⊙ 研究論文

A Study on the Robust Control of Systems Dominantly Subjected to Modeling Errors and Uncertainties

J. H. Kim*

모델링오차와 불확실성을 지배적으로 받는 시스템의 강인한 제어에 관한 연구

김 종 화*

Key words : Fuzzy control rules(퍼지제어규칙), Fuzzification(퍼지화), Defuzzification(비퍼지화), Fuzzy PID control(퍼지 PID 제어), LQG/LTR control(LQG/LTR제어), Target filter loop(목표필터루프), Loop transfer recovery(루프전달회복)

Abstract

In order to control systems which are dominantly subjected to modeling errors and uncertainties, control strategies must deal with the effect of modeling errors and uncertainties. Since most of control methods based on system mathematical model, such as LQG/LTR method, have been developed mainly focused on stability robustness, they can not smartly improve the transient response disturbed by modeling errors and/or uncertainties.

In this research, a fuzzy PID control method is suggested, which can stably improve the transient responses of systems disturbed by modeling errors as well as systems not entirely using mathematical models.

So as to assure the effectiveness of suggested control method, computer simulations are accomplished for some example systems, through the comparison of transient responses.

1. Introduction

Design purpose of control systems lies in assuring nominal stability for plant model, in improving tracking performance for reference input or setpoint and disturbance rejection performance, and lies in assuring stability robustness against modeling errors and uncertainties.

As industry has been developed day by day, the demand for control system design has been changing in the direction to accomplishing more accurate and more faster control by improvement of transient response. In order to sat-

* Dept. of Control and Instrumentation Engineering, Korea Maritime University.

isfy this requirements, the effect of modeling errors or uncertainties must be considered, being accompanied in the process of mathematical modeling for the controlled plant or process.

In the postdecade of 1970, a large number of design methods[1~5] about multivariable control systems assuring stability robustness against modeling errors or uncertainties, have been developed since the singular values of matrix has begun to be applied as the measure of stability robustness of systems, in view of engineering aspects rather than mathematical aspects. The representative method of these is LQG/LTR(Linear Quadratic Gaussian with Loop Transfer Recovery)[6] which is model based compensation technique with design procedure of the first step for target filter loop design and the second step for loop transfer recovery, using singular values and their frequency responses. But LQG/LTR method has the constraint that controlled plant is linear, stabilizable, detectable, nonsingular and minimum phase so that desirable loop shape and loop transfer recovery are assured. In recent, an application in nonlinear systems was tried by Kim[7] named as QLQG/LTR(Quasi - linear Quadratic Gaussian with LTR) and reserches[8,9] about loop transfer recovery for nonminimum phase systems were discussed as well as loop shaping problems[10,11] for singular systems. Also reserch trends on H^∞ control and H^∞/LTR control using the infinite norm of transfer matrix in steady state instead of singular values, are appeared and discussed nowadays.

By the way, many methods about stability robustness are very effective and useful in view of sufficient satisfaction about stability requirement, but these methods do not supply tools which improve transient responses disturbed by modeling errors, uncertainties, or disturbances because of their fixed controller structures.

In this research, general fuzzy logic controllers with variable structures[18] are discussed, which reflect and control the effect due to uncertainties etc.Then a new fuzzy logic controller is suggested named fuzzy PID controller which controller parameters are varying according to input signals of controller. And computer simulations are accomplished to evaluate the performance of suggested control method through the comparison of its responses with those of standard LQG/LTR method. In order to assure the excellence of suggested method, many example systems are employed such as linear system, nonlinear system, and nonminimum phase system.

2. Standard LQG/LTR control method

In general the state equation of controlled plant is described as

$$\dot{x}_p(t)=A_p x_p(t)+B_p u_p$$
$$y_p(t)=C_p x_p(t) \qquad (2.1)$$

where $x_p(t)\in R^n$, $u_p(t)\in R^m$, and $y(t)\in R^r$. And given plant is assumed to be nonsingular so that A_p^{-1} exists. Also it is assumed that design plant model should be selected by adding free integrator on the feedforward path in order to make steady state error be zero if free integrators do not exist in plant transfer matrix.

$$\dot{x}(t)=Ax(t)+Bu(t)$$
$$y(t)=Cx(t) \qquad (2.2)$$

where

$$x(t)=[u_p(t)\; x_p(t)]^T,\; u(t)=\dot{u}_p(t)$$

$$A=\begin{bmatrix} 0 & 0 \\ B_p & A_p \end{bmatrix},\; B=\begin{bmatrix} I \\ 0 \end{bmatrix},\; C=[0 \quad C_p] \qquad (2.3)$$

The structure of standard LQG/LTR control

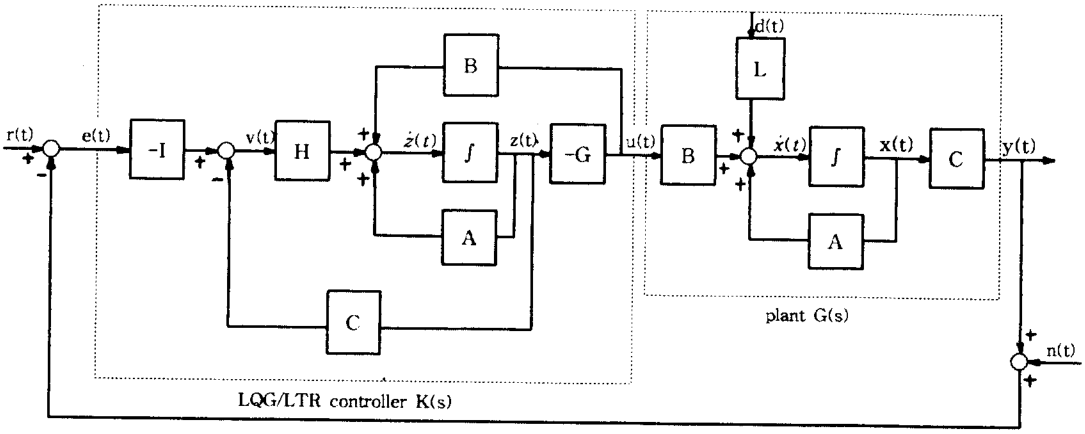

Fig. 2.1 Structure of standard LQG/LTR control system

system is given as Fig. 2.1.

The design procedure of LQG/LTR is divided into two parts. At first, target filter loop is designed to obtain desirable loop shape for design plant model; next, loop transfer recovery is accomplished so that the singular values of loop transfer matrix of control system may be recovered to the singular values of loop transfer matrix of target filter loop.

The transfer matrix of LQG/LTR controller K(s) given as Fig. 2.1 is described as

$$K(s)=G(sI-A+BG+HC)^{-1}H \qquad (2.4)$$

where design parameter H is selected in design process of target filter loop and G is selected in design process of loop transfer recovery.

2.1 Design of target filter loop

In order to design LQG/LTR control system, it is assumed that design plant model should be disturbed by disturbance and sensor noise.

$$\dot{x}(t)=Ax(t)+Bu(t)+L\zeta(t)$$
$$y(t)=Cx(t)+\theta(t) \qquad (2.5)$$

where $\zeta(t)$ is disturbance, $\theta(t)$ is sensor noise, and both are Gaussian processes with zero mean.

$$E[\zeta(t)]=0, \quad E[\zeta(t)^T\zeta(\tau)]=I\,\delta(t-\tau)$$

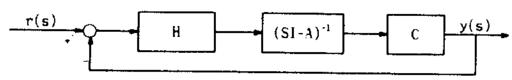

Fig. 2.2 The structure of target filter loop

$$E[\theta(t)]=0, \quad E[\theta(t)^T\theta(\tau)]=\mu I\delta(t-\tau) \qquad (2.6)$$

The blockdiagram of target filter loop is shown in Fig. 2.2.

When filter loop is cut at output or at error signal, the loop transfer matrix is given as

$$G_F(s)=C(sI-A)^{-1}H \qquad (2.7)$$

Then filter gain matrix H as the design parameter of target filter loop is selected by solving Kalman filter problem for the fictitious sensor noise.

$$H=\frac{1}{\mu}PC^T \qquad (2.8)$$

where P is the solution of the following filter algebraic Riccati equation.

$$AP+PA^T+LL^T-\frac{1}{\mu}PC^TCP=0 \qquad (2.9)$$

To solve eq.(2.9) μ and L must be selected, so that μ and L are ultimate design parameters of target filter loop. μ and L can be obtained using Kalman filter frequency domain equality[6]. That is, the approximate loop transfer matrix of target filter loop is given as

$$G_F(s) \cong \frac{1}{\sqrt{\mu}} C(sI-A)^{-1} L \qquad (2.10)$$

Then L is selected in view of obtaining desirable loop shape with respect to reference input tracking performance and modeling error constraint[6].

(1) case in equalizing singular values at low frequency range

$$L = -C_p^T(C_p A_p^{-1} C_p^T)^{-1}$$
$$\text{or } L = -A_p C_p^T(C_p C_p^T)^{-1} \qquad (2.11)$$

(2) case in equalizing at high frequency range

$$L = C_p^T (C_p C_p^T)^{-1} \qquad (2.12)$$

(3) case in equalizing at low and high frequency range simultaneously

$$L = \begin{bmatrix} -(C_p A_p^{-1} B_p)^{-1} \\ C_p^T (C_p C_p^T)^{-1} \end{bmatrix} \qquad (2.13)$$

Another design parameter μ is selected in the way that the singular value shape of target filter loop satisfies demanded cutoff frequency or bandwidth.After μ and L are selected, filter gain matrix H of LQG/LTR controller is computed using eqs.(2.8) and (2.9).

2.2 Loop transfer recovery

Loop transfer recovery can be accomplished by solving cheap control linear quadratic regulator problem[6]. In order for loop transfer recovery to be accomplished, the solution K of control algebraic Riccati equation must be solved as ρ approaches zero, where ρ is selected arbitrarily.

$$KA + A^T K + C^T C - \frac{1}{\rho} K B B^T K = 0 \qquad (2.14)$$

And control gain matrix G of design parameter of LQG/LTR is selected as following expression.

$$G = \frac{1}{\rho} B^T K \qquad (2.15)$$

Then if singular value shape is recovered into that of target filter loop, the design procedure of LQG/LTR controller is completely accomplished.

3. Design of fuzzy logic controller

During the past several years, fuzzy control has emerged as one of the most active and important branch of fuzzy set theory since the invention of the first fuzzy controller using Zadeh's fuzzy logic by Mamdani[12] in 1974. A large number of literature on fuzzy control and application in industrial processes have been growing rapidly, making it difficult to present a comprehensive survey[13~20].

Fuzzy logic controller(FLC) is based on the fuzzy logic which is much closer in spirit to human thinking and natural language than the traditional logical system. Viewed in this perspective, the essential part of the FLC is a set of linguistic control rules related by the dual concepts of fuzzy implication and the compositional rule of inference. Then, in essence, the FLC provides an algorithm which can convert the linguistic control strategy based on expert knowledge into an automatic control strategy.

In particular, the methodology of the FLC appears very useful in case the processes are too complex for analysis by conventional control technique, and in case the available source of information are regarded inexactly or uncertainly. However, at present there is few literature on the systematic procedure for the design of an FLC purely. The trends are appeared in order to design an FLC systematically and to assure global stability with the aid of conventional logical control theories, such as sliding mode control[21] and PI control[22,23].

In this research, the author may try to derive nonlinear fuzzy PID control law with time varying PID gains by development of nonlinear fuzzy PI control theory suggested by H.Ying et.al.,[22,23] in order to control systems which are dominantly subjected to modeling errors or uncertainties, and also to control nonlinear systems.

3.1 Description of a fuzzy logic controller(FLC)

The basic configuration of FLC is described by Fig. 3.1 which compirses four principal components.

1) The fuzzification interface measures the values of input variables of FLC and performs a scale mapping that transfers the range of values of input variables into corresponding universes of discourse. And also it performs fuzzification that converts inputs into suitable linguistic values which may be viewed as labels of fuzzy sets.

2) The knowledge base consists of a data base and fuzzy linguistic control rule base. The data base provides necessary definitions which are used to define linguistic control rules and fuzzy data handling in an FLC. The rule base characterizes the control goals and control strategy of the related system experts by means of a set of linguistic control rules.

3) The decisionmaking logic has the capability of simulating human decisionmaking based on fuzzy concepts and of inferring fuzzy control actions employing fuzzy implication and the rules of inference in fuzzy logic.

4) The defuzzification interface performs a scale mapping which converts the range of values of output variables into corresponding universes of discourse, and performs defuzzification which yields a nonfuzzy control action from an inferred fuzzy control action.

3.2 Derivation of fuzzy PID control law

The blockdiagram of the FLC suggested in this research is described by Fig. 3.2.

Although most popular fuzzy controller developed so far employ two inputs, such as error and rate of change of error("rate" for short) about a setpoint, an additional input named as accelerated rate of change of error("acc" for short) is used for FLC. Using these three inputs, the structure of the FLC is composed of two independent parallel fuzzy control rules and defuzzifier blocks.Then the

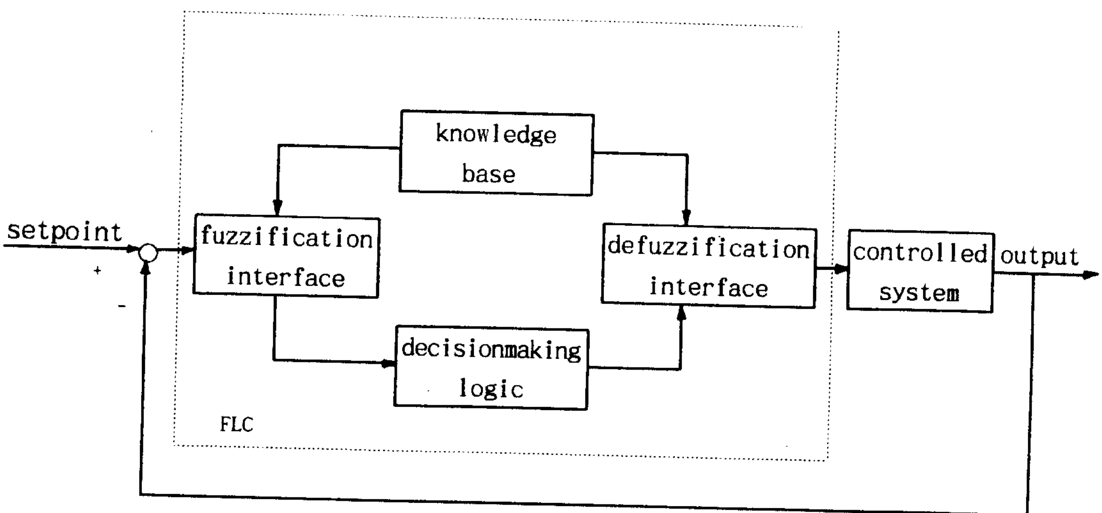

Fig. 3.1 General description of fuzzy logic controller

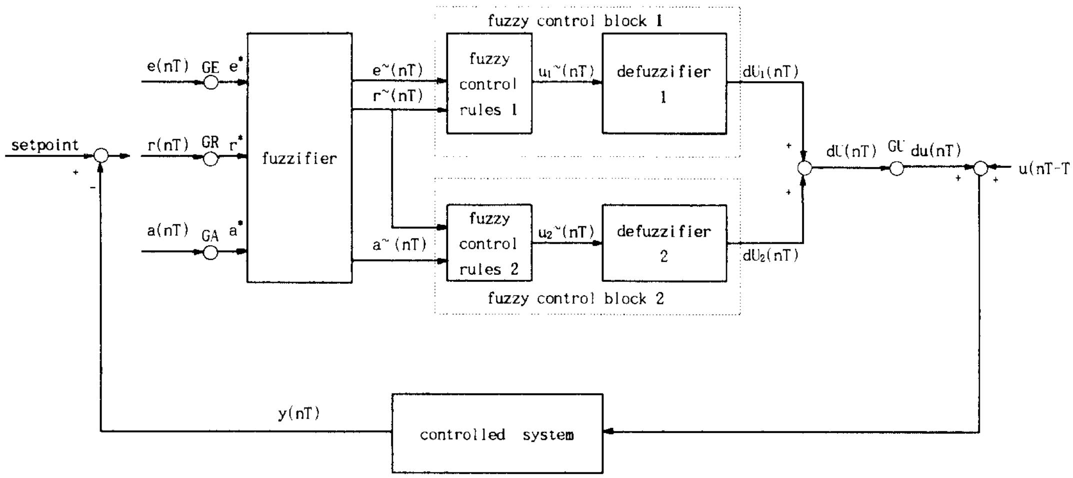

Fig. 3.2 The structure of FLC suggested in this paper

incremental output of the FLC is formed by algebraically adding the two outputs of each fuzzy control block.Here we employ the following notations :

$e(nT) = setpoint - y(nt)$

$e^{\sim}(nT) = F(e^*),\quad e^* = GE \times e(nT)$

$r(nT) = [e(nT) - e(nT-T)]/T$

$r^{\sim}(nT) = F(r^*),\quad r^* = GR \times r(nT)$

$a(nT) = [r(nT) - r(nT-T)]/T$

$\quad = [e(nT) - 2e(nT-T) + e(nT-2T)]/T^2$

$a^{\sim}(nT) = F(a^*), a^* = GA \times a(nT)$

$u(nT) = du(nT) + u(nT-T),$

$du(nT) = GU \times dU(nT)$

$dU(nT) = dU_1(nT) + dU_2(nT)$

where n is positive integer and T is sampling period.

The y(nT), e(nT), r(nT) and a(nT) denote process output, error, rate and acc at sampling time nT, respectively.

GE(gain for error) is the input scaler for error, GR(gain for rate) is the input scaler for rate, GA(gain for acc) is the input scaler for acc and GU(gain for controller output) is the output scaler of the FLC. F(.) means fuzzification of the scaled input signal(.). The du(nT) denotes the incremental output of the FLC at sampling time nT. The dUi(nT) (i=1,2) designates the incremental output of the fuzzy control block i from defuzzification of the fuzzy set "output i" at sampling time nT.

Thus the components of an FLC suggested in this research include :

1) input scalers GE, GR, GA and output scaler GU

2) a fuzzification algorithms for scaled error e*, scaled rate r*, scaled acc a* and outputs for control blocks

3) fuzzy control rules for each control block

4) fuzzy decisionmaking logics to evaluate the fuzzy control rules for each control block

5) a defuzzification algorithm to obtain crisp output of each control block for the control of process.

3.2.1 Fuzzification algorithm for scaled inputs

The fuzzification algorithm for scaled inputs is shown in Fig. 3.3.

The fuzzy set "error" has two members EP(error_positive) and EN(error_negative) ; the fuzzy set "rate" has two members RP (rate_positive) and RN(rate_negative) ; the fuzzy set "acc" also has two members AP(acc_-positive) and AN(acc_negative). The fuzzy set

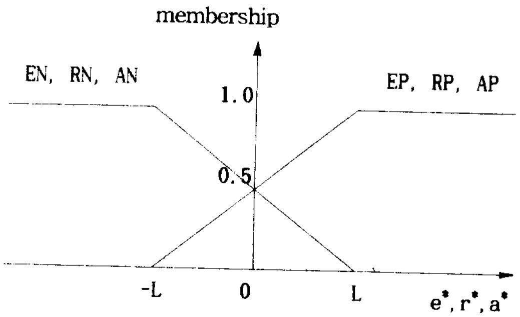

Fig. 3.3 Fuzzification algorithm for the inputs of FLC e*, r* and a*

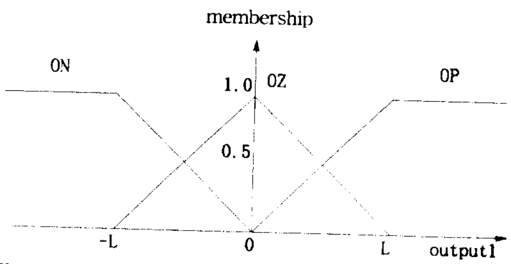

Fig. 3.4 Fuzzification algorithm for the incremental output of fuzzy control block 1 in FLC

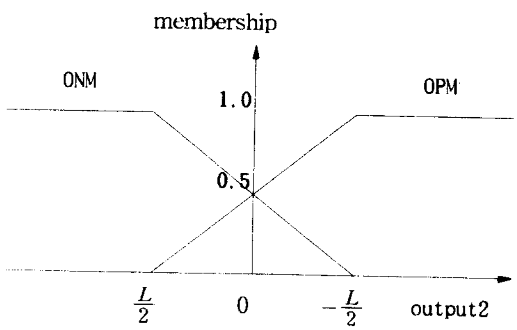

Fig. 3.5 Fuzzification algorithm for the incremental output of fuzzy control block 2

"output1" has three members OP(output_positive), OZ(output_zero) and ON(output_negative) shown as Fig. 3.4 for the fuzzification of incremental output of fuzzy control block 1.

The fuzzy set "output2" has two members OPM(output_positive_middle) and ONM(output_negative_middle) as shown in Fig. 3.5 for the fuzzification of incremental output of fuzzy control block 2.

Although the grades of membership function of the output members may be decided from the fuzzy control rules, the definitions of fuzzy set "output1" and "output2" are necessary for the fuzzification and fuzzy control rules. And it should be noted that the fuzzification algorithm for the output2 is different from that for the output1.

3.2.2 Fuzzy control rules and fuzzy logics for evaluation of the fuzzy control rules

Fuzzy control rules must be made based on expert experience and control engineering knowledge, or based on operator's control action. In this research, fuzzy control rules were made based on expert experience and control engineering knowledge, and each control rule set was composed of four linear fuzzy control rules for each fuzzy control block.For fuzzy control block 1, four linear fuzzy control rules are given as ;

R1 : if error=EP and rate=RP then output=OP
R2 : if error=EP and rate=RN then output=OZ
R3 : if error=EN and rate=RP then output=OZ
R4 : if error=EN and rate=RN then output=ON

For fuzzy control block 2, four linear fuzzy control rules, different from that of fuzzy control block 1, are given as ;

R1′ : if rate=RP and acc=AP then output=OPM
R2′ : if rate=RP and acc=AN then output=ONM
R3′ : if rate=RN and acc=AP then output=OPM
R4′ : if rate=RN and acc=AN then output=ONM

We, then now, apply fuzzy control logic to evaluate each fuzzy control rules. The fuzzy logics with which we are concerned are those of Zadeh and of Lukasiewicz. In evaluating the control rules, it is proper to use the Zadeh AND logic to evaluate the individual control rules, but the Lukasiewicz OR to evaluate the implied OR between control rules R2 and R3 in control block 1.

If μ_A and μ_B represent the grades of membership of an object in fuzzy sets A and B, respectively, then these logics are defined as ;

Zadeh logic :
$AND(\mu_A, \mu_B)=\min(\mu_A, \mu_B)$
$OR(\mu_A, \mu_B)=\max(\mu_A, \mu_B)$
Lukasiewicz logic :
$AND(\mu_A, \mu_B)=\max(0, \mu_A+\mu_B-1)$
$OR(\mu_A, \mu_B)=\min(1, \mu_A+\mu_B)$

The control rules R1 - R4 and R1' - R4' all employ the Zadeh AND of two conditions in the antecedents, such as one on the scaled rate, and the other on the scaled error. Since the Zadeh AND is the minimum of two values, two different conditions arise for each rule in fuzzy control blocks, that is, one when the scaled error is less than the scaled rate and one when the scaled rate is less than the scaled error in control block 1. In the similar manner, two conditions also arises between scaled rate and scaled acc in control block 2.

The eight different combinations of scaled error and scaled rate constituting inputs to the control rules are shown graphically in Fig. 3.6 for the control block 1.

For the control block 2, the eight different combinations of scaled rate and scaled acc are shown in Fig. 3.7.

These combinations of inputs must be considered when the fuzzy control rules are evaluated. The results of evaluating the fuzzy control rules R1, R2, R3, and R4 when scaled error and rate are in [-L, L], are given in Table 3.1.In Table 3.1 μ_{EP} and μ_{EN} (μ_{RP} and μ_{RN}) mean the membership values of EP and EN(RP and RN) in the fuzzy set "error"("rate").

For example, when the values of scaled error e* and rate r* are given, let the membership values obtained by using the fuzzification algorithm shown in Fig. 3.3, be given as μ_{EP} and

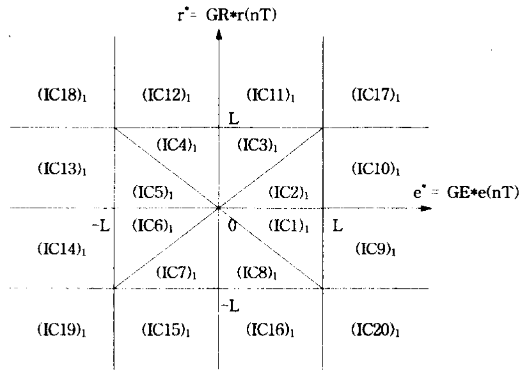

Fig. 3.6 Possible input combinations of e* and r* for control block 1.

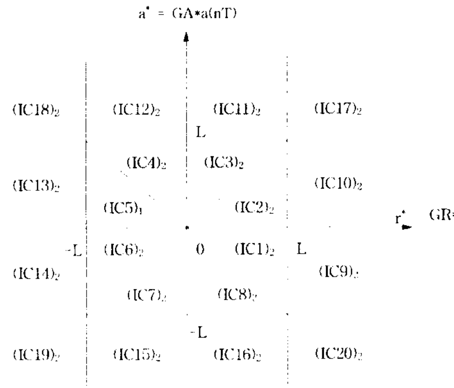

Fig. 3.7. Possible input combinations of r* and a* for control block 2.

Table 3.1 Results of evaluating the fuzzy control rules for all combinations of inputs using Zadeh AND logic when e* and r* are within the interval[-L, L]

Input combination of e* and r*	Membership obtained by evaluating fuzzy control rules			
	R1	R2	R3	R4
$(IC1)_1$	μ_{RP}	μ_{RN}	μ_{EN}	μ_{EN}
$(IC2)_1$	μ_{RP}	μ_{RN}	μ_{EN}	μ_{EN}
$(IC3)_1$	μ_{EP}	μ_{RN}	μ_{EN}	μ_{RN}
$(IC4)_1$	μ_{EP}	μ_{RN}	μ_{EN}	μ_{RN}
$(IC5)_1$	μ_{EP}	μ_{EP}	μ_{RP}	μ_{RN}
$(IC6)_1$	μ_{EP}	μ_{EP}	μ_{RP}	μ_{RN}
$(IC7)_1$	μ_{RP}	μ_{EP}	μ_{RP}	μ_{EN}
$(IC8)_1$	μ_{RP}	μ_{EP}	μ_{RP}	μ_{EN}

μ_{RP}. Then, say in rule R1, the membership value associated with the member, ON, of the fuzzy set "output1" is the Min(μ_{EP}, μ_{RP}). In this way, the membership values listed in Table 3.1. Notice that

$$\mu_{EP} = [r^* + L]/2L = [GE \times e(nT) + L]/2L \quad (3.1)$$
$$\mu_{EN} = [L - GE \times e(nT)]/2L \quad (3.2)$$
$$\mu_{RP} = [GR \times r(nT) + L]/2L \quad (3.3)$$
$$\mu_{RN} = [L - GR \times r(nT)]/2L \quad (3.4)$$
$$\mu_{EP} + \mu_{EN} = 1 \quad (3.5)$$
$$\mu_{RP} + \mu_{RN} = 1 \quad (3.6)$$

Also, be aware that in our case the Lukasiewicz OR reduces to the sum of the grades of membership being ORd, since this sum can never be greater than one for the fuzzy controller under study.

In the same manner, Table 3.2 shows the results of evaluating the fuzzy control rules for all combination of inputs when the scaled rate r^* and acc a^* are within the interval[−L, L], for the case of Fig. 3.7. Notice that

$$\mu_{RP} = [GR \times r(nT) + L]/2L \quad (3.7)$$
$$\mu_{RN} = [L - GR \times r(nT)]/2L \quad (3.8)$$
$$\mu_{AP} = [GA \times a(nT) + L]/2L \quad (3.9)$$
$$\mu_{AN} = [L - GA \times a(nT)]/2L \quad (3.10)$$

Table 3.2 Results of evaluating the fuzzy control rules for all combinations of inputs using Zadeh AND logic when r* and a* are within the interval[−L, L]

Input combination of e^* and r^*	Membership obtained by evaluating fuzzy control rules			
	R1′	R2′	R3′	R4′
(IC1)$_2$	μ_{AP}	μ_{AN}	μ_{RN}	μ_{RN}
(IC2)$_2$	μ_{AP}	μ_{AN}	μ_{RN}	μ_{RN}
(IC3)$_2$	μ_{RP}	μ_{AN}	μ_{RN}	μ_{AN}
(IC4)$_2$	μ_{RP}	μ_{AN}	μ_{RN}	μ_{AN}
(IC5)$_2$	μ_{RP}	μ_{RP}	μ_{AP}	μ_{AN}
(IC6)$_2$	μ_{RP}	μ_{RP}	μ_{AP}	μ_{AN}
(IC7)$_2$	μ_{AP}	μ_{RP}	μ_{AP}	μ_{RN}
(IC8)$_2$	μ_{AP}	μ_{RP}	μ_{AP}	μ_{RN}

$$\mu_{RP} + \mu_{RN} = 1 \quad (3.11)$$
$$\mu_{AP} + \mu_{AN} = 1 \quad (3.12)$$

3.2.3 Defuzzification algorithm

In this research, the center of area method is used as the defuzzification algorithm, which amounts to a normalization of the grades of membership of the members of the fuzzy set being defuzzified to a sum of one.The defuzzified output of a fuzzy set is defined as

$$dU = \frac{\Sigma(\text{membership of member}) \times (\text{value of member})}{\Sigma(\text{memberships})} \quad (3.13)$$

The value, used in the defuzzification algorithm, for example the defuzzification algorithm for the control block 1, is the value for the members of the fuzzy set "output1" which are chosen as those values for which the grade of membership in [−L, L] is unity, that is, those values for which the grade of membership is maximum. Therefore, these values are L for the fuzzy member OP, 0 for the fuzzy member OZ and −L for the fuzzy member ON as shown in Fig. 3.4.

The values used in the defuzzification algorithm for the fuzzy control block 2 are $\frac{L}{2}$ for the fuzzy member OPM and $-\frac{L}{2}$ for the fuzzy member ONM.

When the defuzzification algorithm given as eq.(3.13) is applied to Table 3.1, in case the Lukasiewicz OR is used for the membership of the member OZ of the fuzzy set "output1", the incremental output of the fuzzy control block 1 at sampling time nT, dU1(nT), can be described by the following two equations.

$$\text{If } GR \times |r(nT)| \le GE \times |e(nT)| \le L,$$
$$dU_1(nT) = \frac{0.5 \times L}{2L - GE \times |e(nT)|}[GE \times e(nT) + GR \times r(nT)] \quad (3.14)$$

$$If\ GE\times|e(nT)|\le GR\times|r(nT)|\le L,$$

$$dU_1(nT)=\frac{0.5\times L}{2L-GR\times|r(nT)|}[GE\times e(nT)+GR\times r(nT)] \quad (3.15)$$

These results can be observed with careful examination of Fig. 3.6 and Table 3.1.

If scaled error and/or scaled rate are not within the interval [−L, L] of the fuzzification algorithm shown in Fig. 3.3, the incremental output of the fuzzy control block 1 is as listed in Table 3.3 for the defuzzification algorithm eq.(3.13).

In a similar fashion, when the defuzzification algorithm is applied to Table 3.2, the incremental output of the fuzzy control block 2 at sampling time nT, $dU_2(nT)$, can be given by the following two equations.

$$If\ GA\times|a(nT)|\le GR\times|r(nT)|\le L,$$

$$dU_2(nT)=\frac{0.25\times L}{2L-GR\times|r(nT)|}[GA\times a(nT)] \quad (3.16)$$

$$If\ GR\times|r(nT)|\le GA\times|a(nT)|\le L,$$

$$dU_2(nT)=\frac{0.25\times L}{2L-GA\times|a(nT)|}[GA\times a(nT)] \quad (3.17)$$

If scaled rate and/or scaled acc are not within the interval [−L, L] of the fuzzification algorithm, the incremental output of the fuzzy control block 2 is as listed in Table 3.4.

Table 3.3 The incremental output of the fuzzy controller when e* and/or r* are not within the interval [−L, L] of the fuzzification algorithm

Input combination as shown in Fig. 3.6	Incremental output of the fuzzy control block 1, $dU_1(nT)$
$(IC9)_1$, $(IC10)_1$	[GR×r(nT)+L]/2
$(IC11)_1$, $(IC12)_1$	[GE×e(nT)+L]/2
$(IC13)_1$, $(IC14)_1$	[GR×r(nT)−L]/2
$(IC15)_1$, $(IC16)_1$	[GE×e(nT)−L]/2
$(IC17)_1$	L
$(IC18)_1$, $(IC20)_1$	0
$(IC19)_1$	−L

Table 3.4 The incremental output of the fuzzy controller when r* and/or a* are not within the interval[−L, L] of the fuzzification algorithm

Input combination as shown in Fig. 3.7	Incremental output of the fuzzy control block 2, $dU_2(nT)$
$(IC9)_2$, $(IC10)_2$, $(IC13)_2$, $(IC14)_2$	0.5×GA×a(nT)
$(IC11)_2$, $(IC12)_2$, $(IC17)_2$, $(IC18)_2$	0.5×L
$(IC15)_2$, $(IC16)_2$, $(IC19)_2$, $(IC20)_2$	0.5×L

Consequently, the overall incremental output of the FLC, dU(nT), can be obtained by adding incremental output $dU_1(nT)$ from fuzzy control block 1 and incremental output $dU_2(nT)$ out of fuzzy control block 2.

$$dU(nT)=dU_1(nT)+dU_2(nT) \quad (3.18)$$

Then the crisp value of incremental output, du(nT), can be obtained via multiplying dU(nT) by output scaler GU.

$$du(nT)=GU\times dU(nT) \quad (3.19)$$

Thus far, the process through which the incremental output could be obtained using FLC structure being suggested in Fig. 3.2, was discussed and developed.

Conclusively, the incremental ouput of FLC can be divided into four different forms according to the following conditions :

1) $If\ GR\times|r(nT)|\le GE\times|e(nT)|\le L$ and $GA\times|a(nT)|\le GR\times|r(nT)|\le L,$

$$du(nT)=\frac{0.5\times L\times GU}{2L-GE\times|e(nT)|}[GE\times e(nT)+GR\times r(nT)]+\frac{0.25\times L\times GU}{2L-GR\times|r(nT)|}[GA\times a(nT)] \quad (3.20)$$

2) $If\ GR\times|r(nT)|\le GE\times|e(nT)|\le L$ and $GR\times|r(nT)|\le GA\times|a(nT)\le L,$

$$du(nT)=\frac{0.5\times L\times GU}{2L-GE\times|e(nT)|}[GE\times e(nT)+GR\times r(nT)]$$

$$+\frac{0.25\times L\times GU}{2L-GA\times|a(nT)|}[GA\times a(nT)] \quad (3.21)$$

3) If $GE\times|e(nT)|\leq GR\times|r(nT)|\leq L$ and $GA\times|a(nT)|\leq GR\times|r(nT)\leq L$,

$$du(nT)=\frac{0.5\times L\times GU}{2L-GR\times|r(nT)|}[GE\times e(nT)+GR\times r(nT)]+\frac{0.25\times L\times GU}{2L-GR\times|r(nT)|}[GA\times a(nT)] \quad (3.22)$$

4) If $GE\times|e(nT)|\leq GR\times|r(nT)|\leq L$ and $GR\times|r(nT)|\leq GA\times|a(nT)\leq L$,

$$du(nT)=\frac{0.5\times L\times GU}{2L-GR\times|r(nT)|}[GE\times e(nT)+GR\times r(nT)]+\frac{0.25\times L\times GU}{2L-GA\times|a(nT)|}[GA\times a(nT)] \quad (3.20)$$

If scaled error, rate and/or acc are not within the interval[－L, L] the incremental output of the FLC is obtained from the combinations of incremental outputs for the fuzzy control blocks given as Table 3.3 and Table 3.4.

Here, if we carefully observe eq.(3.20), then we can find important fact described as below.

$$du(nT)=\frac{0.5\times L\times GU\times GE}{2L-GE\times|e(nT)|}\times e(nT)+\frac{0.5\times GU\times GR}{2L-GE\times|e(nT)|}\times r(nT)+\frac{0.25\times L\times GU\times GA}{2L-GR\times|r(nT)|}\times a(nT) \quad (3.24)$$

Let

$$K_i=\frac{0.5\times L\times GU\times GE}{2L-GE\times|e(nT)|}$$
$$K_p=\frac{0.5\times L\times GU\times GR}{2L-GE\times|e(nT)|}$$
$$K_d=\frac{0.25\times L\times GU\times GA}{2L-GR\times|r(nT)|} \quad (3.25)$$

Then we can obtain the following equation and can find that the fuzzy controller in this paper is a nonlinear PID controller with K_p, K_i and K_d changing with error, rate and acc.

$$du(nT)=K_i\times e(nT)+K_p\times r(nT)+K_d\times a(nT) \quad (3.26)$$

This nonlinear PID controller is naturally named as fuzzy PID controller, where K_p is defined proportional gain, K_i defined integral gain and Kd defined derivative gain.

In a similar fashion, K_p, K_i and K_d can be obtained for equations (3.21)~(3.23). We also define the static proportional gain K_{ps}, static integral gain K_{is} and static derivative gain K_{ds} when error, rate and acc are zero. Then they are defined as following from eq.(3.25) and they are always the same through all conditions.

$$K_{ps}=\frac{GU\times GR}{4}$$
$$K_{is}=\frac{GU\times GE}{4}$$
$$K_{ds}=\frac{GU\times GA}{8} \quad (3.27)$$

There are infinitely many combinations of GE, GR, GA and GU so that eq.(3.27) holds true. Once GE, GR and GA are selected, GU can be uniquely determined to satisfy eq.(3.27). Therefore, although most of all the conventional control strategies are not applied to control the process since the mathematical model for the process can not be defined or partially defined, the suggested fuzzy PID controller could be applied only if static proportional gain K_{ps} is selected using process input/output so that K_{ps} may satisfy rising time requirement in control specification. And also, linear PID controller can be, naturally, composed by using K_{ps}, K_{is} and K_{ds} obtained from pairs of GE, GR, GA and GU in eq.(3.27), in spite of the mathematical model for controlled process not being known.

4. Computer simulations

In order to assure the performance and the effectiveness of fuzzy PID controller, computer simulations were executed for the following examples.

(1) Plant transfer function $\frac{2}{s(s+1)}$

This example is an illustration of stable undamped system. It is used to test whether fuzzy PID controller improve transient response or not.

The results were given in Fig. 4.1. As shown in Fig. 4.1 fuzzy PID controller as well as LQG/LTR controller exhibits a good unit step response with nearly zero overshoot, more faster rising time and satisfactory settling time than those of nominal plant. In this viewpoint, the fuzzy PID controller is considered to be designed correctly.

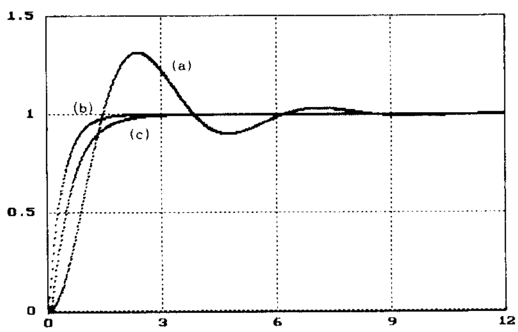

Fig. 4.1 Comparison of unit step responses
(a) nominal plant (b) LQG/LTR (c) fuzzy PID

(2) Nominal plant described by differential equation $\ddot{y}+\dot{y}=0.5y^2+2u$

This example is an illustration of nonlinear system which is diverged exponentially and slowly. This is used to assure that fuzzy PID is more effective than LQG/LTR.The results were shown in Fig. 4.2. As shown in Fig. 4.2, fuzzy PID controller exhibits good transient response

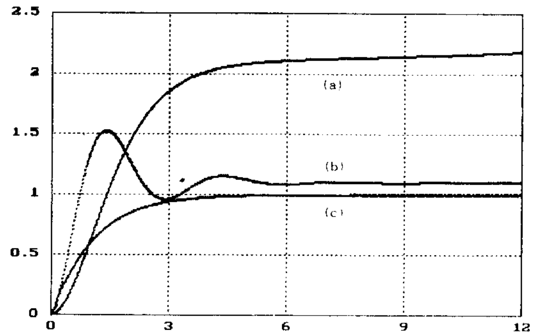

Fig. 4.2 Comparison of unit step responses
(a) nominal plant (b) LQG/LTR (c) fuzzy PID

and control action despite of divergent nominal controlled process. But LQG/LTR exhibits poor transient response and steady state response, because of modeling error caused by linearization, and ultimately does not track setpoint or reference input. In this respect, the fuzzy PID controller was turned out nonlinear controller and exhibited good performance without regard for controlled plant to be linear or nonlinear.

(3) Plant transfer function $\frac{e^{-0.2s}}{s(s+1)}$

This example is an illustration of time delay or nonminimum phase system.

In this case LQG/LTR controller may not accomplish loop transfer recovery effectively and may exhibit poor transient response. How the response of fuzzy PID is ?

The simulation results were given in Fig. 4.3.

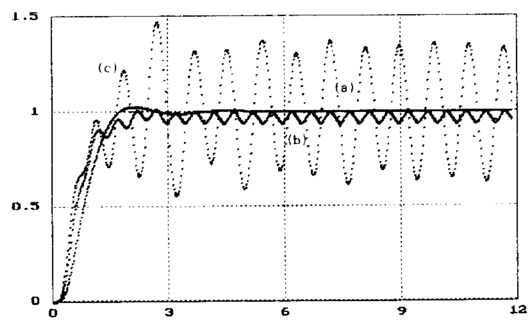

Fig. 4.3 Comparison of unit step responses
(a)nominal plant (b) LQG/LTR (c) fuzzy PID

As was expected, the fuzzy PID controller exhibits better performance than that of LQG /LTR controller. LQG/LTR controller exhibits so worse response that it can't be used a controller for this case. Thus it should be noted that the use of LQG/LTR controller for the control of nonminimum phase system requires much carefulness and observation.

By the way, in the design of fuzzy PID controller suggested in this research, it also was known that the combination of GE, GR, GA based on nominal plant input/output relation and GU based on proportional gain K_{ps} must be selected carefully, especially proportional gain K_{ps} used to decide GU, against the possibility of divergence. By the simulation experience, when controlled process is stable minimum phase system, the selection of K_{ps} may be allowed to be decided the value slightly greater than unity, then the performance may not nearly be different regardless of variant values of K_{ps}. But when controlled process in nonlinear and/or nonminimum phase system, K_{ps} must be selected carefully as the value smaller than unity, which must not generate exceeded control input as GU is varied and must be tuned step by step with small incremental values to obtain stable desired output.

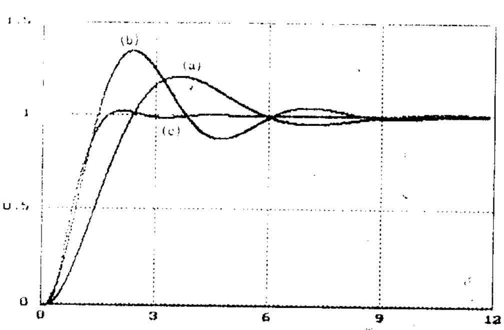

Fig. 4.4 Unit step responses of fuzzy PID according to K_{ps}
(a) Kps=GU=0.17 (b) Kps=GU=0.4
(c) Kps=GU=0.6 when L=5, GE=5, GR=4, and GA=2.5

Fig. 4.4 shows step responses according to the values of Kps and so GU.

5. Conclusion

In this research, a fuzzy PID control algorithm was derived in order to control systems with modeling errors and/or uncertainties. The fuzzy PID controller derived has the characteristics of nonlinear controller, so is named nonlinear PID controller, and is expecially powerful for the linear and nonlinear time invariant systems, but must be carefully designed for the nonmimum phase system. The most important advantage of fuzzy PID controller developed in this paper is that it is possible to design control system whose plant dynamics, so called mathematical model, is not known, by only using the input/output information. Also, linear PID controller can be designed, which is derived under the procedure of fuzzy PID controller , although plant dynamics is not known.The usefulness and effectiveness were assured through the computer simulation for several example systems, in spite of the simple fuzzification and defuzzification algorithms. Thus controller designers can expect effecient design of control system in real time with good transient performance, by only employing serial type microprocessors without computational burden.

Further research works will be concentrated on persuit of more fine control rules development and stability improvement about nonminimum phase system.

References

1) M. G. *Safonov, Stability and Robustness of Multivariable Feedback System*, MIT Press, 1980.
2) J. C. Doyle and G. Stein, "Robustness with Observers," IEEE Trans. in Automatic Control, Vol. AC - 24, pp. 607~611, 1979.

3) J. C. Doyle and G. Stein, "Multivariable Feedback Design : Concepts for a classical/Modern Synthesis," IEEE Trans. on Automatic control, Vol. AC - 26, pp. 4~16, 1981.

4) M. G. Safonov, A. J. Laub, and G. L. Hartmann, "Feedback Properties of Multivariable Systems : The Role and Use of the Return Difference Matrix," IEEE Trans. on Automatic Control, Vol. AC - 26, pp. 63~81, 1981.

5) U. Shaked, "Guaranteed Stability Margins for the Discrete - time Linear Quadratic Optimal Regulator," IEEE Trans. on Automatic Control, Vol. AC - 31, pp. 162~165, 1986.

6) G. Stein and M. Athans, "The LQG/LTR Procedure for Multivariable Feedback Control Design," IEEE Trans. on Automatic Control, Vol. AC - 32, pp. 105~114, 1987.

7) J. S. Kim,"Nonlinear Multivariable Control Using Statistical Linearization and Loop Transfer Recovery," ph.D Dissertation, MIT, 1987.

8) Z. Zhang and J. S. Freudenberg, "Loop Transfer Recovery for Nonminimum phase plants," IEEE Trans. on Automatic Control, Vol. AC - 35, pp. 547~553, 1990.

9) B. M. Chen and A. Saberi, and P. Sannuti, "Necessary and Sufficient Conditions for a Nonminimum Phase Plant to have a Recoverable Target Loop A Stable Compensator Design for LTR," Automatica, Vol. 28, pp. 493~507, 1992.

10) D. McFarlane, K. Glover, "A Loop Shaping Design Procedure Using H^∞ Synthesis," IEEE Trans. on Automatic Control, Vol. AC - 37, pp. 759~769, 1992.

11) M. Saberi, "H^∞/LTR Procedure with Specified Degree of Recovery," Automatica, Vol. 28, pp. 509~517, 1992.

12) E. H. Mamdani, "Application of Fuzzy Algorithm of Simple Dynamic Plant," Proc.IEE, Vol. 121, pp. 1585~1588, 1974.

13) T. J. Procyk and E. H. Mamdani, "A Linguistic Self - Organizing Process Controller," Automatica, Vol. 15, pp. 15~30, 1979.

14) T. Takaki and M. Sugeno, "Fuzzy Identification of Systems and Its Applications to Modeling and Control," IEEE. Trans on Systems, Man, and Cybernetics, Vol. SMC - 15, pp. 116~131, 1985.

15) M. Sugeno and G. T. Kang, "Structure Identification of Fuzzy Model," Fuzzy Sets and Systems, Vol. 28, pp. 15~33, 1988.

16) B. P. Graham and R. B. Newell, "Fuzzy Adaptive Control of a First - order Process," Fuzzy sets and Systems, Vol. 31, pp. 47~65, 1989.

17) W. Siler and H. Ying, "Fuzzy Control Theory : The Linear Case," Fuzy sets and Systems, Vol. 33, pp. 275~290, 1989.

18) C. C. Lee, "Fuzzy Logic in Control Systems : Fuzzy Logic Controller - Part I," IEEE Trans. on Systems, Man, and Cybernetics, Vol. SMC - 20, pp. 404~418, 1990.

19) M. M. Gupta and J. Qi, "Design of Fuzzy Logic Controller Based on Generalized T - Operators," Fuzzy Sets and Systems, Vol. 40, pp. 473~489, 1991.

20) K. Tanaka and M. Sugeno, "Stability Analysis and Design of Fuzzy Control Systems," Fuzzy Sets and Systems, Vol. 45, pp. 135~156, 1992.

21) G. C. Hwang and S. C. Lin, "A Stability Approach to Fuzzy Control Design for Nonlinear Systems," Fuzzy Sets and Systems, Vol. 48, pp. 279~287, 1992.

22) H. Ying, W. Siler, and J. J.Buckley, "Fuzzy Control Theory : A Nonlinear Case," Automatica, Vol. 26, pp. 513~520, 1990.

23) H. Ying, "Practical Design of Nonlinear Fuzzy Controller with Stability Analysis for Regulating Processes with Unknown Mathematical Models," Automatica, Vol. 30, pp. 1185~1195, 1994.