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Derivation of Design Low Flows by Transformation Method

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Abstract□It is shown that two step power transformation is more efficient for the normalization of frequency distribution with the coefficient of skewness of zero in comparison with others including SMEMAX and power transformations. It is confirmed that the design low flows calculated using power and two step power transformations used in this study are generally nearer to the observed data as compared with those of SMEMAX transformation at all return periods in the applied watersheds of the Kum, Naktong and Yongsan rivers in Korea.

Keywords□Coefficient of skewness, SMEMAX transformation, power transformation, two step power transformation, design low flows.

I. Introduction

Hydrologic drought can be defined as the deficiency in water supply in comparison with water demand. Yevjevich[16,17] used the theory of runs as the basis for an objective definition of droughts. Drought occurs when there is a deficit in water including not only precipitation but also surface water runoff and storage from the hydrologic point of view. Estimation of design low flows is one of the requirements for the safe design of hydraulic structures for coping with droughts. Probabilistic approaches for the estimation of low flows of a return period of n year have been described by several hydrologists. Different probability distributions have been found to fit the low flows of different streams. Especially, extreme events including annual minimum series are rarely distributed normally, but usually asymetrically[10]. Data from extreme value series form their own distribution. Therefore, it is better to find the best fitting distribution for the given data instead of fitting a known distribution to the data. When theoretical or empirical distributions are found to be inappropriate, the given data could be reconstituted by some transformations such that transformed series follows a particular distribution. Bethalamy[1] and Chander[3] et al. have suggested using the power and SMEMAX transformations to normalize skewed data respectively. Gupta et al.[6] also proposed two step power transformation method for the normalization of skewed flood data. This study is mainly conducted to get low flows of a return period of n year by the normalization of frequency distribution through SMEMAX, power and two step power transformations and to compare with the results calculated by those methods using annual low flows of applied watersheds along Kum, Naktong and Yongsan rivers in Korea.

II. Transformation Method

1. Power and Two Step Power Transformation

Box and Cox[2)] have suggested the following transformation which is called as power transformation for normalizing the skewed flood series :

$$Y_i = \frac{X_i^{\lambda} - 1}{\lambda}, \text{ in which } \lambda \fallingdotseq 0 \qquad (1)$$

where X_i=ith variate of a given series, i. e. original skewed flows, Y_i=ith variate of power transformation series, and λ=parameter of power transformation such that Y_i have zero skewness. Transformation equation (1) hold for $X_i>0$.

The parameter, λ may be estimated by trial and error method such that the skewness of the transformed series is reduced to zero or nearly zero. The value of λ generally ranges from -1.0 to 1.0. The histogram of the transformed series which the skewness is reduced to zero or nearly zero becomes symmetrical about the vertical axis passing through the mean. It is generally seemed that the skewness of the power transformed annual maximum series becomes nearly zero[3)]. Power transformation in this study is required so that the given annual minimum series can be normalized with the coefficient of skewness of zero. For this requirement, a further transformation procedure of power transformation is needed to get normalization of skewed data. This double transformation is referred to as two step power transformation in this study. The coefficient of skewness of skewed data should be equalled to zero or become nearly to zero in this two step approach. The two limbs of the transformed distribution obtained by power transformation of the first step are equally stretched or reduced by the following transformation :

$$T_i = (|Y_i - \overline{Y}|)^{\gamma} \qquad (2)$$

where γ=parameter of TSPT, T_i=ith variate of two step power transformed series, $\overline{Y}$=mean of power transformed series.

Algorithm value of λ and γ, parameters of power and two step power transformations respectively should be obtained by iteration such that the skewness of the transformed series tend to be zero. When the normal distribution is obtained by two step power transformation, low flows according to the return period can be calculated by the following equation :

$$T_p = \overline{T} + K_p \cdot S_t \qquad (3)$$

where T_p=low flows according to pth return period in the two step power transformed series, K_p=frequency factor according to the pth return period, and S_t=standard deviation of the two step power transformed series.

X_p, the value corresponding to pth return period in a given series may be calculated by using the following equations :

$$X_p = (\lambda \cdot Y_p + 1)^{1/\lambda} \qquad (4)$$

where

$$Y_p = \overline{Y} + (T_p)^{1/\gamma} \qquad (5)$$

where X_p=low flows according to pth return period in a given series, Y_p=value corresponding to pth return period in the two step power transformed series. A flow chart and program with functions for two step power transformation are shown in Fig. 1 and Table 1 respectively.

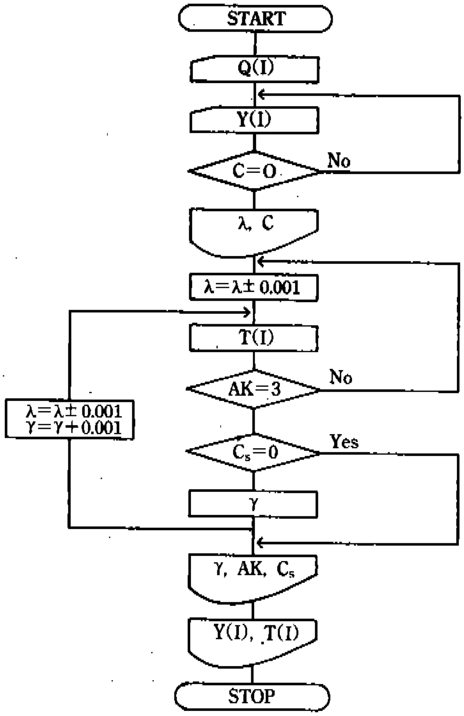

Fig. 1. Flow chart of two step power transformation

Table 1. Program and it's description of variables and parameters for two step power transformation

Item	Description
Q(I)	Low flows of given data
Y(I)	Variate of transformed series
λ	Parameter of power transformation
C	Coefficient of skewness for Y(I)
T(I)	Variate of two step power transformed series
γ	Parameter of two step power transformation
AK	Kurtosis for T(I)
C_s	Coefficient of skewness for T(I)

2. SMEMAX Transformation

SMEMAX transformation transforms a skewed annual minimum series for normalization using the smallest, median and largest value of the given series. SMEMAX is based on the median value, a relatively stable figure that is not affected by chance outliers. But, the resulting series can still have appreciable skewness, even though median value is equidistant from the smallest and the largest value.

Ⅲ. Data used for Application

Four watersheds were selected are Gyu Am, and Og Cheon of Kum river, Jeong Am and Ma Reuk of Naktong and Yongsan rivers respectively in Korea. Annual low flows of applied watersheds are used to estimate low flows of a return period of n year. Physical characteristics for the watersheds are shown in Table 2.

Ⅳ. Results and Discussion

1. Basic Statistics

Basic statistics of observed annual low folws are obtained from applied watersheds as shown in Table 3.

Those were within the range of 3.13 to 21.05, 1.78 to 8.04, 0.37 to 0.57 and 0.299 to 0.855 for the mean, standard deviation, coefficient of variation and coefficient of skewness respectively.

2. Efficiency of Transformation

Efficiency of transformation for the normalization of frequency distribution can be judged by checking whether the coefficient of skewness becomes nearly to zero. It can be seen from Table 4 that SMEMAX transformation reduces the skewness in comparison with those of original data but still cannot be considered as a normally distributed series and in power transformation the skewness could be brought to closer zero to a satisfactory degree than those of original and SMEMAX transformed data. Two step power transformation

Table 2. Gauging watersheds and their physical characteristics

River	Watershed	Area (km^2)	Length of main stream (km)	Average basin width (km)	Shape factor	Observed duration (yrs)	Location
Kum	Gyu Am	8273	339.2	24.48	0.07	28	Long. 126° 53′ Lati. 36° 16′
	Og Cheon	2943	169.3	17.38	0.10	16	Long. 127° 39′ Lati. 36° 16′
Naktong	Jeong Am	2990	166.5	17.96	0.11	18	Long. 128° 17′ Lati. 35° 19′
Yongsan	Ma Reuk	684	56.0	12.21	0.22	25	Long. 126° 49′ Lati. 35° 08′

Table 3. Basic statistics

River	Watershed	Observed years (N)	Mean ($\overline{X}$)	Standard deviation (S)	Coefficient of variation (Cv)	Coefficient of skewness (C_s)
Kum	Gyu Am	28	21.05	8.04	0.38	0.442
	Og Cheon	16	9.50	3.48	0.37	0.299
Naktong	Jeong Am	18	5.22	2.47	0.47	0.411
Yongsan	Ma Reuk	25	3.13	1.78	0.57	0.855

Table 4. Effect of different transformations on coefficient of skewness

River	Watershed	Observed duration (yrs)	Original data	SMEMAX transformed data	Power transformed data		TSPT transformed data		
			Skewness	Skewness	Lambda (λ)	Skewness	Lambda (λ)	Gramma (γ)	Skewness
Kum	Gyu Am	28	0.442	−0.216	0.407	−0.00059	0.852	0.696	−0.00045
	Og Cheon	16	0.299	−0.004	0.602	−0.00070	0.415	1.748	0.00007
Naktong	Jeong Am	18	0.411	0.22	0.216	−0.00052	0.350	0.245	0.00009
Yongsan	Ma Reuk	25	0.855	0.04	0.342	0.00069	0.460	0.236	0.00008

of original data series was also attempted for the skewness to be brought as close to zero in this study. The value of λ obtained in power transformation was used as initial trial value and thereafter it was subsequently modified with determination of γ iteratively. Consequently, the skewness of the two step power transformation could be brought to closer zero in comparison with those of SMEMAX and power transformed series.

3. Prediction of Low Flows by SMEMAX Transformation

1) Basic Statistics

Basic statistics calculated by SMEMAX transformation are within the range of 2.2 to 11.4, 1.33 to 5.94 and −0.216 to 0.22 for the mean, standard deviation and coefficient of skewness respectively as shown in Table 5.

2) Predicted Low Flows Calculated by SMEMAX Transformation

Low flows according to the return period were calculated by SMEMAX transformation for applied watersheds as shown in Table 6.

4. Prediction of Low Flows by Power Transformation

1) Basic Statistics

Basic statistics calculated by power transformation are within the range of 1.2 to 5.9, 0.699 to 1.438, −0.00070 to 0.00069 and 0.216 to 0.602 for the mean, standard deviation, coefficient of skewness and coefficient of transformation respectively as shown in Table 7.

Table 5. Basic statistics calculated by SMEMAX transformation

River	Watershed	Mean	Standard deviation	Coefficient of skewness
Kum	Gyu Am	11.4	5.94	−0.216
	Og Cheon	3.8	2.58	−0.004
Naktong	Jeong Am	2.2	1.82	0.22
Yongsan	Ma Reuk	2.6	1.33	0.04

The frequency factor, K for normality of the skewed distribution are given in Table 8 for the corresponding return period.

2) Predicted Low Flows Calculated by Power Transformation

Prediction of low flows, Y(T) for the return period, T can be made from :

$$Y(T)=(\lambda\cdot Z(T)+1)^{1/\lambda} \qquad (6)$$

in which $Z(T)=\bar{Z}-K\sigma_z$, $\bar{Z}$ and σ_z are the mean and the standard deviation of the transformed Z series respectively. Low flows of a return period of n year by power transformation were calculated as shown in Table 9.

Table 6. Predicted low flows calculated by SMEMAX transformation for the applied watersheds(m^3/sec)

River	Watershed	Return period(yrs)					
		5	10	20	50	100	200
Kum	Gyu Am	19.16	14.40	11.98	9.99	7.75	6.25
	Og Cheon	9.19	6.60	5.25	4.13	2.87	2.04
Naktong	Jeong Am	5.04	3.17	2.19	1.38	0.47	0.13
Yongsan	Ma Reuk	2.62	1.72	1.25	0.87	0.43	0.14

Table 7. Basic statistics calculated by power transformation

River	Watershed	Mean ($\bar{Z}$)	Standard deviation(σ_z)	Coefficient of skewness(C_s)	Coefficient of transformation(λ)
Kum	Gyu Am	5.9	1.336	−0.00059	0.407
	Og Cheon	4.7	1.438	−0.00070	0.602
Naktong	Jeong Am	1.9	0.699	−0.00052	0.216
Yongsan	Ma Reuk	1.2	0.850	0.00069	0.342

Table 8. Frequency factors according to the return period(yrs)

Return period	2	5	10	20	50	100
Frequency factor	0	0.8416	1.2816	1.6449	2.0538	2.3264

Table 9. Predicted low flows calculated by power transformation for the applied watersheds(m^3/sec)

River	Watershed	Return period(yrs)					
		5	10	20	50	100	200
Kum	Gyu Am	20.15	14.13	11.44	9.51	7.85	6.63
	Og Cheon	9.25	6.51	5.22	4.26	3.16	2.65
Naktong	Jeong Am	4.80	3.08	2.42	1.93	1.49	1.25
Yongsan	Ma Reuk	2.81	1.62	1.16	0.85	0.57	0.43

Table 10. Basic statistics calculated by two step power transformation

River	Watershed	Mean ($\overline{T}$)	Standard deviation(S_t)	Coefficient of transformation(λ)	Coefficient of transformation(γ)	Coefficient of skewness(C_s)
Kum	Gyu Am	4.82	1.807	0.852	0.696	−0.00045
	Og Cheon	56.29	9.321	0.415	1.748	0.00007
Naktong	Jeong Am	1.76	0.037	0.350	0.245	0.00009
Yongsan	Ma Reuk	1.72	0.039	0.460	0.236	0.00008

Table 11. Predicted low flows calculated by two step power transformation for the applied watersheds(m^3/sec)

River	Watershed	Return period(yrs)					
		5	10	20	50	100	200
Kum	Gyu Am	20.18	14.06	11.41	9.54	7.81	6.92
	Og Cheon	9.25	6.50	5.23	4.26	3.28	2.68
Naktong	Jeong Am	4.78	3.08	2.40	1.94	1.49	1.25
Yongsan	Ma Reuk	2.81	1.63	1.16	0.85	0.57	0.42

5. Prediction of Low Flows by Two Step Power Transformation

1) Basic Statistics

Basic statistics calculated by two step power transformation are within the range of 1.72 to 56.29, 0.037 to 9.321, 0.350 to 0.852, 0.236 to 1.748 and −0.00045 to 0.00009 for the mean, standard deviation, coefficient of transformation λ and γ, and coefficient of skewness respectively as shown in Table 10.

2) Predicted Low Flows Calculated by Two Step Power Transformation

The frequency factor, K_p according to the pth return period for normality of the skewed distribution are given in Table 8. Prediction of low flows, X_p according to the return period can be made from Equation (2) to Equation (4). Low flows of a return period of n year by two step power transformation were calculated as shown in Table 11.

6. Comparison of Design Low Flows Calculated by Different Transformation Methods

Design low flows according to the return period were calculated by SMEMAX, power and two step power transformation methods as shown in Table 12.

Comparing the relative suitabilities of the trans-

Table 12. Comparison of design low flows calculated by SMEMAX, Power and Two step power transformation methods (m^3/sec)

River	Watershed	Method	Return period(yrs)					
			5	10	20	50	100	200
Kum	Gyu Am	SMEMAX	19.16	14.40	11.98	9.99	7.75	6.25
		Power	20.15	14.13	11.44	9.51	7.85	6.63
		TSPT	20.18	14.06	11.41	9.54	7.81	6.92
	Og Cheon	SMEMAX	9.19	6.60	5.25	4.13	2.87	2.04
		Power	9.25	6.51	5.22	4.26	3.16	2.65
		TSPT	9.25	6.50	5.23	4.26	3.28	2.68
Naktong	Jeong Am	SMEMAX	5.04	3.17	2.19	1.38	0.47	0.13
		Power	4.80	3.08	2.42	1.93	1.49	1.25
		TSPT	4.78	3.08	2.40	1.94	1.49	1.25
Yongsan	Ma Reuk	SMEMAX	2.62	1.72	1.25	0.87	0.43	0.14
		Power	2.81	1.62	1.16	0.85	0.57	0.43
		TSPT	2.81	1.63	1.16	0.85	0.57	0.42

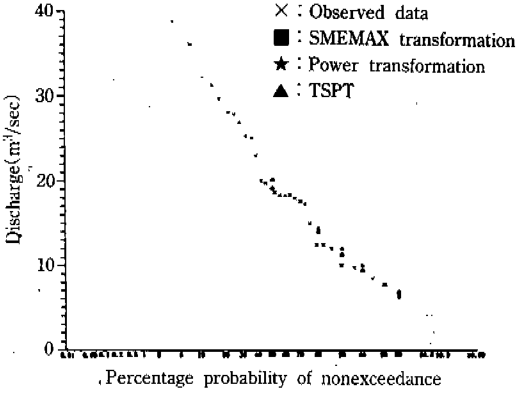

Fig. 2. Comparison of estimated low flows at Gyu Am watershed in the Kum river

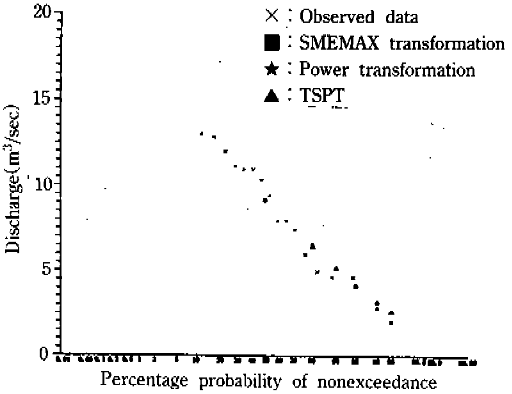

Fig. 3. Comparison of estimated low flows at Og Cheon watershed in the Kum river

formation methods including SMEMAX, power and two step power transformations, these results are plotted along with the observed data on a normal probability paper as shown in Fig. 2 to Fig. 5.

The plotting position is based on the weibull formula in which the probability of nonexceedance is calculated as $P=1-m/(n+1)$, in which n is the sample size and m is the rank commencing with the largest value. It can be seen from Fig. 2 to Fig. 5 that the design low flows computed by power and two step power transformations except SMEMAX transformation used in this study are generally much closer to the observed data at all return periods. It also has shown that computed design low flows based on two step power transformation are almost equal to those of power transformation.

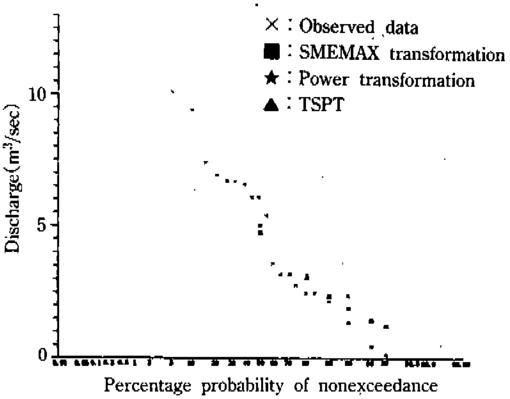

Fig. 4. **Comparison of estimated low flows at Jeong Am watershed in the Naktong river**

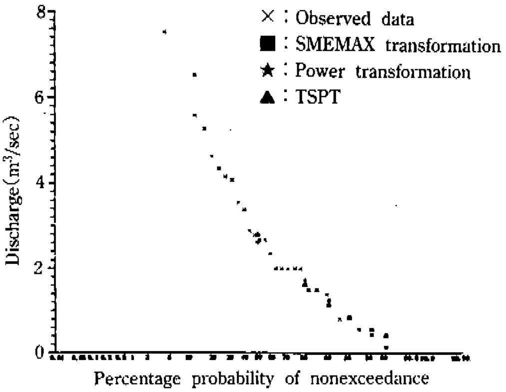

Fig. 5. **Comparison of estimated low flows at Ma Reuk watershed in the Yongsan river**

V. Conclusions

It has been concluded that two step power transformation seems to be more efficient for the normalization of frequency distribution than power and SMEMAX transformation in making the coefficient of skewness nearer to zero. Design low flows of different return periods estimated by two step power transformation were compared with the results computed by power and SMEMAX transformations with annual low flows of four watersheds along Kum, Naktong and Yongsan rivers in Korea. The results were analyzed and summarized as follows.

1. It is proved that two step power transformation is more efficient to make the coefficient of skewness into zero than those of SMEMAX and power transformations.
2. Design low flows according to the return period were derived by SMEMAX, power and two step power transformation methods for the applied watersheds.
3. Judging by the relative suitabilities of the three transformation methods, it was confirmed that the design low flows calculated using two step power and power transformation methods are nearer to the observed data as compared with those of SMEMAX transformation method at all return periods. It has also shown that design low flows based on two step power transformation are almost equal to those of power transformation.

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