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Comparisons of Multivariate Quality Control Charts by the Use of Various Correlation Structures

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Abstract

Several quality control schemes have been extensively compared using multivariate normal data sets simulated with various correlation structures. They include multiple univariate CUSUM charts, multivariate EWMA charts, multivariate CUSUM charts and Shewhart T^2 chart. This paper considers a new approach of the multivariate EWMA chart, in which the smoothing matrix has full elements instead of only diagonal elements. Performance of the schemes is measured by average run length(ARL), coefficient of variation of run length(CVRL) and rank in order of signaling of off-target shifts in the process mean vector. The schemes are also compared by noncentrality parameter. The multiple univariate CUSUM charts are generally affected by the correlation structure. The multivariate EWMA charts provide better ARL performance. Especially, the new EWMA chart shows remarkable results in small shifts.

1. Introduction

The evolution of modern technology is radically affecting data-acquisition equipment and on-line computers used in industrial process. Due to innovations in industrial quality control(QC), it is now common to monitor several correlated quality characteristic measurements rather than a single measurement. For QC processes using multiple cha-

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racteristics together, both multivariate and multiple univariate control charts can be utilized to detect changes in the mean level of the processes. Multivariate control chart methods are based on multivariate statistics which involve information on the interdependence between the separate measurements. When the statistic exceeds a given threshold, the chart gives an out-of-control signal for corrective action. Another approach for multiple measurements is to use multiple univariate control charts that operate separate univariate charts for each measurement being monitored. If any of the multiple charts indicates a change in the mean, corrective action is then required.

Multivariate control chart techniques for the mean level of a sequence of observations can be interpreted as repeated significance tests of the hypothesis for the unknown mean vector μ

$$H_0 : \mu=\mu_0 \qquad H_1 : \mu\neq\mu_0 \qquad (1)$$

where μ_0 is a target value vector of the mean level. Let $X_n=(X_{1,n}, X_{2,n},\cdots, X_{p,n})'$ denote the p-component vector of quality characteristic measurements where $X_{i,n}$ is the observation on the variable at time n. These successive observations are often sample mean vectors. A typical assumption is that the successive $\{X_n, n=1,2,\cdots\}$ are independent and identically distributed with a multivariate normal distribution whose covariance matrix Σ is known and may be estimated using data collected from in-control processes. For simplicity, it can be also assumed without loss of generality that $\mu_0=(0, 0,\cdots,)'=0$ and Σ is a normalized matrix with diagonal elements 1. Under these assumptions, it is well known that the null hypothesis of (1) should be rejected[8] if

$$T^2 = X'_n\Sigma^{-1}X_n > h = \chi^2(p, \alpha) \qquad (2)$$

where $\chi^2(p, \alpha)$ is the upper 100α percentage point of the χ^2 distribution with p degrees of freedom. This suggests a multivariate approach of monitoring several correlated measurements simultaneously to detect statistically significant shift in the mean level of sequence away from the target value $\mu_0=0$. This technique is usually called a "Shewhart type" T^2 chart in order to distinguish it from other methods, multivariate cumulative Sum(CUSUM) and Exponentially Weighted Moving Average(EWMA) charts, which also employ the Hotelling T^2 statistic. It is referred to simply as Shewhart in this study.

Two multivariate CUSUM charts were suggested by Crosier[2]. The first method is a CUSUM procedure of T that is the positive square root of quadratic form of (2) and a scalar representation of the multivariate observation. Using T has a more meaningful scale than using T^2. But a CUSUM T^2 is statistically more efficient. Another CUSUM scheme is based on the statistics

$$C_n = [(S_{n-1}+X_n)'\Sigma^{-1}(S_{n-1}+X_n)]^{1/2}$$

and

$$S_n = 0, \text{ if } C_n \le k$$
$$=(S_{n-1}+X_n)(1-k/C_n), \text{ if } C_n > k$$

for n=1, 2,⋯ where S_0 and $k > 0$. Given a reference value k and a threshold value h, the multivariate CUSUM signals when $Y = \{S'_n\Sigma^{-1}S_n\}^{1/2} > h$. In this paper, TCUSUM denotes the first CUSUM procedure and the second CUSUM scheme is referred to as T scale.

Lowry et al. [10] extended the univariate EWMA procedure to the multivariate process by defining EWMA vectors

$$Y_n = RX_n + (I-R)Y_{n-1} \qquad (3)$$

for n=1, 2,⋯ where $Y_0=0$ and smoothing matrix R is a diagonal matrix whose diagonal elements are $\{0 \le r_i \le 1,\ i=1, 2,\cdots, p\}$. Unless there is reason to differently weight the quality characteristic measurements related to the normalized covariance matrix Σ, all diagonal elements of the weight matrix can be set to the equal value, that is, $r_1=r_2=\cdots=r_p=r$. This EWMA scheme, denoted by DEWMA, gives an out-of-control signal for a given threshold h as soon as $T^2=Y_n'\Sigma_{Y_n}^{-1}Y_n > h$ where

$$\Sigma_{Y_n} = \{r[1-(1-r)^{2n}] / (2/r)\}\Sigma. \qquad (4)$$

In QC schemes for a multivariate normal process, it is most prevalent to use multiple univariate CUSUM procedures simultaneously to monitor the mean levels of variables that jointly measure the quality of the process. Woodall and Ncube [13] described how a p-component multivariate normal process can be monitored with p two-sided univariate CUSUM charts. The ith univariate CUSUM is operated for a given reference value k_i and threshold h_i by forming the cumulative sums

$$U_{i,n} = \max(0,\ U_{i,n-1}+X_n-k_i) \text{ and } L_{i,n} = \min(0,\ L_{i,n-1}+X_n+k_i)$$

where $0 \le U_{i,n} < h_i$, and $k_i \ge 0$. Under the assumption of the target vector and covariance matrix, the reference value and threshold are given the same values for every variable. The multiple univariate CUSUM chart signals an out-of-control condition when any of the p two-sided schemes produces an out-of-control signal, that is,

$$MCX = \max_{i} \left[\max_{t} (U_{i,t}, -L_{i,t}) \right] > h \text{ for a given } h. \qquad (5)$$

In this paper, QC schemes for multvariate normal processes have been extensively examined via simulation. This study also includes three other schemes as well as the five schemes previously described. One is a multivariate EWMA method which uses a nondiagonal smoothing matrix R and is referred to FEWMA. The others, called MCW and MCZ[6], are multiple univariate CUSUM chart schemes of (5), based on principal components and regression-adjusted components of the data. This study has concentrated on the quantitative analysis for the performances of the QC schemes in various correlation structures of multivariate measurements rather than the qualitative examination to the results. The schemes have been applied to multivariate data of 2, ·, and 10 variables using six different types of correlation structure and all the experiments have been quantitatively analyzed on the basis of 10,000 Monte Carlo simulation runs. In Section 2, the approaches to examine the multivariate QC chart schemes are discussed and the QC chart schemes mentioned in this section are compared with each other in Section 3, 4 and 5. Finally, Section 6 contains some conclusions.

2. Considerations in examining QC schemes for multivariate normal processes

In this paper, the QC schemes have been examined in three aspects: operating scheme, performance measure, measurement characteristics. The multivariate QC chart schemes can be categorized by their weighting functions over time, which are shown in Figure 1[9]. Shewhart chart give a whole weight to only each measurement observed at the present time. The weights are uniformly distributed over some period for the observations up to present time in the CUSUM chart, while the observations from the present time to the past are exponentially and decreasingly weighted in the EWMA chart. QC chart schemes can also be grouped according to the number of charts used in the operation. Multivariate QC schemes that use a scalar measure combining joint effects of the variables, monitor the process simply by a single chart, but multiple univariate QC chart schemes may have the advantage of interpretability in terms

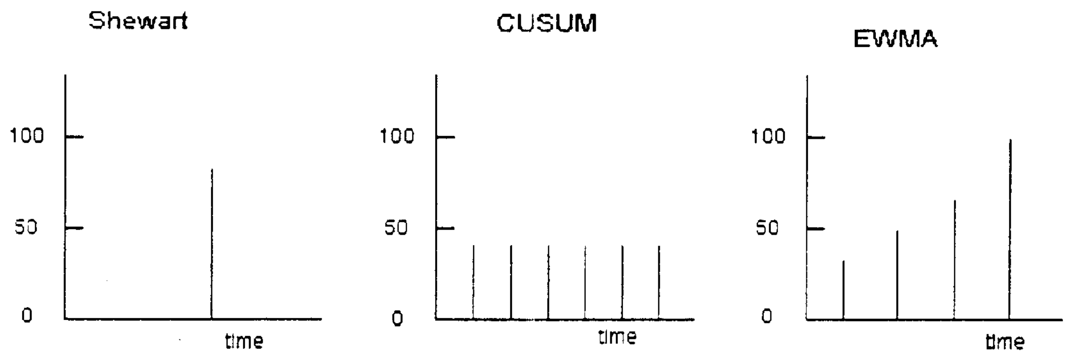

Figure 1. Data Weighting for the Shewhart, CUSUM and EWMA Charts

of individual variables. Crosier[2] compared TCUSUM and TScale with Shewart. In this paper, the QC chart schemes in the same category are first compared with each other: the multiple univariate CUSUM charts of MCX, MCZ and MCW in Section 3, the multivariate EWMA charts of DEWMA and FEWMA in Section 4. Next, the comparison between all eight QC chart schemes is accomplished in Section 5.

The performance of QC chart schemes is usually evaluated by the Average Run Length(ARL), which is the average number of successive observations without an out-of-control signal. Several authors described the Markov chain approaches or the integral equation approaches to estimate the run length properties of univariate control charts[1][3][13]. Unfortunately, the probability distribution of run length for most multivariate QC procedures is intractable. Monte Carlo simulation method is often used to compare the ARL performance of several QC schemes for monitoring multivariate normal processes[2][6][10]. Different multivariate QC schemes are designed to have the same robustness such that the in-control ARL of each scheme is the same for the data with on-target characteristics by simulation, and then the relative performance of various schemes can be evaluated by comparing their ARLs of out-of-control signals for the data with off-target characteristics. Another performance measure is the coefficient of variation(CV)[14] which is defined a function of ARL and standard deviation of run length(SDRL), that is, CV = SDRL/ARL. It expresses relative variation of run lengths as a measure of dispersion for the run lengths. This study also suggests "signaling-order" as a measure to compare the performance of different QC chart schemes. This statistic indicates the rank of the corresponding scheme in order of giving out-of-control signals when several QC charts are operated simultaneously for an identical process. It is considered as a relative measure for the run lengths of out-of-control signals, while the ARL is an absolute measure.

According to the environment of the process to be controlled, the observed measurements have different quality characteristics. The characteristics are usually determined by the mean vector and covariance matrix for the multivariate normal process. In QC using the zero target mean and normalized covariance matrix, they depend on the correlation structure of the process. Doganaksoy et al.[4] used three correlation structures for identification of out-of-control quality characteristics in a multivariate manufacturing environment. These structures correspond respectively with the cases that all variables are positively correlated with mixed sign.

The performance of QC chart schemes varies according to off-target amount of the mean shift in the process. The shift amount can be measured by the noncentrality parameter

$$\eta_c = (\mu' \Sigma^{-1} \mu)^{1/2} \qquad (6)$$

which is a directionally-invariant distance of off-target shift from the on-target mean level. This measure may not be appropriate for the QC chart schemes which are sensitive to the directions occurring a shift such as MCX.

However, it is not a significant problem in the scope of comparison in this study. This study experiments with QC chart schemes for various correlation structures of having only positive relation and joining positive and negative relation with three levels of relational strength and six levels of noncentrality.

3. Multiple Univariate CUSUM Charts

The technique most frequently used for detection of a change in the mean of a normally distributed variable is a CUSUM chart scheme that is a set of sequential procedures based on likelihood ratios. To detect a shift in the mean level for a multivariate normal process, a univariate CUSUM chart can use a linear combination of the variables, which has the standard normal distribution when $\mu_0=0$[7]. The multiple univariate CUSUM chart scheme of (5), MCX was suggested by extending the univariate CUSUM procedure to the multivariate normal process in Woodall and Ncube[13]. In the multiple chart scheme, p two-sided CUSUM schemes are operated simultaneously to detect a shift in the mean vector of p-variate normal distribution and the performance of the collection of individual schemes is evaluated. MCX is in practice often applied to correlated observations but usually without analyzing the resulting ARL performance.

Jackson[5] and Pignatiello and Runger[12] recommended monitoring the principal components with multiple univariate charts. The multivariate normal variables can be transformed to independent principal components by the spectral decomposition of the covariance matrix. For $X_n \sim N(0, \Sigma)$, the vector of principal components.

$$W_n = \Lambda^{1/2} C X_n$$

where C is the matrix of eigenvector and Λ the diagonal matrix of eigenvalues of Σ and each component has the standard normal distribution.

With the idea that the departures from target in the multivariate QC process may be expected to affect only a minority of the variables, Hawkins[6] proposed a measure which is the vector of scaled residuals from the regression of each variable on all others. Realization X_n is transformed to regression-adjusted vector Z_n by

$$Z_n = [\ \text{diagonal}(\Sigma^{-1})\ ]^{-1/2} \Sigma^{-1} X_n$$

The linearly transformed vectors provide the possibility of separate control of the individual variables in X_n. MCW and MCZ denote the multiple univariate CUSUM chart schemes applied to W_n and Z_n respectively. They are compared with MCX for the data which are simulated with various correlation structures.

The structures used in this study are categorized into two classes; positive and mixed types. The correlation between the variables in the variables in the positive type are all positive. In the mixed type, the ith and jth variables have negative relation if (i+j) is odd, otherwise they have positive relation. All experiments of this study assume that the absolute relational strength between the variables is uniform and have been examined for three level of the strength of 0.2, 0.5 and 0.8. P−2, P−5 and P−8 denote the positive types and M−2, M−5 and M−8 the mixed types of the absolute magnitudes of correlation of 0.2, 0.5 and 0.8. Each scheme has been examined for six different magnitudes of the mean shift according to the noncentrality parameters of 0.1, 0.2, 0.4, 0.8 1.6, 3.2 respectively, equally shifting the mean of each variable in the positive direction from the target mean, $\mu_0=0$ and using 2, 4, and 10 variables and given threshold values. Each threshold, h used in this study is set to the value which results in the ARL of 300 in 10,000 simulation runs with no means shift. In this section, the performances of the three schemes, MCX, MCW and MCZ have been evaluated by comparing their ARLs at a given reference value, k=0.5. Table 1 contains the results of applying the three chart scheme to the multivariate normal simulated data. Each result has been obtained from 10,000 runs. The ARLs of MCX and MCZ vary in the different correlation structures, while MCW is little affected by the structures. If two variables negatively correlated each other simultaneously change mean in the same direction, the process actually exhibits smaller effects than the original magnitudes of shifts in the variables by giving negative effects each other. It is true for the opposite case. Due to this fact, MCX quickly detects changes in the mean vector with the positive correlation, but it needs longer run lengths for the data of correlation structures involving negative correlation. In MCZ, the original variables are rescaled to unit variance by regressing a variable on all other variables. The rescaled variables correspond to the residuals resulting from eliminating the effects of all other variables by regression. If the effects of the positively correlated variables are eliminated, the magnitude of the transformed variable are expected to relatively be smaller and for the negative correlation, it will be magnified. In contrast with MCX, MCZ has a good ARL performance for the data of the mixed type, but the regression adjustment seems to be not appropriate for the data of positive correlation. Because the mean of each variable is shifted with an equal magnitude according to the noncentrality, the magnitude of shift becomes larger and ARLs are then decreased as the number of the variables is smaller. Although MCW robustly detects out-of-control situation without regard to the correlations structure, principal component analysis is often unattractive in the multivariate QC process, where interpretation is concerned rather than monitoring of the signal. It is difficult to interpret a physical meaning for the complicate linear transformation of the original variables. But, Hawkins[6] mentioned that "in some problems, the principal components will be more interpretable measurements—typically when the vector of measurements conforms at least approximately to the factor-analysis model." The comparison of the three schemes is more clearly shown in Figure 2 which depicts the curve of ARLs when the value of noncentrality parameter is 0. 4 and h values for the in-control ARL = 300. IND in Figure 2 denotes the independent structure with zero correlation. Based on the results, a multiple univariate chart scheme is suggested both to identify the variable of out-of-control and to give quick signals for shifts in the mean level. The chart scheme for the multivariate QC of p component with the largest eigenvalue to the original vector.

When it is hard to find a significant detect from the on-target value due to the negative correlation between the variables, the principal component may give an out-of-signal for the shifts mixed in the variables. It may not be adequate to examine MCX, MCZ with the data simulated in the basis of the noncentrality parameter which is directionally-variant and the noncentrality parameter which is directionally-variant and invariant chart schemes for the problems in the multivariate QC processes where there are simultaneous mean shifts in the variables. The analysis of MCX and MCZ for directionally-variant shifts was studied in Hawkins[6].

Table 1. ARLs of MCX, MCZ and MCW with simulated multivariate data of six different correlation structure types for 10,000 runs

(Number of Variables=2)

η	M−8			M−5			M−2			P−2			P−5			P−8		
	X	Z	W	X	Z	W	X	Z	W	X	Z	W	X	Z	W	X	Z	W
0.1	203	74	91	147	80	92	113	89	92	89	115	91	80	148	91	74	203	91
0.2	153	41	49	93	44	49	66	50	50	50	67	50	43	95	49	40	155	49
0.4	101	21	25	52	23	24	35	26	24	26	35	25	23	52	24	21	103	25
0.8	59	12	13	26	13	13	18	14	13	14	18	13	13	27	13	12	59	13
1.6	30	7	8	14	8	8	10	8	8	8	10	8	8	14	8	7	30	8
3.2	16	4	5	8	5	5	6	5	5	5	6	5	5	8	5	5	16	5

(Number of Variables=4)

η	M−8			M−5			M−2			P−2			P−5			P−8		
	X	Z	W	X	Z	W	X	Z	W	X	Z	W	X	Z	W	X	Z	W
0.1	217	92	110	164	99	110	130	108	113	96	146	101	81	196	102	74	255	102
0.2	171	51	61	110	56	62	79	63	63	54	92	54	45	144	54	40	221	54
0.4	120	26	32	64	29	32	43	33	33	29	51	27	24	90	27	21	173	27
0.8	73	14	16	33	16	16	22	18	17	16	27	14	13	49	14	12	118	14
1.6	39	8	9	17	9	10	13	10	10	9	14	8	8	25	8	7	70	8
3.2	20	5	6	10	6	6	8	6	6	6	9	5	5	14	5	5	36	5

(Number of Variables=10)

η	M−8			M−5			M−2			P−2			P−5			P−8		
	X	Z	W	X	Z	W	X	Z	W	X	Z	W	X	Z	W	X	Z	W
0.1	261	167	172	223	168	173	194	170	175	106	249	136	78	282	136	70	295	138
0.2	234	113	115	178	113	118	140	114	118	62	210	71	44	165	71	39	291	71
0.4	192	67	67	125	66	68	87	67	68	34	158	33	24	236	33	21	283	33
0.8	142	36	35	76	36	35	48	37	36	19	103	17	14	192	17	12	265	17
1.6	94	20	19	42	20	19	26	20	19	12	58	10	9	136	10	7	231	10
3.2	56	12	11	23	12	12	15	12	12	8	31	6	6	83	6	5	186	6

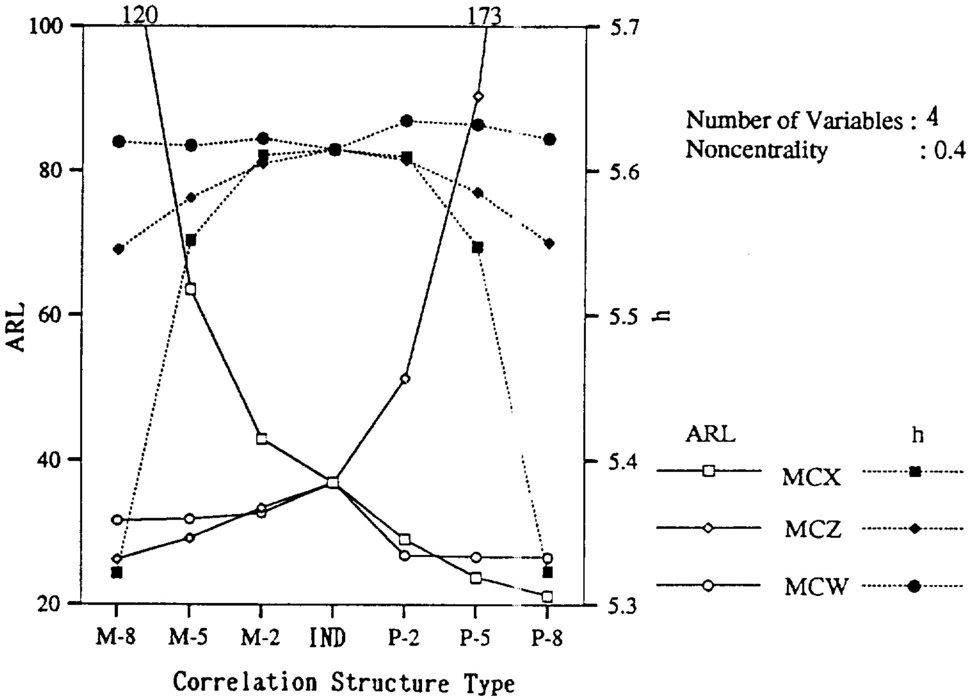

Figure 2. ARLs and values of h of maximum CUSUMs with simulated of seven covariance correlation structure types for 10,000 runs

4. Multivariate EWMA charts

Recently, there has been interested in using the EWMA chart scheme to detect shifts in the mean level of processes. Crowder[3] and Lucas and Saccucci[11] have thoroughly investigated properties of the EWMA chart and have suggested design strategies. Lowry et al. [10] proposed the multivariate EWMA chart scheme of (4) using a smoothing matrix of diagonal form. This approach for the design of multivariate EWMA charts is a straightforward generalization of the strategy for the univariate chart. A natural extension is to use a smoothing matrix having full elements in (3) if there exist interactions between the variables in the multivariate process. This study examines the multivariate EWMA charts using smoothing matrices of general form, in which the smoothing components associated with each variable are equivalent under the assumption that the scale of each variable is uniform. If r_{ij}

is the (i, j)th element of R in (3), the smoothing matrix for the p variate process is formed in this study so that $r_{ii}=r_{on}$ for $i=1, 2,\cdots, p$ and $r_{ij}=r_{off}$ off i, j=1, 2,$\cdots$, p and $i\neq j$. It is not appropriate to use off-diagonal elements greater than the on-diagonal element in the same row of the smoothing matrix and there is no special reason for the smoothing terms to have a negative sign. To prevent the covariance matrix of the EWMA vector of (3) from being ill-conditioned, the row sums of R are constantly fixed with the smoothing weight r so that

$$\sum_{j=1}^{p} r_{ij} \text{ and } \sum_{i\neq j} cr_{ii} \text{ for } \forall_i (\text{that is, } r_{off} = \frac{c}{p-1} r_{on})$$

where c is the ratio of the on-diagonal weight and the sum of the off-diagonal weights and $0 \leq c < 1$. Given r and c, then $r_{on}=c_{on}$ and $r_{off}=c_{off}$ where

$$c_{on} = \frac{1}{1+c} \text{ and } c_{off} = \frac{c}{(p-1)(1+c)}$$

For example, for $p=4$, $r=0.1$ and $c=0.5$,

$$R=\begin{bmatrix} 0.067 & 0.011 & 0.011 & 0.011 \\ 0.011 & 0.067 & 0.011 & 0.011 \\ 0.011 & 0.011 & 0.067 & 0.011 \\ 0.011 & 0.011 & 0.011 & 0.067 \end{bmatrix}$$

DEWMA corresponds to c=0. If the smoothing matrix has off-diagonal elements, the covariance matrix of the EWMA vector of (3) is more complicated and is recursively calculated in the EWMA scheme of (4) :

$$\sum_{Y_n} = R\sum R+(1-R)\sum(1-R) \text{ for } n=1, 2,\cdots \quad \text{where } \sum_{Y_0}=0$$

The ARL performances of DEWMA charts depend on the mean vector μ and covariance matrix Σ only through the value of the noncentrality parameter η_c of (6). It may not be true for the case of FEWMA. However, the simulation experiments have shown that the ARL performance of FEWMA is varied only according to the magnitude of noncentrality at least for the correlation structures and the smoothing matrices considered in this study. For several typical cases, the results of the experiments of 10,000 simulation runs are illustrated in Table 2 for ARLs and Table 3 for hs. The experiments in Section 4 also show the performance of FEWMA is little affected by the correlation

Table 2. ARLs of EWMA using diagonal and full smoothing matrices for simulated data of 4 variables with different correlation structure types from 10,000 simulation runs

Correlation structure type	η_c	0.2						0.8					
	r	0.1			0.5			0.1			0.5		
	c	0	25%	75%	0	25%	75%	0	25%	75%	0	25%	75%
M−5		47	42	33	136	132	115	13	12	10	37	35	29
IND		47	42	33	136	132	114	13	12	10	37	35	30
p−5		47	42	33	136	131	117	13	12	10	37	36	30

Table 3. Estimated values of h according to ARL=300 in 10,000 simulation runs of EWMA using diagonal and full smoothing matrices with 4 variables

Correlation structure type	r	0.1					
	c	0	25%	75%	0	25%	75%
M−5		3.73	3.59	3.35	3.95	3.89	3.70
IND		3.73	3.59	3.35	3.95	3.90	3.71
P−5		3.73	3.59	3.34	3.95	3.90	3.71

Table 4. Estimated values of h according to ARL=300 in 10,000 simulation runs of EWMA using diagonal and full smoothing matrices for multivariate data(Results were obtained by averageing h values for seven different correlation structure types)

No. Vars	r	0.1				0.2				0.5			
	c	0	25%	50%	75%	0	25%	50%	75%	0	25%	50%	75%
2		3.12	3.06	3.00	2.93	3.26	3.21	3.16	3.10	3.36	3.34	3.31	3.25
4		3.73	3.59	3.46	3.34	3.86	3.75	3.64	3.52	3.95	3.90	3.82	3.71
10		4.92	4.61	4.43	4.28	5.04	4.79	4.61	4.43	5.12	4.98	4.83	4.64

Table 5. ARLs of EWMA using diagonal and full smoothing matrices for simulated multivariate data with seven different correlation structure types(Results were obtained from 10,000 simulation runs for each correlation structure type respectively)

(Number of Variables=2)

η	r	0.1				0.2				0.5			
	c	0	25%	50%	75%	0	25%	50%	75%	0	25%	50%	75%
0.1		67	64	61	58	95	92	89	84	161	160	156	149
0.2		37	25	33	31	53	51	48	45	105	104	100	94
0.4		19	19	18	17	26	25	24	23	56	55	53	50
0.8		11	10	10	9	13	12	12	11	25	25	24	22
1.6		6	6	5	5	7	6	6	6	10	10	10	10
3.2		3	3	3	3	4	4	3	3	5	5	4	4

(Number of Variables=4)

η	r	0.1				0.2				0.5			
	c	0	25%	50%	75%	0	25%	50%	75%	0	25%	50%	75%
0.1		86	78	70	62	120	114	106	95	193	190	182	173
0.2		47	42	38	33	70	66	60	52	136	132	125	115
0.4		24	22	20	18	35	32	29	26	79	77	72	65
0.8		13	12	11	10	16	15	14	12	37	35	33	29
1.6		7	6	6	5	8	8	7	6	15	14	14	12
3.2		4	4	3	3	4	4	4	4	6	6	6	5

(Number of Variables=4)

η	r	0.1				0.2				0.5			
	c	0	25%	50%	75%	0	25%	50%	75%	0	25%	50%	75%
0.1		120	105	88	72	167	156	139	118	234	228	220	203
0.2		68	57	46	38	105	96	83	66	186	180	168	149
0.4		35	29	23	19	55	49	41	32	124	118	107	90
0.8		18	14	12	10	25	22	18	14	64	61	54	43
1.6		9	8	7	6	[illegible]	10	8	7	26	25	22	17
3.2		5	4	4	3	6	5	4	4	9	9	8	6

structure type. Table 4 and 5 contain the results, which were obtained from 10,000 simulation runs for seven correlation structures including IND respectively, of average h values for the in-control ARL=300 and ARLs for six different levels of the noncentrality for three levels of the smoothing weight. As shown in table 2 and 5, the EWMA chart schemes with smaller values of r are more effective in detecting small shifts in the mean, and their performance by using the off-diagonal smoothing weight is more evident in the cases of larger number of variables and the results of signal-ordering. Appendix demonstrates the effectiveness of FEWMA. There seems no case for using off-diagonal smoothing weight to hold a special meaning in QC and it may complicate physical interpretation on the EWMA vector to commingle the variables through the full smoothing matrix. FEWMA may not be consistent in the ARL performance for the same degree of noncentrality for some complicated correlation structures and is computationally inefficient compared to DEWMA. However, it have some practical advantages of improving performance in detecting a shift in the process level for specially subtle changes over large number of variables. The use of FEWMA in the multivariate QC processes can be grounded on the existence of interaction between the variables.

5. Comparative Performances of QC Chart Schemes for Multivariate Processes

The eight QC chart schemes described in the previous sections are comprehensively compared in their performances in detecting a mean shift in this section. All the chart schemes are designed to give out-of-control signals when the test statistics have greater values than the h values corresponding to in-control ARL of 300. TCUSUM uses the reference value equivalent to the value of square root of number of variables and the reference values of TScale, MCX, MCW and MCZ are set to 0.5. Both DEWMA and FEWMA are designed with r = 0.1 and FEWMA with c=0.5. Lowry et al.(1992) discussed optimal values of r for the DEWMA chart and suggested using r=0.1 to detect small shifts. All the results are obtained from 10,000 simulation runs respectively for simultaneous mean shifts in the variables according to the noncentrality parameters.

Figure 3 illustrates the comparison of ARLs and CVs between all eight schemes in six different correlation structure types using the data of 4 variables with the shift of η from the on-target mean vector. As described in Section 3, the performances of MCX and MCZ vary depending on the correlation structure types. MCX has better performance, while MCZ is less effective in detecting shifts from the on-target as the variables are more positively correlated each other. If there exists negative correlation between the variables, MCX is not effective and MCZ has better performance. Though the multiple univariate chart schemes, MCX and MCZ are less effective than some multivariate chart schemes in consideration of the ARL performance, the usefulness of MCX and MCZ is to identify the variable resulting in signaling the out-of-control. The values of CV represent the variations of run lengths relative to the corresponding ARLs. The CV values of MCX and MCZ are proportional to the ARLs, and are similar to the values of MCW in the correlation structures which result in better performance of the two schemes. The CV values of the multivariate control chart schemes except TScale are distributed in a similar range. TScale has the best CV values, implying that the ARLs of the scheme are most stabilized. Figure 5 also shows the results of ARLs and CVs of six different QC schemes whose performances are independent on the correlation structure types, using 2,4 and 10 variables and fine levels of the noncentrality. These results show that Shewhart performs worst in the average run lengths as well as in the variation of run lengths among the eight QC chart schemes. Performances of both ARL and CV are more dis-

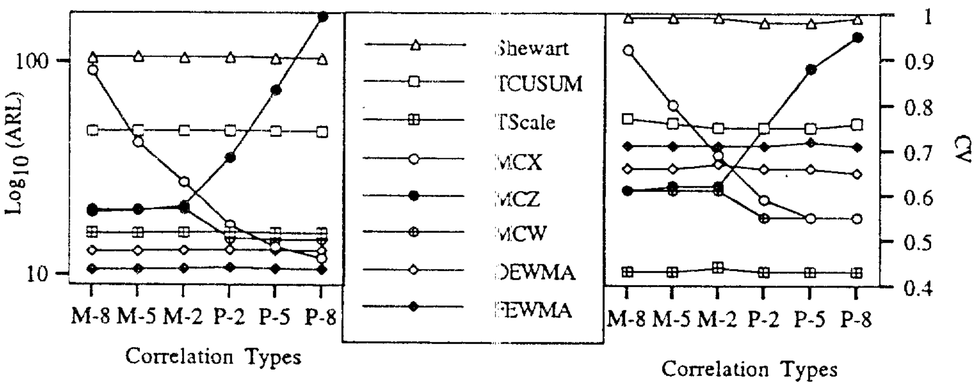

Figure 3. Results of ARLs and CVs of eight different MQC techniques for six different correlation structure types with 4 variables and η =0.8

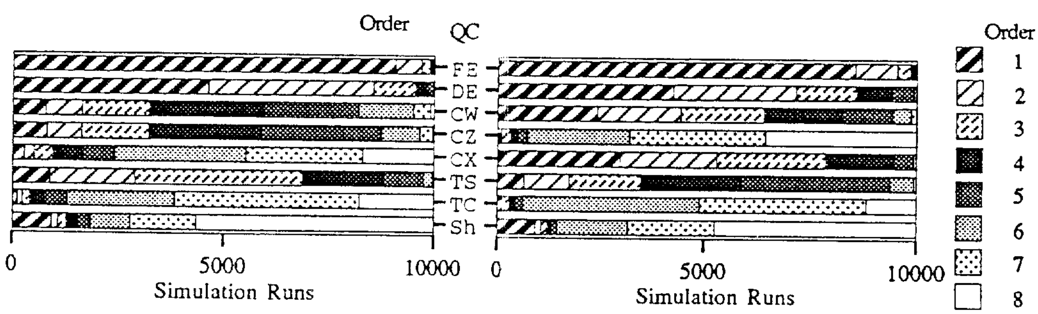

Figure 4. Results of signaling-orders of eight different MQC techniques for correlation structure of M−5 type and P−5 type with 4 variables and η =0.8

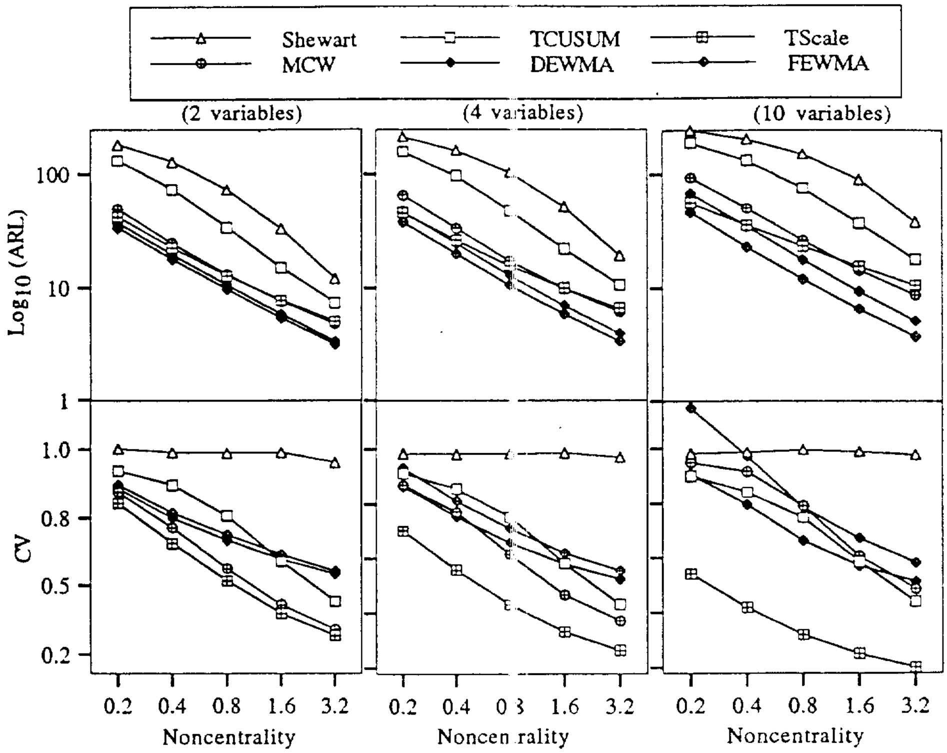

Figure 5. Results of ARLs and CVs of si: different MQC techniques with 2, 4 and 10 variables

tinctive in larger number of variables. The E.VMA charts are more sensitive to shifts from the on-target than the other schemes, while tl e ARLs of TScale have the least variation. The six schemes for the multivariate process gene ally have better ARL performance in order of FEWMA, DEWMA, TScale, MCW, TCUSUM, Shewart. Though MCW is a little more effective than TScale for larger mean shifts, it is insigr ificant and the CV performance of TScale is superior to that of MCW.

Figure 4 displays the results of signal-orderi: g of the chart schemes for two typical types of correlation structures and the noncetrality pa rameter of 0.8 using 4 variables. As shown in Figure 5, though the ARL performances of DEWMA and FEWMA are not much different, FEWMA gives much quicker signal than EWMA. It is clear that the EWMA chart schemes, especially FEWMA, are more effective in de ecting an initial out-of-control conditions. The detailed results of ARLs, CVs and signal-ordei ing ranks are contained in Table A-1 to A-6 of Appendix.

6. Conclusions

When comparing multivariate QC schemes, interpretation of the signal may be more important than performance of the multivariate scheme. In practice, a process control engineer would want to find an assignable cause for the signal and to adjust the process control variables that will bring the process back on-target. The multiple univariate charts are appropriate for this purpose. But, the performances of MCX and MCZ are dependent on the correlation structure of data and inferior to some fine multivariate QC chart schemes such as multivariate EWMA charts and TScale. MCW has a problem in interpreting out-of-control signals same as the multivariate control chart schemes. Signals in a multivariate QC scheme using a single chart may not have any meaningful interpretation of physical processes. But, it may not be possible to provide a corrective action on the single variable which results in signaling without affecting one or more of the other variables. Since the ability to partition or isolate the problems for the target solution may be limited, all the available information should be used to evaluate the process and identify an appropriate corrective action. Such information would include the relationships between the variables. MCX and MCZ are directionally variant and the other six chart schemes are directionally-invariant approaches. Unlike the directionally-variant chart, the directionally-invariant chart does not lose sensitivity in detecting multiple shifts of small amount in the process parameter level even if the shift in one direction is insignificant. A disadvantage of using a multivariate directionally-invariant chart is that it may not always be clear as to what cause the chart to signal an off-target condition. To overcome the disadvantages of both directionally-variant and invariant approaches, the combination of two approaches can be used by exploiting the merits of both schemes. Even though the relative performance of FEWMA is independent on the correlation structures in the experiments of this study, it is not mathematically true. There is no special ground on using the off-diagonal smoothing weight in the QC process. However, the performance of FEWMA is more effective in detecting simultaneous shifts in several variables from the on-target mean values, especially very superior to the other chart schemes if the process is initially out of control.

Appendix

Table A-1. Results of ARLs and coefficients of variation of eight multivariate QC techniuqes for six different types of correlation structure with 2 variables from 10,000 runs

QC		Shewhart		TCUSUM		TScale		MCOX		MCZ		MCW		DEWMA		FEWMA	
Type	η	ARL	CV	ARL	CV	ARL	CV	ARL	CV	ARL	CV	ARL	CV	ARL	CV	ARL	CV
M-8	0.0	300	0.98	301	0.97	304	0.97	302	0.98	301	0.96	306	0.99	301	1.01	304	1.04
	0.4	129	0.99	73	0.88	22	0.66	101	0.90	21	0.71	24	0.71	19	0.75	18	0.77
	0.8	72	0.98	34	0.75	13	0.52	59	0.88	12	0.58	13	0.56	11	0.67	10	0.69
	1.6	33	0.98	15	0.59	8	0.40	30	0.75	7	0.44	8	0.43	6	0.59	5	0.61
	3.2	12	0.94	7	0.45	5	0.32	16	0.52	5	0.35	5	0.34	3	0.54	3	0.55
M-5	0.0	299	0.99	300	0.96	305	0.96	301	0.96	302	0.98	305	0.97	302	1.01	304	1.03
	0.4	128	1.00	73	0.87	22	0.65	52	0.83	23	0.71	25	0.72	19	0.75	18	0.77
	0.8	73	0.98	34	0.76	13	0.52	26	0.71	13	0.57	13	0.57	11	0.67	10	0.69
	1.6	33	0.99	15	0.59	8	0.40	14	0.53	8	0.44	8	0.43	6	0.60	5	0.62
	3.2	12	0.96	7	0.44	5	0.32	8	0.38	5	0.34	5	0.34	3	0.54	3	0.55
M-2	0.0	300	0.99	302	0.96	303	0.96	303	0.98	302	0.98	302	0.95	303	1.01	301	1.03
	0.4	129	0.99	73	0.87	23	0.66	35	0.79	26	0.74	25	0.71	20	0.75	18	0.77
	0.8	73	0.98	34	0.76	13	0.52	18	0.63	14	0.58	13	0.56	11	0.66	10	0.68
	1.6	33	0.99	15	0.60	8	0.40	10	0.46	8	0.44	8	0.43	6	0.59	5	0.61
	3.2	12	0.95	7	0.44	5	0.32	6	0.35	5	0.35	5	0.34	3	0.54	3	0.56
P-2	0.0	300	0.98	298	0.96	304	0.96	302	0.97	308	0.99	304	0.96	302	1.01	303	1.04
	0.4	130	0.99	73	0.86	23	0.65	26	0.73	36	0.78	25	0.71	20	0.75	18	0.77
	0.8	73	0.99	34	0.75	13	0.52	14	0.58	19	0.63	13	0.56	11	0.66	10	0.68
	1.6	33	0.99	15	0.59	8	0.40	8	0.44	10	0.47	8	0.43	6	0.60	5	0.61
	3.2	12	0.95	7	0.44	5	0.32	5	0.34	7	0.35	5	0.34	3	0.54	3	0.56
P-5	0.0	300	0.98	302	0.97	303	0.97	304	0.98	307	0.98	300	0.99	301	1.00	300	1.04
	0.4	129	0.98	74	0.87	23	0.66	23	0.72	53	0.84	25	0.72	20	0.75	18	0.77
	0.8	72	1.00	34	0.76	13	0.52	13	0.57	27	0.71	13	0.56	11	0.67	10	0.69
	1.6	33	0.99	15	0.59	8	0.40	8	0.43	14	0.52	8	0.43	6	0.59	5	0.61
	3.2	12	0.95	7	0.44	5	0.32	5	0.34	8	0.38	5	0.34	3	0.55	3	0.56
P-8	0.0	299	0.99	297	0.98	303	0.96	304	0.98	302	0.98	297	0.98	302	1.01	298	1.03
	0.4	127	0.99	74	0.87	23	0.65	22	0.72	104	0.93	25	0.72	20	0.75	18	0.77
	0.8	73	1.00	34	0.76	13	0.52	12	0.57	60	0.87	13	0.57	11	0.67	10	0.69
	1.6	33	1.00	15	0.58	8	0.40	7	0.44	31	0.74	8	0.44	6	0.60	5	0.62
	3.2	12	0.96	7	0.45	5	0.32	5	0.34	16	0.52	5	0.34	3	0.54	3	0.55

Table A−2. Results of signaling-order of eight multivariate QC techniuqes for two different types of correlation structure with 2 variables from 10,000 runs

Rank		1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
η	QC	Type M-5								Type P-5							
0.8	Sh	1288	94	164	95	96	1106	1374	5783	1304	117	195	89	88	1106	1471	5630
	TC	241	114	187	181	152	2602	4745	1778	222	120	185	178	175	2593	4704	1823
	TS	1985	1497	2986	2507	872	137	16	0	2064	1452	2996	2484	857	135	11	1
	MX	405	182	385	336	402	4202	2752	1336	2483	1321	2862	1549	1506	248	28	3
	MZ	2520	1325	2761	1526	1589	249	30	0	369	191	368	310	394	4188	2813	1367
	MW	2293	1369	2834	1351	1044	881	198	30	2413	1375	2831	1321	1068	788	171	33
	DE	6784	2388	483	216	103	19	6	1	6714	2380	519	247	116	21	2	1
	FE	9076	630	171	76	33	12	2	0	9050	671	162	81	26	8	2	0
3.2	Sh	2580	170	784	205	107	1160	828	4166	2563	170	808	216	113	1166	821	4143
	TC	108	72	604	768	437	4208	3294	509	86	66	603	755	465	4157	3341	527
	TS	766	467	3869	3205	1310	366	16	1	769	471	3893	3134	1382	336	14	1
	MX	72	43	355	384	224	2521	3330	3071	1160	607	4550	2770	800	105	8	0
	MZ	1176	612	4537	2825	753	92	5	0	65	30	364	383	254	2560	3245	3099
	MW	1377	677	5081	2230	372	191	61	11	1371	679	5076	2224	410	162	59	19
	DE	8358	1568	64	9	1	0	0	0	8371	1553	70	5	0	1	0	0
	FE	9862	133	3	2	0	0	0	0	9859	132	8	1	0	0	0	0

Table A−3. Results of ARLs and coefficients of variation of eight multivariate QC techniuqes for six different types of correlation structure with 4 variables from 10,000 runs

QC		Shewhart		TCUSUM		TScale		MCX		MCZ		MCW		DEWMA		FEWMA	
Type	η	ARL	CV	ARL	CV	ARL	CV	ARL	CV	ARL	CV	ARL	CV	ARL	CV	ARL	CV
M-8	0.0	294	0.99	299	0.96	297	0.96	296	0.98	300	0.96	294	0.97	299	1.01	299	1.08
	0.4	163	0.99	97	0.87	26	0.55	141	0.95	36	0.77	38	0.77	24	0.75	20	0.80
	0.8	104	0.99	48	0.77	16	0.43	91	0.92	20	0.61	20	0.61	13	0.66	11	0.71
	1.6	52	0.98	22	0.58	10	0.33	52	0.84	11	0.45	11	0.45	7	0.58	6	0.62
	3.2	19	0.97	11	0.43	7	0.27	27	0.69	7	0.35	7	0.35	4	0.52	3	0.56
M-5	0.0	296	0.99	298	0.96	299	0.96	294	0.95	298	0.97	298	0.97	301	1.02	300	1.08
	0.4	164	0.99	96	0.86	26	0.56	78	0.88	37	0.77	39	0.77	25	0.76	20	0.82
	0.8	105	0.99	48	0.76	16	0.43	42	0.80	20	0.62	20	0.61	13	0.66	11	0.71
	1.6	52	0.98	22	0.58	10	0.34	22	0.61	12	0.46	11	0.45	7	0.58	6	0.62
	3.2	19	0.97	11	0.43	7	0.27	12	0.43	7	0.35	7	0.34	4	0.52	3	0.56
M-2	0.0	296	0.99	300	0.95	299	0.95	296	0.96	297	0.96	297	0.97	302	1.01	299	1.08
	0.4	165	0.99	97	0.86	26	0.56	52	0.82	39	0.78	39	0.77	25	0.76	20	0.83
	0.8	104	0.99	48	0.75	16	0.44	27	0.69	21	0.62	20	0.61	13	0.67	11	0.71
	1.6	52	0.99	22	0.58	10	0.34	15	0.51	12	0.46	11	0.45	7	0.59	6	0.62
	3.2	19	0.97	11	0.43	7	0.27	9	0.37	8	0.34	7	0.34	4	0.53	3	0.56
P-2	0.0	293	0.99	303	0.94	298	0.94	297	0.96	299	0.96	300	0.97	304	1.01	300	1.07
	0.4	163	0.99	97	0.85	26	0.56	31	0.73	68	0.86	28	0.70	25	0.75	20	0.81
	0.8	105	0.98	48	0.75	16	0.43	17	0.59	36	0.75	15	0.55	13	0.66	11	0.71
	1.6	52	0.99	22	0.59	10	0.33	10	0.43	19	0.57	8	0.41	7	0.58	6	0.62
	3.2	19	0.97	11	0.43	7	0.27	6	0.33	11	0.40	5	0.33	4	0.53	3	0.56
P-5	0.0	293	0.99	300	0.95	296	0.95	298	0.98	293	0.98	297	0.98	303	1.01	297	1.08
	0.4	164	0.97	97	0.85	26	0.56	24	0.69	124	0.92	28	0.70	25	0.75	20	0.81
	0.8	104	0.98	48	0.75	16	0.43	13	0.55	74	0.88	15	0.55	13	0.66	11	0.72
	1.6	53	0.99	22	0.58	10	0.33	8	0.41	39	0.76	8	0.41	7	0.58	6	0.62
	3.2	19	0.98	11	0.44	7	0.27	5	0.33	20	0.56	5	0.33	4	0.52	3	0.55
P-8	0.0	295	1.00	297	0.95	297	0.96	301	0.98	298	0.96	300	0.97	303	1.02	295	1.08
	0.4	166	0.99	97	0.87	26	0.56	21	0.70	214	0.98	28	0.71	25	0.77	20	0.82
	0.8	104	0.99	48	0.76	16	0.43	12	0.55	162	0.95	15	0.55	13	0.65	11	0.71
	1.6	51	0.99	22	0.58	10	0.33	7	0.42	106	0.91	8	0.41	7	0.58	6	0.62
	3.2	20	0.98	11	0.43	7	0.26	5	0.33	61	0.84	5	0.32	4	0.52	3	0.55

Table A-4. Results of signaling-order of eight multivariate QC techniuqes for two different types of correlation structure with 4 variables from 10,000 runs

Rank		1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
η	QC	Type M-5								Type P-5							
0.8	Sh	928	153	205	264	293	955	1554	5648	932	145	134	98	149	1708	2097	4737
	TC	117	117	180	347	530	2557	4377	1775	111	69	104	138	198	4276	3927	1177
	TS	888	2015	3973	1904	970	220	28	2	647	1087	1747	2395	3500	566	58	0
	MX	286	234	432	687	772	3116	2768	1705	2985	2319	2565	1598	464	63	6	0
	MZ	825	803	1599	2648	2852	937	292	44	99	91	122	193	221	2490	3250	3534
	MW	784	859	1592	2692	2261	1309	420	83	2425	2004	1991	1830	1176	442	112	20
	DE	4635	3904	999	317	118	25	2	0	4253	2923	1298	851	496	73	6	0
	FE	9070	616	195	76	30	12	1	0	8551	988	300	105	37	16	3	0
3.2	Sh	1712	266	935	262	305	924	990	4606	1766	268	410	208	433	1476	2066	3373
	TC	13	26	463	793	975	3844	3281	705	9	27	85	190	538	5814	2076	261
	TS	62	234	5570	2717	1079	301	36	1	54	166	619	1514	5899	1645	102	1
	MX	30	30	465	468	514	2107	3290	3096	1131	1489	4183	2790	398	9	0	0
	MZ	137	227	3205	2938	2413	838	230	12	5	3	32	101	120	1009	3191	5539
	MW	190	333	3802	3047	1600	746	232	50	1345	1673	4187	2333	414	45	3	0
	DE	6412	3453	131	2	1	1	0	0	6441	3222	276	57	4	0	0	0
	FE	9889	109	1	1	0	0	0	0	9857	140	3	0	0	0	0	0

Table A-5. Results of ARLs and coefficients of variation of eight multivariate QC techniuqes for six different types of correlation structure with 10 variables from 10,000 runs

QC		Shewhart		TCUSUM		TScale		MCX		MCZ		MCW		DEWMA		FEWMA	
Type	η	ARL	CV	ARL	CV	ARL	CV	ARL	CV	ARL	CV	ARL	CV	ARL	CV	ARL	CV
M-8	0.0	300	1.00	301	0.93	298	0.90	301	0.96	298	0.98	298	0.96	299	1.03	299	1.26
	0.4	207	0.99	133	0.85	36	0.42	193	0.94	67	0.84	67	0.83	35	0.80	23	0.96
	0.8	151	1.00	76	0.74	23	0.32	143	0.92	36	0.71	35	0.69	18	0.66	12	0.79
	1.6	89	0.99	37	0.59	16	0.25	94	0.89	20	0.52	19	0.50	9	0.56	7	0.68
	3.2	39	0.98	18	0.44	11	0.20	56	0.84	12	0.37	11	0.36	5	0.51	4	0.58
M-5	0.0	300	1.00	300	0.93	299	0.91	298	0.98	297	0.97	297	0.97	300	1.03	294	1.27
	0.4	208	0.98	133	0.84	36	0.42	125	0.90	68	0.84	68	0.82	35	0.79	23	0.95
	0.8	152	1.00	76	0.75	23	0.32	77	0.87	36	0.70	35	0.68	18	0.65	12	0.79
	1.6	90	0.99	37	0.58	16	0.25	42	0.76	20	0.52	19	0.50	9	0.57	7	0.67
	3.2	38	0.98	18	0.44	11	0.20	23	0.57	12	0.37	11	0.36	5	0.51	4	0.58
M-2	0.0	299	0.99	299	0.94	296	0.91	295	0.97	295	0.97	295	0.96	302	1.03	296	1.25
	0.4	207	0.98	134	0.84	36	0.41	88	0.86	69	0.84	69	0.85	35	0.80	23	0.98
	0.8	153	1.00	76	0.76	23	0.32	49	0.77	37	0.71	36	0.71	18	0.66	12	0.79
	1.6	90	0.99	37	0.58	16	0.25	26	0.60	20	0.52	20	0.51	9	0.57	7	0.67
	3.2	38	0.98	18	0.44	11	0.20	15	0.43	12	0.38	12	0.37	5	0.52	4	0.58
P-2	0.0	302	0.99	300	0.93	295	0.91	300	0.97	301	0.97	294	0.96	299	1.03	299	1.27
	0.4	208	1.00	134	0.83	36	0.42	34	0.72	158	0.93	33	0.69	35	0.79	23	0.98
	0.8	153	0.98	76	0.75	23	0.32	19	0.55	104	0.90	17	0.52	18	0.66	12	0.78
	1.6	90	0.99	37	0.58	15	0.25	12	0.41	58	0.80	10	0.39	9	0.58	7	0.67
	3.2	38	0.97	18	0.44	11	0.20	8	0.31	31	0.64	6	0.31	5	0.52	4	0.58
P-5	0.0	304	0.99	300	0.93	294	0.91	299	0.97	301	0.98	298	0.98	299	1.03	301	1.25
	0.4	208	0.99	132	0.85	35	0.42	24	0.66	239	0.96	33	0.68	35	0.81	23	0.99
	0.8	152	1.00	76	0.74	23	0.32	14	0.52	192	0.94	17	0.52	18	0.67	12	0.79
	1.6	88	0.98	37	0.59	15	0.25	9	0.39	137	0.92	10	0.39	9	0.57	7	0.67
	3.2	38	0.97	18	0.43	11	0.20	6	0.31	83	0.86	6	0.31	5	0.51	4	0.58
P-8	0.0	298	1.00	300	0.92	298	0.91	299	0.97	301	0.98	295	0.96	301	1.02	303	1.23
	0.4	209	0.99	133	0.85	35	0.42	21	0.67	284	0.97	33	0.67	34	0.81	23	0.99
	0.8	154	1.00	75	0.76	23	0.32	12	0.53	267	0.96	17	0.51	18	0.67	12	0.80
	1.6	89	0.99	37	0.59	15	0.25	7	0.40	232	0.95	10	0.39	9	0.58	7	0.67
	3.2	38	0.98	18	0.44	11	0.20	5	0.32	186	0.94	6	0.31	5	0.51	4	0.59

Table A-6. Results of signaling-order of eight multivariate QC techniuqes for two different types of correlation structure with 4 variables from 10,000 runs

Rank		1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
η	QC	Type M-5								Type P-5							
0.8	Sh	730	271	317	406	438	969	1710	5159	708	192	152	192	288	1934	2951	3583
	TC	69	111	254	584	963	2475	3808	1736	39	66	77	155	311	4673	3663	1016
	TS	295	1487	3992	2368	1332	441	73	12	29	167	634	2072	5699	1191	199	9
	MX	205	298	553	798	1008	2285	2591	2262	4092	3409	1964	452	72	9	2	0
	MZ	337	610	1357	2322	2794	1683	679	218	56	53	98	144	179	1577	2724	5169
	MW	370	701	1449	2581	2393	1591	722	193	1814	2263	2541	2371	778	193	38	2
	DE	2443	5299	1548	521	142	36	11	0	1828	2266	1823	2676	1143	231	32	1
	FE	9015	685	186	75	27	8	2	2	7210	1374	1044	261	94	15	2	0
3.2	Sh	1004	378	935	315	318	740	1246	5064	982	261	184	131	916	1447	3816	2263
	TC	1	12	347	871	1122	3478	3368	801	1	1	13	18	676	6319	2815	157
	TS	0	6	4449	3069	1709	597	156	14	0	0	0	18	7703	1955	320	4
	MX	10	32	582	597	657	1795	3250	3077	1800	3299	3530	1256	114	1	0	0
	MZ	25	106	2384	2619	2828	1384	562	92	1	3	4	18	97	471	2055	7351
	MW	30	151	3355	3002	1946	1057	361	98	1616	2707	2908	2424	343	2	0	0
	DE	3767	5992	238	3	0	0	0	0	3709	4197	981	1076	37	0	0	0
	FE	9896	103	1	0	0	0	0	0	9690	254	54	2	0	0	0	0

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