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추계학적 최적화방법에 의한 기존관수로시스템의 병열관로 확장
Stochastic Optimization Approach for Parallel Expansion of the Existing Water Distribution Systems

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Abstract

The cost of a looped pipe network is affected by a set of loop flows. The mathematical model for optimizing the looped pipe network is expressed in the optimal set of loop flows to apply to a stochastic optimization method. Because the feasible region of the looped pipe network problem is nonconvex with multiple local optima, the Modified Stochastic Probing Method is suggested to efficiently search the feasible region. The method consists of two phase: i) a global search phase(the stochastic probing method) and ii) a local search phase(the nearest neighbor method). While the global search sequentially improves a local minimum, the local search escapes out of a local minimum trapped in the global search phase and also refines a final solution. In order to test the method, a standard test problem from the literature is considered for the optimal design of the parallel expansion of an existing network. The optimal solutions thus found have significantly smaller costs than the ones reported previously by other researchers.

요 지

管網狀配管(Looped networks)시스템에서 관수로시스템의 전체비용은 閉回路流量(Loop flows)에 따라 영향을 받는다. 따라서 관망상배관의 最適設計를 위한 수학적모형을 推計學的 最適化방법에 적용하기 위하여 폐회로 유량의 攝動(Perturbations)으로 전체비용이 變하게 하였다. 관망상 배관문제의 分析可能영역은 수많은 局地解(Local optimum)를 갖는 비볼록(Nonconvex)이므로 분석가능영역의 효율적인 探査를 위하여 修定推計學的 探査방법을 제안하였으며 이 방법은 局部探査단계(Global search phase)와 局地探査단계(Local search phase)로 구성되어 있다. 국부탐사에서는 점차적으로 국지해를 증진시키며 국지탐사에서는 국부탐사단계에서 교착상태에 있는 국지해로 부터 벗어나게 하거나 최종국지해를 증진시킨다. 제안한 방법의 효율성을 검정하기 위하여 참고문헌에 있는 기존관수로시스템의 병열관로(Parallel pipe line) 확장문제를 標本으로 채택하여 제안한 방법을 적용한 결과 먼저 발표된 연구자들의 비용보다 적은 비용으로 설계할 수 있었다.

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1. Introduction

Pipe network optimization methods should incorporate a scheme for optimal layout selection including optimal pipe diameter selection. A comprehensive review of optimization of water distribution system is given in Lancy and Mays (1989). If it is assumed that the layout of a looped network is known, the flow variables and/or the head variables are used to optimize the cost of the given layout. The conventional optimization methods are divided into the following four groups based on Kessler and Shamir (1991), which are i) optimization of optimal heads and optimal flows simultaneously, ii) optimization of flows with respect to fixed heads, iii) optimization of hydraulic heads with respect to fixed flows, and iv) optimization of flows or heads by solving alternatively for flow and head variables.

The methods in the first group are usually based on nonlinear programming or enumeration techniques. Watanatada (1973) used the Quasi-Newton method and Shamir (1974) used penalty functions to solve the problems. These techniques are relatively complicated, converge slowly, and only a local optimum is possible. Gessler (1982) presented a model based on an enumeration scheme to find the best solution. Given a set of possible pipe sizes, the method performs cost and hydraulic feasible tests on all potential combinations to find the best solution. Such enumeration method may be computationally burdensome if many possibilities cannot be discarded apriori.

The methods in the second group solve the optimal flow rates for fixed heads. If the values of the heads are known, the problem is reduced to a concave problem with a separable objective function and linear constraints. Quindry et al. (1981) presented an algorithm in which the heads were fixed in the subproblem and updated after each iteration. The subproblem converted the nonlinear cost function to a piece-wise linear function. The subproblem was based on the work of Lai and Schaake (1969) and determined optimal continuous diameter for a link. A gradient term was derived from the node equation and directed the change in the head distribution for the next iteration.

The methods in the third group solve the optimal heads for fixed flows. If the flows are known, the problem is reduced to a convex problem with a separable convex objective function and linear constraints. Therefore the problem provides a global minimum. Alperovits and Shamir (1977) presented a two level algorithm that is called the linear programming gradient(LPG) method. Kessler and Shamir (1989) introduced the projection of the gradient of objective function onto the constraint surface to improve the search procedure and obtained a local optimal solution.

The methods in the fourth group use a decomposition technique. Fujiwara and Khang (1990) and Kessler and Shamir (1991) proposed a two-phase decomposition method. These methods attempt to find the near global optimum. It is not uncommon to find real problems which have cost or profit functions defined over a nonconvex feasible region involving local optima and the pipe network optimization problem falls into this group.

In this paper an existing municipal water distribution system is considered. The mathematical optimization model for the pipe network is nonlinear and nonconvex, which may have several local optima. While conventional optimization methods find only local optima, stochastic optimization schemes adapt the local optimum seeking methods to migrate among local optima to find the best one. A variety of general global optimization strategies has been suggested and comprehensive reviews are given in Torn and Zilinskas (1987), Rinnooy Kan and Timmer (1989), and Schoen (1991).

2. Stochastic Optimization Approach for Pipe Networks

2.1 Model Formulation

The pipe network problem(P1) for fixed head nodes may be stated as follows: Problem P1:

$$\text{Minimize} \sum_{(i,j)} \sum_{m=1}^{M} C_{(i,j)m}\, x_{(i,j)m} \qquad (1)$$

Subject to:

$$\sum_{j} Q_{(i,j)} - \sum_{j} Q_{(j,i)} = q_i \quad \text{for } i \in \{N\text{-}S\} \qquad (2)$$

$$H_s - H_k^{\min} - \sum_{(i,j)\in r_k} \pm \sum_{m} J_{(i,j)m} x_{(i,j)m} \geq 0$$
$$\text{for } s \in S \text{ and } k \in \{N\text{-}S\} \qquad (3)$$

$$\sum_{(i,j)\in p} \pm \sum_{m} J_{(i,j)m} x_{(i,j)m} = b_p \quad \text{for } p \in p \qquad (4)$$

$$\sum_{m} x_{(i,j)m} = L_{(i,j)} \quad \text{for} (i,j) \in L \qquad (5)$$

$$x_{(i,j)} \geq 0 \qquad (6)$$

where, N= the number of nodes
M=the number of commercial pipe diameters
S=the set of fixed head nodes
$\{N\text{-}S\}$=the set of junction nodes
L=the set of links
$Q_{(i,j)}$=the steady state flow rate through link $(i,j) \in L$
$L_{(i,j)}$=the length of link$(i,j) \in L$
$C_{(i,j)m}$=the cost per unit length of a pipe of d_m
r_k=the path from a fixed head node(source) to demand node, k
P=the set of paths connecting fixed head nodes and basic loops
b_p=the head difference between the fixed head nodes for path p connecting them; and it is zero corresponding to loops
H_s=the fixed head
$H_k^{\min}$=the minimum head at node $k \in \{N\text{-}S\}$
q_i=the supply at node i which is positive; if it is demand, it is negative.

Let $J_{(i,j)m}$ be the hydraulic gradient for segment m(partial length of a link with diameter d_m) which is given by

$$J_{(i,j)m} = K[Q_{(i,j)}/C]^{1.85} d_m^{-4.87} \qquad (7)$$

in which: K=10.7 for $Q_{(i,j)}$ in cubic meters per second(*cms*) and d_m in meters; C=Hazen-Williams Coefficient. The pipe network problem may be stated as follows: In Problem(P1) pipe cost objective function (1) is minimized; constraint (2) represents steady state flow continuity; constraint (3) is the minimum head restriction; constraint (4) represents the sum of head losses in a path which is zero for loops; constraint (5) dictates that sum of segment lengths must equal link length; constraint (6) is the non-negativity on segment lengths.

2.2 Modified Stochastic Probing Method

As stated in chapter 1, the general mathematical model for a pipe network is a nonlinear and nonconvex programming, which may have several local optima. Deterministic optimization methods have been applied to the pipe network problems to find only local optima. For the given mathematical model(P1), the model reduces to a linear programming problem if a set of fixed link flows are hydraulically feasible. However, the model still defines over a nonconvex feasible region involving several local optima because the cost of the model mainly depends upon a set of fixed link flows. The objective of the stochastic optimization method is to find the solution in a feasible region for which the mathematical model contains its smallest value, a near global minimum.

Since the cost of the fixed flows model is affected by the fixed link flows, the following Problem P2 is suggested for the solution of Problem P1. Problem(P2): Minimize $LP(\boldsymbol{Q})$ or $LP(\boldsymbol{q})$ where $\boldsymbol{Q}$ is a set of link flows and $\boldsymbol{q}$ is a set of loop flows. It should be noted that LP denotes the solution of problem P1 given a set of link flows $\boldsymbol{Q}$ or a set of loop flows $\boldsymbol{q}$. The linear programming model, $LP(\boldsymbol{Q})$, is equivalent to $LP(\boldsymbol{q})$ because a set of link flows is determined by a set of loop flows in the looped network. The set of loop flows $\boldsymbol{q}$ is treated as the set of decision variables in the stochastic optimization method. The stochastic optimization method is then to find a near global minimum $\boldsymbol{q}^*$ such that $LP(\boldsymbol{q}^*) < LP(\boldsymbol{q})$ for all $\boldsymbol{q} \in \boldsymbol{R}^n$ where $\boldsymbol{R}^n$ is a vector space and n is the number of loops. Therefore, the given model has been converted into an unconstrained model in the stochastic optimization. The modified stochastic probing method is employed to search the feasible region of a pipe network.

The modified stochastic probing method consists of a global search phase(the stochastic probing method) and a local search phase (the nearest neighbor method). These two phases are used to obtain a near global minimum of a looped pipe network. First the global search is conducted with the stochastic probing method. For a given starting location of loop flows $\boldsymbol{q}^o$, the stochastic probing method is sequentially used to find a local minimum $\boldsymbol{q}$. When the stochastic probing method does not improve a local minimum $\boldsymbol{q}$, the nearest neighbor method is then used to escape out of a local minimum $\boldsymbol{q}$, and also to refine a final optimal solution obtained with the global search. The stochastic probing method is to search the location of a near global minimum $\boldsymbol{q}^*$. The method begins with the construction of a probing distribution, with density $p(\cdot \mid \boldsymbol{q}^o, \sigma^o)$ where $\boldsymbol{q}^o$ location of loop flows and σ is a scale of loop flows, respectively. The costs of $LP(\boldsymbol{q})$ are evaluated at a few loop flows $\boldsymbol{q}$ sampled from the distribution. Since the probing distribution is a normal distribution, the generated loop flows outside of a $100(1-\alpha)\%$ confidence interval for the given parameters are simply discarded. It is assumed that the cost of simulating sample loop flows are negligible compared to those of evaluating $LP(\boldsymbol{q})$. The updating location of loop flows $\boldsymbol{q}$ and scale σ are based on Gibbs-like distribution and the entropy of the current distribution, respectively. Thus, the method is related to the simulated annealing in that the updating location of loop flow is on basis of Gibbs distribution. The method iteratively improves the cost of $LP(\boldsymbol{q})$ by searching successive better loop flows.

Once an optimal local minimum $\boldsymbol{q}^*$ is obtained by the global search, the local search is conducted to improve the current local minimum $\boldsymbol{q}^*$. That is, the local search is designed to move from the current local minimum $\boldsymbol{q}^*$ to a better solution $\boldsymbol{q} \neq \boldsymbol{q}^*$ with $LP(\boldsymbol{q}) < LP(\boldsymbol{q}^*)$. In this phase $\boldsymbol{q}$ is any point in a neighborhood of $\boldsymbol{q}^*$ such that $|\boldsymbol{q}-\boldsymbol{q}^*| < \delta$ and $LP(\boldsymbol{q}) < LP(\boldsymbol{q}^*)$, where δ is a prescribed critical distance. The procedure of the modified stochastic probing method is described next.

Global Search Phase

Step 0.(*Initialization*) Select an initial location of loop flow $\boldsymbol{q}^o$ and a scale factor σ^o of a probing distribution. Assign a value for the level of significance α.

Step 1.(*Generation step*) At stage n($n \geq 0$), generate k independent, identically distributed loop flows $\boldsymbol{q}_{n1}, \cdots \boldsymbol{q}_{nk}$ from $p(\cdot \mid \boldsymbol{q}^o, \sigma)$. Let $\boldsymbol{q}_{no} = \boldsymbol{q}_n$ and $\boldsymbol{q}^n = (\boldsymbol{q}_{no}, \cdots, \boldsymbol{q}_{nk})$

Step 2.(*Update location*)The updated location $\boldsymbol{q}_{n+1}$ is chosen from a point that is the highest probability in the following Gibbs-like distribution;

$$P_r(q_{n+1} = q_{ni}) = \frac{1}{Z} e^{-B(\phi, 1, q^n)}, \quad i = 0, 1, \cdots, k$$

where, $Z = \sum_{1=0}^{k} e^{-B(\phi, 1, q^n)}$ *and*

$$B(\phi, i, q^n) = \frac{LP(q_{ni}) - LP(q_{no})}{\min(LP(q_{ni}),\ LP(q_{no})) - LP_{min}}$$

in which $LP(\cdot)$ is the cost of the linear program(P1) given a set of loop flows.

Step 3.(*Updated scale*) The rule for scale reduction is of the form

i) $\boldsymbol{q}_{n+1} = \boldsymbol{q}_n$ and σ_n if $f(\boldsymbol{q}_{n+1}) \geq f(\boldsymbol{q}_n)$ Call the Local Search Phase

ii) $\sigma_{n+1} = w_n \sigma_n$ if $f(\boldsymbol{q}_n)$, where $w_n = \dfrac{Ent(n)}{log(k+1)}$

The scale reduction factor w_n is based on the entropy of the current distribution:

$$Ent(n) = -\sum_{1=0}^{k} P_r(\boldsymbol{q}_{n+1} = \boldsymbol{q}_{ni}) log\ P_r(\boldsymbol{q}_{n+1} = \boldsymbol{q}_{ni})$$

where $P_r(\cdot)$ is given in the current distribution.

Step 4.(*Stopping rule*)

i) If $\sigma_{n+1} < \sigma_n$, increase n by one and go to step 1.

ii) If $\sigma_{n+1} = \sigma_n$ and no improvement has been obtained in the last few iterations, save $\boldsymbol{q}_n^* = \sigma_n$ and $f(\boldsymbol{q}_n^*)$ and stop; otherwise go to Step 1.

Local Search Phase

Step a.(Initialization) Choose the current probing distribution with density $p(\cdot \mid q_n, \sigma_n)$. Assign values for sample size $Mmax$ and critical distance δ. Set $M=1$.

Step *b*. If $Mmax = M$, save q_n and $LP(q_n)$. Go to step 1 of the Global Search Phase. Let q_n^o be a random neighbor of q_n. Generate q_n^o from the density function $p(\cdot \mid q_n, \sigma_n)$.

Step c. i) If $|\boldsymbol{q}_n^o - \boldsymbol{q}_n| \leq \delta$ and $LP(\boldsymbol{q}_n^o) - LP(\boldsymbol{q}_n) \leq 0$ $\boldsymbol{q}_n = \boldsymbol{q}_n^o$.Increase M by one and go to step b.

ii) If $|\boldsymbol{q}_n^o - \boldsymbol{q}_n| \leq \delta$ or $LP(\boldsymbol{q}_n^o) - LP(\boldsymbol{q}_n) > 0$, go to step b.

2.3 Parallel Expansion of Existing Networks

In the present formulation(Problem P1) the parallel expansion of existing pipes is considered. Adaptation of Problem(P1) for existing networks is as follows. For existing pipes, the pipe diameters are fixed and parallel links may be required for carrying additional flow in order not to increase the head loss. The flow from node i to node j, for a parallel system, is denoted by $Q_{(i,j)}=Q_{(i,j),O}+Q_{(i,j)N}$ in which subscripts O and N indicate Old and New respectively. In the present study it is suggested that Problem(P1) be solved with the restriction that an existing pipe is not undersized. If an existing pipe is undersized, the flow becomes open channel flow rather than pipe flow because the hydraulic radius of the existing pipe is larger than that of the undersized pipe. From the solution to Problem(P1) the head loss for the each pipe can be obtained by multiplying the optimal segment lengths, $x_{(i,j)m}$'s and the corresponding head gradients $J_{(i,j)m}$'s Because optimal head loss, friction parameter and existing pipe size are known, the flow in an existing link and therefore the flow for the parallel pipe can be computed. The parallel pipe is in turn decomposed into discrete diameter pipes in series to design the least cost. The procedure of the parallel expansion of existing pipe networks is suggested as follows.

Step 1. Call the procedure Modified Stochastic Probing Method to solve Problem(P1) to get the best loop flows.

Step 2. Detect link that is required to have a parallel link.

Step 3. Compute flow in the existing link.

Step 4. Calculate flow and continuous diameter for new parallel link.

Step 5. Decompose continuous diameter into adjacent discrete diameters in series.

3. Application to a Typical Pipe Network

The New York city water supply problem is solved to expand the parallel links of the system with the proposed procedure. The solution for the New York water distribution system problem as shown in Figure 1 has been attempted before by Schaake and Lai (1969), Quindry et al.(1981), Gessler(1982), Bhave(1985), Morgan and Goulter(1985), and Fujiwara and Khang(1990). These authors report improvements in the optimal solution progressively. Schaake and Lai, Quindry et al., Gessler, Bhave, and Morgan and Goulter obtained optimal costs of $77.611(106), $63.581(106), $41.2(106), $40.18(106), and $39.229(106), respectively. This progression is the result of finding better local optima with the improvements in the solution algorithms. As given in the previous studies, the following data will be used: Hazen-Williams; C=100 for all links; conversion factor K=10.7 for flows in *cms* and diameters in meters; exponents for discharge and diameter are 1.85 and -4.87 respectively. The cost per unit length of pipes is given by $1.1d^{1.24}$ in which d is the diameter in inches. Link length, diameter for existing pipes, minimum heads, and demands are given in Table 1.

For known flow rates, Problem(P1) is a linear program. To solve the New York city water supply problem with the proposed procedure, First, the algorithm TREESEARCH (Loganathan et al, 1990) is applied to obtain the optimal tree layout and the optimal tree link flows. The global tree network is obtained by deleting link 10 and 20. The optimal tree link flows are given in Table 2. The procedure Modified Stochastic Probing method is then implemented to search the feasible region beginning with the perturbed optimal tree link flows as the initial flows. The optimal loop flows in $cms(\Delta Q_1, \Delta Q_2)=(4.0776, 0.2378)$ are obtained with the method. Once the optimal link flows are obtained, the problem(P1) is again solved with the restriction

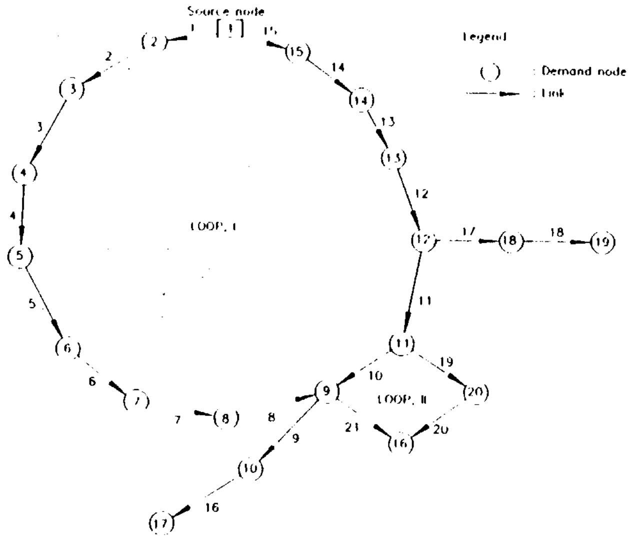

Fig. 1. New York Water Distribution System

Table 1. Node and Link Data of New York City Water Distribution Network

Node Number	Demand cms	Minimum Head m	Link Number	Length m	Diameter m
1	-57.1293	91.44	1	3535.68	4.572
2	2.6165	77.72	2	6035.04	4.572
3	2.6165	77.72	3	2225.04	4.572
4	2.4975	77.72	4	2529.84	4.572
5	2.4975	77.72	5	2621.28	4.572
6	2.4975	77.72	6	5821.68	4.572
7	2.4975	77.72	7	2926.08	3.353
8	2.4975	77.72	8	3810.00	3.353
9	4.8167	77.72	9	2926.08	4.572
10	0.02832	77.72	10	3413.76	5.182
11	4.8138	77.72	11	4419.60	5.182
12	3.3159	77.72	12	3718.56	5.182
13	3.3159	77.72	13	7345.68	5.182
14	2.6165	77.72	14	6431.28	5.182
15	2.6165	77.72	15	4724.40	5.182
16	4.8138	79.25	16	8046.72	1.829
17	1.6282	83.15	17	9509.76	1.829
18	3.3159	77.72	18	7315.20	1.524
19	3.3159	77.72	19	4389.12	1.524
20	4.8138	77.72	20	11704.32	1.524
			21	8046.72	1.829

Table 2. Optimal Tree Link Flows for New York System

Link Number	Flow *cms*	Link Number	Flow *cms*
1	29.005	12	19.5755
2	26.3885	13	22.8914
3	23.7720	14	25.5079
4	21.2745	15	28.1243
5	18.7769	16	1.6282
6	16.2794	17	6.6318
7	13.7818	18	3.3159
8	11.2843	19	4.8139
9	1.6565	20	0.0
10	0.0	21	4.8139
11	9.6277		

that an existing pipe should not be undersized. The solution to Problem(P1) then shows which link is required to add a parallel link.

Parallel expansion of this system is discussed in the following. From the solution to Problem(P1) as shown in Table 3, links 7, 16, 17, 18, 19, and 21 are required to have parallel links because one of the split segments of each link is larger than an existing diameter of link. From the solution to Problem(P1), the optimal head loss $h_{f(i,j)}$ and the optimal(total) flows $Q_{T,(i,j)}$ are known. Therefore the flow in an existing link is solved by the following equation:

$h_{f(i,j)} = 10.7L_{(i,j)}(\boldsymbol{Q}_{(i,j)O}/C)^{1.852}D_{(i,j)O}^{-4.87}$ where $\boldsymbol{Q}_{(i,j)O}$ is the flow in existing link, $D_{(i,j)O}$ is the existing diameter of pipe, and $L_{(i,j)}$ the length of link. Once the flow in the existing link is computed, the flow in the parallel pipe $\boldsymbol{Q}_{(i,j)N}$ is computed from $\boldsymbol{Q}_{T,(i,j)} = \boldsymbol{Q}_{,(i,j)o} + \boldsymbol{Q}_{,(i,j)N}$ and therefore the diameter for the parallel pipe can be computed from $h_{f(i,j)} = 10.7L_{(i,j)}(\boldsymbol{Q}_{(i,j)N}/C)^{1.852}D_{(i,j)N}^{-4.87}$ in which subscript N indicates new parallel pipe. The parallel pipe is then decomposed into discrete diameter pipes in series as:

$$h_{f(i,j)} = 10.7L_{(i,j)}(\boldsymbol{Q}_{(i,j)N}/C)^{1.852}D_{(i,j)N}^{-4.87}$$
$$= 10.7L_{1(i,j)}(\boldsymbol{Q}_{(i,j)N}/C)^{1.852}D_{1(i,j)}^{-4.87}$$
$$+ 10.7L_{2(i,j)}(\boldsymbol{Q}_{(i,j)N}/100)^{1.85}D_{2(i,j)}^{-4.87}$$

and it follows

$$LD_{(i,j)N}^{-4.87} = L_1D_{1(i,j)}^{-4.87} + L_2D_{2(i,j)}^{-4.87}$$

in which subscripts 1 and 2 denote split segments with $D_{1(i,j)} < D_{(i,j)N} < D_{2(i,j)}$.

Finally the solution for this system is rechecked for hydraulic feasibility. Complete results are documented in Tables 4, 5 and 6.

The solution reported by Fujiwara and Khang (1990), is infeasible to their own set of parameters. This is easily verified by using the optimal flows and optimal diameters of Fujiwara and Khang(see their Tables 5 and 6, p. 547). The solution fails to yield zero head loss for the large loop and violates minimum heads for nodes 16, 17, and 19. For example they report flow in link 10 to be 3.6614 *cms* and its diameter is 5.182 meters. Therefore, using the given parameters and length of the link 3,413.76 m, the head loss in link 10 is

$h_{f10} = 10.7(3,413.76)(3.6614/100)^{1.85}(5.182)^{-4.87} = 0.027m$, Using the head value at node 11, $H_{11} = 83.314$ m, the head value at node 9 is obtained as $H_9 = 83.287$ m.

This value is clearly different from 83.226 m obtained from

$h_8 = 10.7(3,810)(6.9829/100)^{1.85}(3.353)^{-4.87} = 0.81m$ and the head at node 8,

$H_8 = 84.036$ m proving the improper diameter selection for the reported parameter values.

Table 3. Solution to Problem(P1)

Link	Flow cms	Diameter m	Length m	Headloss m
1	24.9247	4.572	3535.68	1.7496
2	22.3109	4.572	6035.04	2.4323
3	19.6944	4.572	2225.04	0.7132
4	17.1969	4.572	2529.84	0.6309
5	14.6993	4.572	2621.28	0.4877
6	12.2018	4.572	5821.68	0.7681
7	9.7042	3.353	454.03	0.1768
7	9.7042	4.572	2472.05	0.2134
8	7.2067	3.353	3810.00	0.8595
9	1.6565	4.572	2926.08	0.0091
10	3.8398	5.182	3413.76	0.0274
11	13.7054	5.182	4419.60	0.3932
12	23.6531	5.182	3718.56	0.9083
13	26.9690	5.182	7345.68	2.2860
14	29.5855	5.182	6431.28	2.3744
15	32.2020	5.182	4724.40	2.0422
16	1.6282	2.743	4112.43	0.1585
16	1.6282	3.048	3934.29	0.0914
17	6.6318	2.743	6422.27	3.3010
17	6.6318	3.048	3087.49	0.9510
18	3.3159	2.438	7315.20	1.8532
19	5.0517	2.134	3053.95	3.2278
19	5.0517	2.438	1335.17	0.7376
20	0.2379	1.524	11704.32	0.2225
21	4.5760	1.829	328.97	0.6126
21	4.5760	2.438	7717.75	3.5448

However, links 7, 16, 19, and 21 can be easily redesigned with the given information. By setting the sum of head losses equal to zero for the larger loop one can obtain the head loss for link 7. Because the flow in link 7 is 9.4805 *cms*, one can obtain the equivalent diameter for link 7 which should be decomposed to account for the existing link of diameter 3.353 meters and a parallel pipe is provided. Also, Table 5 of Fujiwara and Khang contains typographical errors for flows in links 5 and 12 which should be 14.4756 *cms* and 23.8768 *cms* to satisfy continuity. By setting minimum heads for nodes 16, 17, and 19 and using the reported flows with the above correction and the rest of the optimal diameters, the necessary links are redesigned. However, the cost of this feasible design for the reported flow pattern in Fujiwara and Khang(Table 5, p. 547) is around 40.0 million dollars. In this study, the procedure Modified Stochastic Probing Method yields a cost of 38.041 million dollars which is an improvement over the current best result of 39.229million dollars by Morgan and Goulter (1985), as shown in Figure 2.

Table 4. Optimal Heads for New York System

Node Number	Demand cms	Min. Head m	Optimal Head m
1	-57.1293	91.44	
2	2.6165	77.72	89.69
3	2.6165	77.72	87.26
4	2.4975	77.72	86.54
5	2.4975	77.72	85.91
6	2.4975	77.72	85.43
7	2.4975	77.72	84.66
8	2.4975	77.72	84.27
9	4.8139	77.72	83.41
10	0.0283	77.72	83.40
11	4.8139	77.72	83.44
12	3.3519	77.72	83.83
13	3.3519	77.72	84.74
14	2.6165	77.72	87.02
15	2.6165	77.72	89.40
16	4.8139	79.25	79.25
17	1.6282	83.15	83.15
18	3.3159	77.72	79.58
19	3.3159	77.72	77.72
20	4.8139	77.72	79.47

4. Summary

The modified stochastic probing method ite-

Table 5. Optimal Solution for New York System

Link No.	Length *m*	Optimal flow, *cms*	Head Loss, *m*	Existing *m*	New Dia. *m*	Split pipe Length, *m*	Existing Links $ million	New Links $ millon
1	3535.68	24.9274	1.7496	4.572			7.9924	
2	6035.04	22.3109	2.4323	4.572			13.6422	
3	2225.04	19.6944	0.7132	4.572			5.0297	
4	2529.84	17.1969	0.6309	4.572			5.7187	
5	2621.28	14.6993	0.4877	4.572			5.9254	
6	5821.68	12.2018	0.7681	4.572			13.1599	
7	2926.08	9.7042	0.1768	3.353	3.048	2713.35	4.5024	4.0394
			0.2134		3.353	212.73		
8	3810.00	7.2067	0.8595	3.353			5.8625	
9	2926.08	1.6565	0.0091	4.572			6.6144	
10	3413.76	3.8398	0.0274	5.182			9.0048	
11	4419.60	13.7054	0.3932	5.182			11.6580	
12	3718.56	23.6531	0.9083	5.182			9.8088	
13	7345.68	26.9690	2.2860	5.182			19.3764	
14	6431.28	29.5855	2.3744	5.182			16.9644	
15	4724.40	32.2020	2.0422	5.182			12.4620	
16	8046.72	1.6282	0.1585	1.829	2.438	6241.10	5.8344	8.6329
			0.0914	−	2.743	1805.62		
17	9509.76	6.6318	3.3010	1.829	2.438	9444.49	6.8952	9.8696
			0.9510	−	2.743	65.27		
18	7315.20	3.3159	1.8532	1.524	2.134	7085.60	4.2240	6.4449
				−	2.438	229.60		
19	4389.12	5.0517	3.2278	1.524	1.829	4171.82	2.5344	3.2152
			0.7376	−	2.134	217.30		
20	11704.32	0.2379	0.2225	1.524			6.7584	
21	8046.72	4.5760	0.6126	1.829	1.829	8013.41	5.8344	5.8394
			3.5448	−	2.134	33.31		
						Total cost: $ million	179.8028	38.0413

Table 6. Optimal Flows of Links Needed Parallel Pipes for New York System

Link Number	Existing Link Flow *cms*	Parallel Pipe Flow *cms*
7	5.4224	4.2818
16	0.5004	1.1279
17	2.1147	4.5171
18	0.9625	2.3534
19	1.9139	3.1378
20	2.2866	2.2894

ratively improves a current optimal solution by searching successive better loop flows and allows to escape from the region of attraction of local minimum at each stage. As shown in the procedure of the method, one of the advantages in this method avoids the cooling schedule of annealing compared with the simulated annealing method. The New York City water distribution system is re-solved with

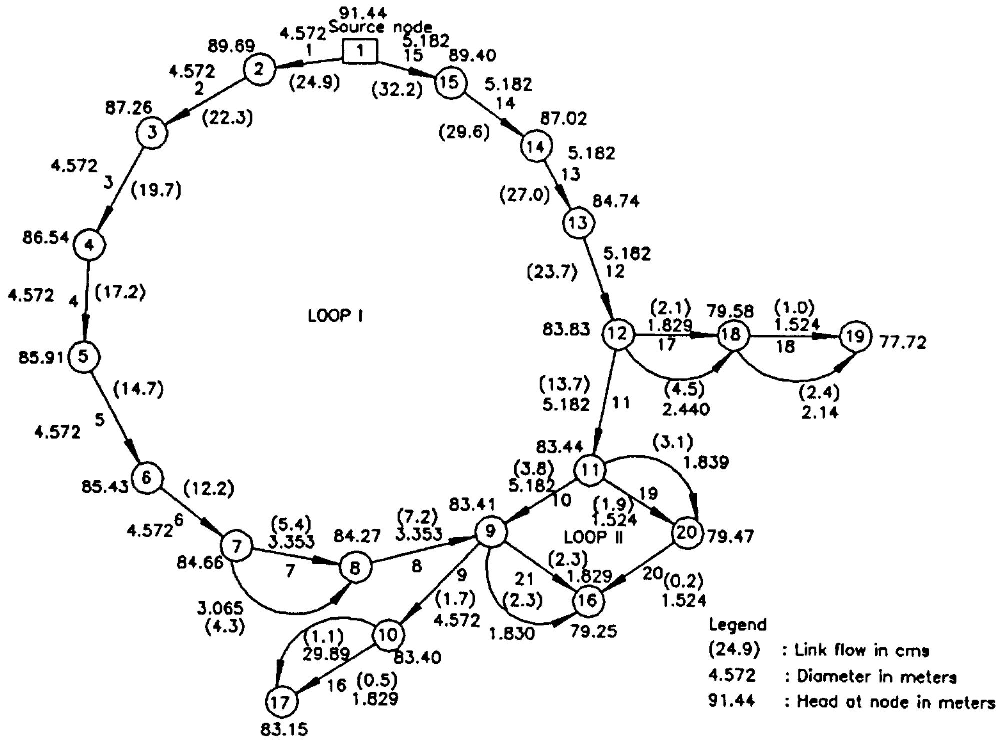

Table 2. Optimal Solution for New York System by Proposed Method

the present approach, yielding least cost designs which are better than the previously reported designs in the literature. The results from implementing the method to the sample pipe network show that even the extra values of locations(loop flows) are initially chosen, the convergence for a near global optimum is successful.

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