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Minimum Weight Design for Bridge Girder using Approximation based Optimization Method

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Abstract□Weight minimization for the steel bridge girders using an approximation based optimization technique is presented. To accomplish this, an optimization oriented finite element program is used to achieve continuous weight reduction until the optimum is reached. To reduce computational cost, approximation techniques are adopted during the optimization process. Constraint deletion as well as intermediate design variables and responses are also used for higher quality of approximations and for a better convergence rate. Both the reliability and the effectiveness of the underlying optimization method are reviewed.

Keywords□Approximation based optimization method, minimum weight design, finite element method, constraint deletion, intermediate design variables, integrated optimization package, efficiency, reliability

I. Introduction

In the community of structural analysis, the finite element method has emerged as the most commonly used approach. Structural optimization, on the other hand, has just started gaining popularity. The growing number of applications using various types of optimization methods for better and more efficient designs demonstrates the increasing confidence in optimization within the community. In general, an optimization problem is stated as : [4,11~13]

Find the set of design variables **X**, that will

$$\text{Minimize } F(\mathbf{X}) \quad (1)$$

Subject to

$$G_j(\mathbf{X}) \leq 0 \quad j=1, M \quad (2)$$

$$H_k(\mathbf{X})=0 \quad k=1, L \quad (3)$$

$$X_i^L \leq X_i \leq X_i^U \quad i=1, N \quad (4)$$

where $F(\mathbf{X})$ is the design objective, $G_j(\mathbf{X})$ and $H_k(\mathbf{X})$ are inequality and equality constraints respectively, and X_i^U, X_i^L are upper and lower bounds respectively for the design variable X_i.

Prior to 1974, finite element method based structural optimization was performed by a simple coupling between the finite element analysis and the optimization algorithm. This approach, however, was found to be impractical. This is because the optimization process usually requires the objective, constraints and their corresponding gradients with respect to design variables be evaluated many times, and each time a complete finite element analysis has to be performed.[12,13] Furthermore, the number of constraints is usually high in real applications, thus making the gradient calculations even more costly. As a result, the computational resources needed in achieving an optimal design for real applications is beyond people's acceptance.

To resolve this difficulty, the concept of creating an approximate problem in structural optimization was introduced by Schmit and Miura in mid 1970s.[16] Their proposed approach which is based on the Taylor series expansion provides a simple and yet powerful tool in structural optimization. Since then, the approximation approach has been continually studied and developed, and considerable progress has been made in recent years. High quality approximations maintain the essential features of the analysis problems and can be quickly evaluated during the optimization process. The objective of using approximation techniques is to create an optimal design with a computational cost comparable to finding only an acceptable design using traditional methods.

In order to make best use of the most advanced approximation techniques, it is desired to have a software which has both analysis and optimization capabilities from the beginning rather than adding optimization to an existing analysis program.[13] Based on this concept, fully incorporated structural optimization programs—in which the approximation method and finite element analysis are integrated—have been developed. One successful example of these developments is the GENESIS program, which was developed by the second and third authors.[3] It is fair to say that structural optimization technology has matured to become a practical design tool and can be efficiently applied to a wide range of design tasks.

Nevertheless, it is observed that structural optimization is still recognized as a relatively new field. This may be due to the fact that most designers/engineers are not familiar with the concepts involved in optimization. Therefore, it is desired to introduce structural optimization to various types of real world design tasks. In this paper, structural optimization is applied to the design of simple steel plate girders which are the major components in bridges and in many other structures. The convergence rate, reliability and effectiveness of the underlying structural optimization program are examined.

II. Finite Element Analysis[11]

The finite element method requires solving the following matrix equation ;

$$\mathbf{KU}=\mathbf{P} \tag{5}$$

where the stiffness matrix $\mathbf{K}$ is the summation of all the element stiffness matrices $\mathbf{k}_i$

$$\mathbf{K}=\sum_{i=1}^{NE}\mathbf{k}_i \tag{6}$$

and NE is the number of finite elements in the model. The element stiffness matrix $\mathbf{k}_i$ is derived from the stiffness matrix $\mathbf{k}_i^0$ which is formed in element's local coordinate system, followed by a geometric transformation using matrix $\mathbf{T}$:

$$\mathbf{k}_i=\mathbf{T}^T\mathbf{k}_i^0\mathbf{T} \tag{7}$$

The matrix $\mathbf{P}$ contains load vectors from all the loading cases, and the matrix $\mathbf{U}$ contains their corresponding nodal displacements. The stiffness matrix $\mathbf{K}$ is a function of design variables which represents some structural features. The load matrix **P** itself can also be a function of design variables, as in the case of structural gravity loads. The displacement matrix **U** can be solved using the following equation.

$$\mathbf{U}=\mathbf{K}^{-1}\mathbf{P} \tag{8}$$

In practice, however, we do not actually invert the stiffness matrix **K**, but decompose **K** and solve the equation using forward and backward substitutions. The element force is calculated from displacement results and the element stiffness matrix associated with that element, followed by the transformation to its local coordinate system :

$$\mathbf{u}^0=\mathbf{TU} \tag{9}$$

where **U** and $\mathbf{u}^0$ contain displacements of those nodes associated with the element of interest. Specifically, the element force is given as :

$$\mathbf{f}=\mathbf{k}_i^0\mathbf{u}^0 \tag{10}$$

From element force, the stress results and their corresponding constraints can be determined. Displacement constraints can be evaluated directly from the displacement results.

III. Design Problem Description

The bridge to be designed is modeled with 494 quadrilateral elements representing the concrete slab, and 130 beam elements representing the simple steel plate girders(Fig. 1). Each beam element is given an "I" section which is defined with six degree of freedom. The model is totally fixed at both ends. According to the *Korea Standard Code of Road Bridge*, two types of loads, namely the dead load and the live load, are involved in the design of road bridge. Structural weights of the concrete slab, balustrade and girders are considered as the dead load, while uniform loads exerted by people walking on two sides of the bridge, along with the standard truck load are the live load. The standard truck load involves two loading conditions, as shown in Fig. 1(a) and (b) respecti-

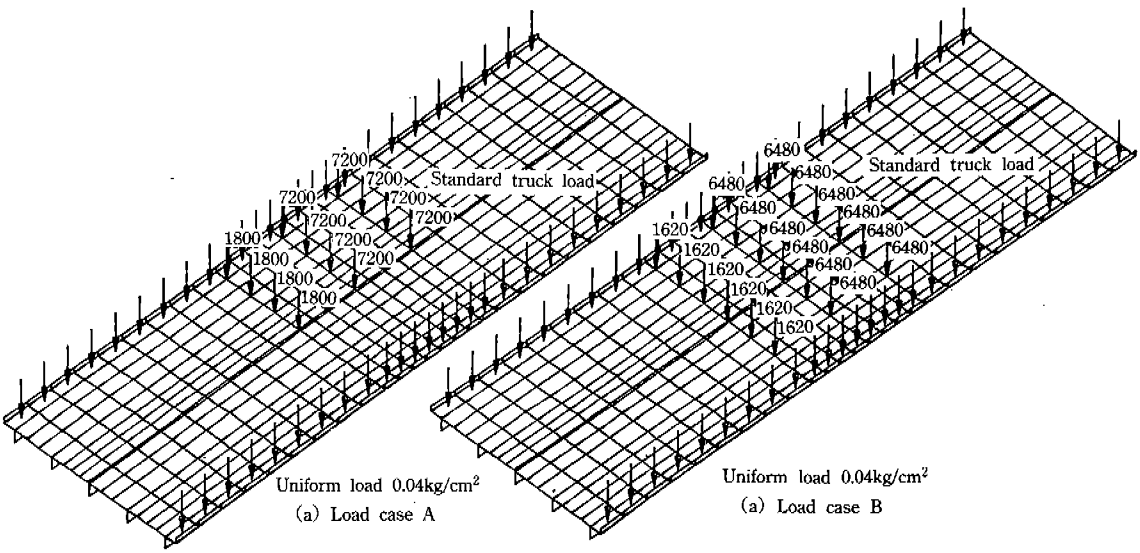

Fig. 1. Finite element model of bridge

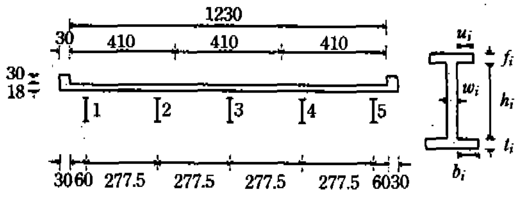

Fig. 2. The outline of cross section of bridge

vely. The total number of degree of freedom in the analysis is 3810.

The cross section of the bridge to be designed is shown in Fig. 2. The concrete slab and balustrade are supported by five simple steel plate girders. It is wished to minimize the girders' weight by changing their sectional dimensions, and at the same time satisfy all the requirements specified in the *Korea Standard Code of Road Bridge.*

1. Design Variables

As shown in Fig. 2, six design variables are considered for the steel plate girder : widths of upper and lower flanges(u_i, b_i), thicknesses of upper and lower flanges(f_i, t_i), height and thickness of the vertical web(h_i, w_i). Due to symmetry of the bridge, 18 design variables are employed. Since all girders should have the same height, 2 out of the 18 design variables are treated as dependent variables.

2. Design Objective

The objective function to be minimized is the total weight of steel plate girders. It is given in Eq.(11).

$$F(\mathbf{X})=\sum_{i=1}^{5}[2(b_i t_i+u_i f_i)+w_i(f_i+h_i+t_i)] \qquad (11)$$

3. Design Constraints

The compressive and tensile stresses in the upper and lower flanges, the shear stress at the center of each girder's web, and the deflections of the concrete slab are constrained according to the *Korea Standard Code of Road Bridge*. The minimum dimensions of flanges and webs allowed by the Code are also specified as side constraints or synthetic[15] constraints accordingly. The tensile and compressive stress constraint are given as :

$$G_1(\mathbf{X})=\frac{\sigma^L-\sigma_{ijk}}{|\sigma^L|}\leq 0 \qquad (12)$$

$$G_2(\mathbf{X})=\frac{\sigma_{ijk}-\sigma^u}{|\sigma^u|}\leq 0 \qquad (13)$$

where i is the element number, j is the stress component, k is the loading condition, and σ^L and σ^u are allowable compressive and tensile stresses respectively. The shear stress constraint is :

$$G_3(\mathbf{X})=\frac{\tau_{ij}-\tau^u}{|\tau^u|}\leq 0 \qquad (14)$$

where i is the element number, j is the loading condition, and τ^u is the shear stress limit. The displacement constraint is also imposed as follows :

$$G_4(\mathbf{X})=\frac{\delta_{ij}-\delta^u}{|\delta^u|}\leq 0 \qquad (15)$$

where i is the node number, j is the loading condition, and δ^u is the allowable slab deflection.

These structural responses are calculated from the finite element analysis. The constraints based on the minimum dimensions of the girders given in the *Korea Standard Code of Road Bridge* are :

$$G_5(\mathbf{X})=\frac{t_{min}-t_i}{|t_{min}|}\leq 0 \qquad (16)$$

$$G_6(\mathbf{X})=\frac{t_{min}-w_i}{|t_{min}|}\leq 0 \qquad (17)$$

$$G_7(\mathbf{X}) = \frac{t_{min} - f_i}{|t_{min}|} \leq 0 \quad (18)$$

$$G_8(\mathbf{X}) = \frac{u_i/16 - f_i}{|u_i/16|} \leq 0 \quad (19)$$

$$G_9(\mathbf{X}) = \frac{b_i/16 - t_i}{|b_i/16|} \leq 0 \quad (20)$$

$$G_{10}(\mathbf{X}) = \frac{h_i/152 - w_i}{|h_i/152|} \leq 0 \quad (21)$$

$$G_{11}(\mathbf{X}) = (f_{i+1} + h_{i+1} + t_{i+1}) + (f_i + h_i + t_i) = 0 \quad (22)$$

where t_{min} is the minimum thickness of the web and flanges. In total, there are 3308 design constraints. The problem is solved with GENESIS program on a SUN SPARC I workstation.

IV. Optimization Procedure

The optimization procedure using the proposed approximation method, as shown in Fig. 3, is to : [12,13]

1. Evaluate the initial design with a full finite element analysis.

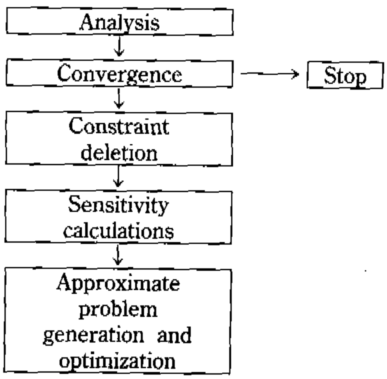

Fig. 3. Optimization procedure

2. Evaluate all constraint functions and rank them according to their critical levels. Retain only the critical and potentially critical constraints for further consideration during this cycle.

3. Call the gradient evaluation routines(sensitivity analysis) to calculate derivatives of the retained set of structural responses. They may be calculated as gradients of intermediate responses in terms of intermediate variables.

4. Using these gradients, construct an approximation problem which can be solved with a general purpose optimization code. The GENESIS program uses the DOT optimizer.[2] Note that the optimization could be linear or nonlinear, and can be modified in various ways as described in Refs. 7, 11~14. During this optimization process, move limits are imposed on the design variables to insure the reliability of the approximation.

5. Update the analysis data and call the analysis program to evaluate the quality of the proposed design. Terminate if the solution has converged to an acceptable optimum. Otherwise, repeat from step 2.

The overall computation process consists of an outer loop and an inner loop(Fig. 4). The outer loop includes finite element analysis, constraint deletions, gradient calculations, and creating an approximation problem. The inner loop includes solving the approximate optimization problem. One cycle of the outer loop is normally referred to as one design cycle. On the other hand, many iterations are needed in solving approximate optimization problem in the inner loop. Typically, less than ten design cycles are required, while 20 or more iterations are required to solve each approximate problem in the inner loop. For each design cycle, it is required to conduct a full finite element analysis and gradient calculations of the retained

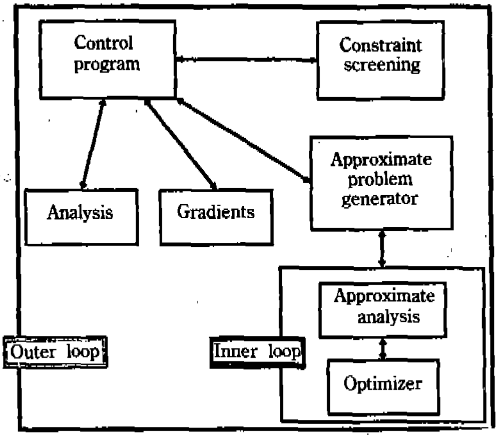

Fig. 4. Program organization

responses. The key therefore is to create a high quality approximation which can be rapidly evaluated in the inner loop, and thus reduce the number of design cycles(full finite element analysis) and the cost of computation.[12,13]

The constraint deletion, combined with the use of intermediate variables and responses, can create a high quality approximation with a reasonable cost. An example of this implementation is in the GENESIS program which has been developed from the beginning to be an optimization oriented program, rather than just adding optimization capability to the analysis module. This includes providing analytical gradient calculation in an efficient manner, usage of basis vectors in shape optimization, constraint deletions, synthetic functions[15] and formal approximations in an organized fashion. It is clear that finite element analysis combined with an advanced optimization technique such as approximation methods provides a powerful tool in structural optimization.

V. Results and Discussion

This design optimization problem was solved with four different starting designs. From the results, three issues were investigated.

1. Global vs. Local Optimum

It is difficult to prove that the solution from structural optimization program is the global optimum rather than a local optimum. This is due to the fact that most structural optimization problems are not convex problems in general. A common way to overcome this difficulty is to solve the optimization problem from different starting points, and choose the best one from them. If most solutions obtained from this approach converge to within a close range, it is quite possible that a global optimum is reached. Otherwise, the best solution can be used to at least improve from the current design. Four starting designs were used in this paper for this purpose. The initial design variable values for these cases are shown in Table 1.

2. Efficiency

The main reason to use approximation methods is to increase the efficiency by reducing the number of detailed finite element analyses needed to reach the optimum. This can be verified by looking at the convergence histories provided by the four cases of different starting designs(Fig. 5). It can

Table 1. Starting points

Cases	Initial value of design variables (cm)					
	b_i	t_i	u_i	f_i	h_i	w_i
A	34.00	2.00	29.00	2.00	160.00	2.00
B	27.65	1.70	24.15	1.70	140.00	1.70
C	21.30	1.40	19.30	1.40	120.00	1.40
D	19.95	1.10	14.45	1.10	100.00	1.10

be seen that even though the starting points are very different from each other, the objective function converges to within a close range. Table 2 summarizes the value of design variables when the optimum is reached in those four cases, as well as the maximum difference of the design variables and their corresponding percentage variation compared to the minimum one in four cases. Even though only 2.1% difference is found for the objective, 37.6% to 6.0% differences were observed for the design variables in the four cases of starting designs.

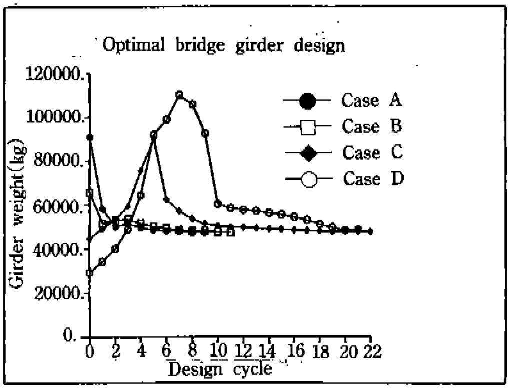

Fig. 5. Design objective convergence history

3. Effectiveness

In addition to the efficiency, the effectiveness of the approximation method was also studied. In cases A and B, the girders' weights were reduced and converge to the optimum in 9 and 11 design cycles respectively. In cases C and D, on the other hand, the constraint violations of 63%

Table 2. Efficiency and reliability of optimization procedure

Design variables	Dimen. (cm)	Opt. value of design variables (cm)				Maximum difference (cm)	Percent. of max. difference(%)
		Case A	Case B	Case C	Case D		
X(1)	b_1	10.030	10.030	12.867	13.521	3.491	34.8
X(2)	t_1	0.800	0.800	0.807	0.848	0.048	6.0
X(3)	u_1	10.000	10.000	12.829	13.480	3.480	34.8
X(4)	f_1	6.008	6.709	4.873	5.922	1.836	37.6
X(5)	h_1	159.940	153.250	157.080	141.330	18.610	13.1
X(6)	w_1	1.055	1.011	1.036	0.932	0.123	13.1
X(7)	b_2	10.030	10.032	12.003	11.596	1.971	19.6
X(8)	t_2	0.803	0.802	0.801	0.890	0.089	11.1
X(9)	u_2	10.000	10.002	11.967	11.561	1.967	19.6
X(10)	f_2	5.067	5.773	4.444	6.039	1.595	35.8
X(11)	h_2	160.880	154.190	157.510	141.170	19.710	13.9
X(12)	w_2	1.061	1.017	1.039	0.932	0.129	13.8
X(13)	b_3	12.282	11.403	13.975	14.643	3.240	28.4
X(14)	t_3	0.802	0.802	0.876	0.967	0.165	20.5
X(15)	u_3	12.245	11.093	13.933	14.600	3.507	31.6
X(16)	f_3	4.072	5.066	3.797	4.719	1.296	33.4
X(17)	h_3	161.870	154.890	158.090	142.410	19.46	13.6
X(18)	w_3	1.068	1.022	1.043	0.939	0.129	13.7
Obj. function (kg)		47439	47444	47684	48465	1026	2.1
No. of analysis		9	11	22	21		

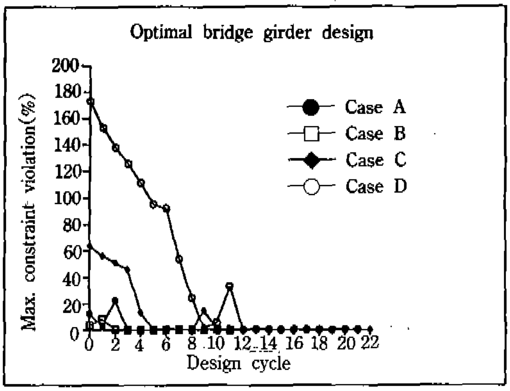

Fig. 6. Maximum constraint violation history

and 173% respectively had to be overcome before any improvement to be made(Fig. 6). From these results, it can be seen that approximation method is quite effective for the optimal design of this type of structures. It can be expected that at least some improvements can be made from the current designs which are usually calculated by hand or determined based on experience.

VI. Conclusions

Using bridge girder design as an example, the approximation based optimization method, coupled with the finite element analysis, was evaluated in this paper. Compared to the simple coupling of finite element analysis and optimization algorithms deveoped in 1960s, it was found that approximatioan based approach was efficient and reliable, and proved to be practical. Until recently, however, few real bridge structures has been designed using this technique. Therefore, it was intended in this paper to demonstrate that the approximation based optimization technique, combined with finite element analysis, is a powerful tool. Several integrated software—such as GENESIS—have been developed based on this philosophy and have been successfully used in real structural designs.

The bridge girder weight minimization design involves 18 design variables and 3308 design constraints. The approximation method, constraint deletion and intermediate design variables and responses were used in the optimization process. From the results it was shown that even though starting points are very different, the object function converges to an almost identical value.

References

1. Bloebaum, C. L., Variable Move Limit Strategy for Efficient Optimization, Proceedings of the AIAA/ASME/ASCE/AHS/ASC 32nd Structures, Structural Dynamics and Materials Conference, Washington DC, April 1991, pp. 431-437.
2. DOT User's Manual, VMA Engineering, Colorado Springs, CO, 1993.
3. GENESIS User's Manual, VMA Engineering, Colorado Springs, CO, 1993.
4. Haftka, R. T., Gurdal, Z. and Kamat, M. T., Elements of Structural Optimization, Kluwer Academic Publishers, 1990.
5. Kirsch, U., Improved First-Order Approximations for Structural Optimization, Proceedings of the 35th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Washington DC, April 1994, pp. 1717-1727.
6. Oda, J., Yamazaki, K., Sakamoto, J. and Junpei, A., Optimum Design of Plate and Shell Structures using the Variable Thickness Element, Proceedings of the 33rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Washington DC, April 1992, pp. 2545-2551.

7. Salajegheh, E. and Vanderplaats, G. N., An Efficient Approximation Method for Structural Synthesis with Reference to Space Structures, International Journal of Space Structures, Vol. 2, No. 3, 1986/87, pp. 165-175.
8. Sepulveda, A. E. and Schmit, L. A., An Approximation Based Global Optimization Strategy for Structural Synthesis, Proceedings of the AIAA/ASME/ASCE/AHS/ASC 32nd Structures, Structural Dynamics and Materials Conference, Washington DC, April 1991, pp. 303-315.
9. Shyy, Y. -K., Thomas, H. L. and Fleury, C., A Pseudo Line Search Algorithm for the Approximation Concepts Approach to Structural Optimization, Proceedings of the AIAA/ASME/ASCE/AHS/ASC 32nd Structures, Structural Dynamics and Materials Conference, Washington DC, April 1991, pp. 316-321.
10. Thomas, H. L., Shyy, Y. -K., Vanderplaats, G. N. and Miura, H., GENESIS-A Modern Shape and Sizing Structural Design Tool, Proceedings of the 33rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Washington DC, April 1992, pp. 2796-2804.
11. Vanderplaats, G. N., Numerical Optimization Techniques for Engineering Design : with Application, McGraw-Hill, 1984.
12. Vanderplaats, G. N., Efficient Structural Optimization Techniques, Proceedings of the Fourth International Conference on Computing in Civil and Building Engineering, July 29-31, 1991, Tokyo, Japan.
13. Vanderplaats, G. N., Thirty Years of Modern Structural Optimization, Advances in Engineering, Software, 16, 1993, pp. 81-88.
14. Vanderplaats, G. N. and Salajegheh, E., A New Approximation Method for Stress Constraints in Structural Synthesis, AIAA J., Vol. 27, No. 3, March 1989, pp. 352-358.
15. Vanderplaats, G. N., Miura, H., Cai, H. D. and Hansen, S. R., Structural Optimization using Synthetic Functions, Proceedings, 1989 AIAA /ASME/ASCE/AHS 30th Structures, Structural Dynamics and Materials Conference, AIAA, Washington DC, 1989.
16. Vanderplaats, G. N. and Han, S. H., Arch Shape Optimization using Force Approximation Method, Structural Optimization 2, 1990, pp. 193-201.
17. Vanderplaats, G. N. and Salajegheh, E., An Approximation Method for Structural Synthesis with Discrete Sizing and Shape Variables using Duality Theory, Proceedings of the 35th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Washington DC, April 1994, pp. 417-426.
18. Yoshida, N. and Vanderplaats, G. N., Structural Optimization using Beam Elements, AIAA J., Vol. 26, No. 4, April 1988, pp. 454-462.