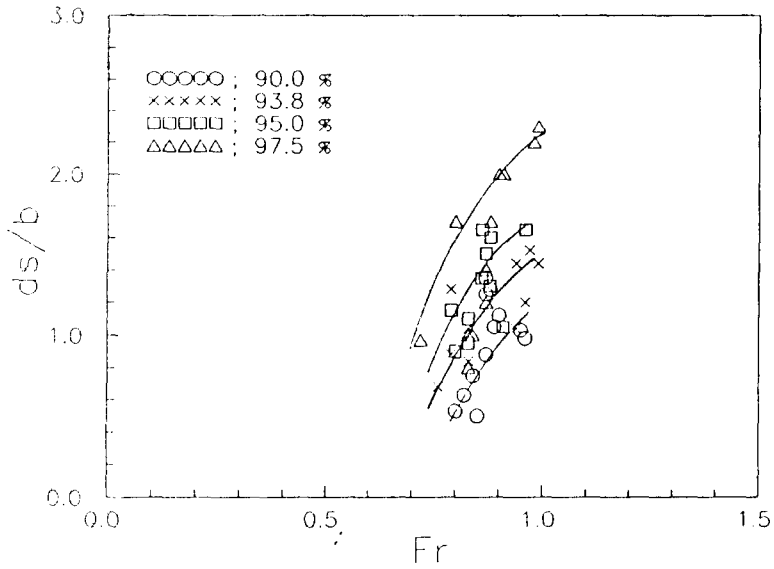


Fig. 8. Maximum Scour Depth Variation by Opening Ratio Increment

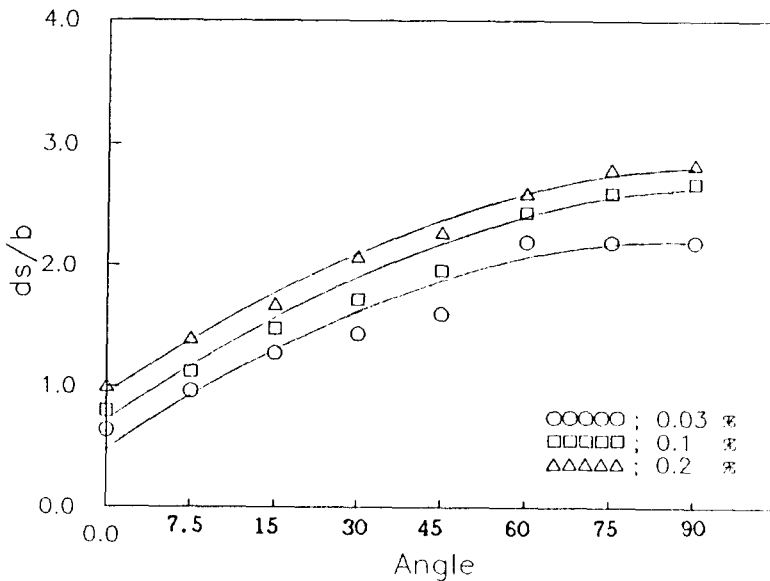


Fig. 9. The Relationship of Maximum Scour Depth for Angle of Attack

4. The Comparison of Pier Scour Depth Between Free Surface Flows and Pressure Flows.

The experimental results mentioned in the previous chapter are based upon the free flow. However, the maximum scour depth in the pressure flow is different from the free flow. In this chapter, the maximum scour depths in the pressure flow, which is indicated when the bridge deck is submerged, were compared with that in the free flow. The comparison is shown in Fig. 10. As shown in the figure, the maximum scour depth in the pressure flow is almost twice as much as that in the free flow. Therefore, the more concerns for safe should be given compared to the free flow.

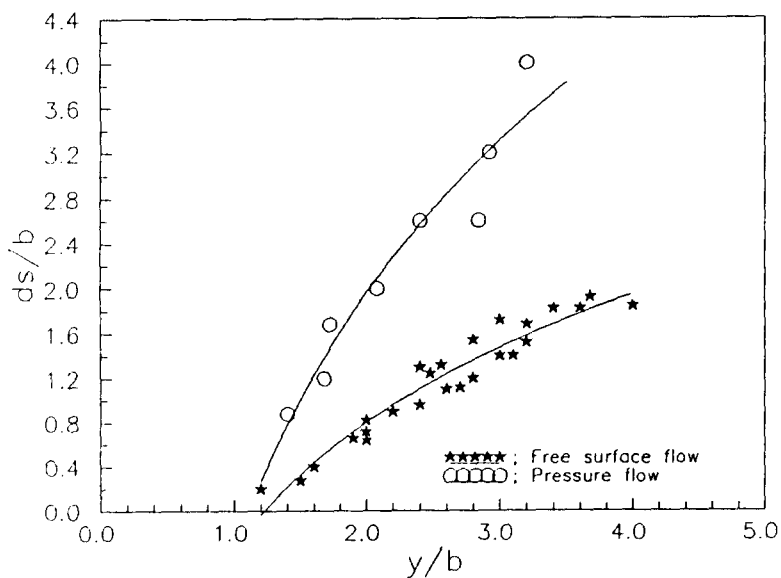


Fig. 10. The Comparison of Pier Scour Depth between Free Surface Flows and Pressure Flows.

5. Conclusions

In this paper, the experiments for maximum scour depth at piers in the free and pressure flow conditions were conducted using the pier model, which was obtained from the pier at San Gye bridge in the Bocheong stream. Froude similarity law was accepted as the major similarity law. The ranges of the channel bed slopes were selected by considering the roughness coefficients in the prototype stream and the model flume. After that, the channel bed slope for the experiment was adjusted by considering the fall velocity and the shear velocity. Also, the final adjustment of the channel bed slope for the experiment was conducted using the measurement data in the field. The conclusions obtained through the experiments are as follows.

First, the maximum scour depth in the pressure flow is almost twice as much as that in the free

flow at San Gye bridge in the Bocheong stream .

Second, the maximum scour depth variations are shown in the figures depending upon the variation of water depth, Froude number, opening ratio and the angle of attack in the free flow condition. Therefore, the figures can be utilized for the design and safety check for the existing piers.

Third, the maximum scour depth in the same water depth is increased depending upon increasing the channel bed slope. However, the increasing rate of the maximum scour depth in the same channel bed slope is reduced by increasing the water depth.

Fourth, the maximum scour depth is increased by increasing Froude numbers and opening ratios.

Fifth, the maximum scour depth is increased by increasing the angle of attack. However, the changing rate of the maximum scour depth is very small in case over 60 degree.

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