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Multi-objective Scheduling with Stochastic Processing Times

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Abstract

A multi-objective, single-stage scheduling problem with stochastic processing times is considered where the objective is to simultaneously minimize the expected value and the variance of total flowtime, and the mean probability of tardiness. In cases where processing times follow normal distributions, a method using pairwise interchange of two jobs(**PITJ**) is proposed to generate a set of the approximate efficient schedules. The efficient schedules are not dominated by the criterion vectors of any other permutation schedules in the feasible region. Numerical experiments performed to ascertain the effectiveness of **PITJ** algorithm are also reported in the results.

Keywords:
multi-objective scheduling, stochastic processing times, flowtime and tardiness measures

1. Introduction

While a lot of scheduling methods have been developed for optimizing various kinds of single performance measures (see Conway et al. [4], Baker[1] and French[7]), we can not directly apply them to multi-objective scheduling which is of relatively recent origin. Exploiting the results obtaind for single-objective scheduling, most investigators on multi-objective scheduling attempt to minimize a combination of total flowtime and some measures of tardiness. Some of the earlier noteworthy studies are Smith[16], Emmons[5], [6], Heck and Roberts[8] and Burns[2]. Other recent attempts have been presented by Van Wassenhove and Gelders[17], Sen and Gupta[14], Nelson, Sarin and Daniels[12] and Sen, Raiszadeh and Dileepan [15].

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Van Wassenhove and Gelders have presented a pseudo-polynomial algorithm to enumerate all of the efficient points for the bicriterion problem of total flowtime and maximum tardiness, using modified Smith's rule. Sen and Gupta have presented a branch and bound algorithm to minimize a linear combination of total flowtime and maximum tardiness of a given number of jobs on a single machine. Sen, Raiszadeh and Dileepan have solved the two criteria of total flowtime and range of lateness using the same approach as that Sen and Gupta. Nelson, Sarin and Daniels have presented algorithms for generating all of the efficient schedules to various problems with more than one criterion among the three- objective such as mean flowtime, the maximum tardiness and the number of tardy jobs.

Although these methods deal with scheduling problems with deterministic processing times, processing times are randomly distributed in most pratical production systems. Scheduling with stochastic processing times is more complex than with deterministic processing times. Therefore, there are few papers on such a scheduling with stochastic processing times except for an approach to minimize the expected makespan or the expected variance of the completion times[3],[13]. Recently, Jung, Nagaswa and Nishiyama[9],[10], [11] have solved a multi-objective single-stage scheduling problem with stochastic processing times.

We consider a three-objective single-stage scheduling problem with stochastic processing times which follow normal distributions. The objectives to be minimized are the expected value and the variance of total flowtime, and the mean probability of tardiness. The expected value of total flowtime is related to the expected value of total in-process inventory cost. The variance of total flowtime has a close relation to the difference between the planned completion times and the actual completion times which may make bad influence on the succeeding process. The mean probability of tardiness is one of performance measures to evaluate the degree of meeting demand within due date.

This paper is organized as follows: In section 2, a multi-objective scheduling model with stochastic processing times is formulated. An algorithm for heuristically generating a set of efficient schedules is proposed in section 3. Section 4 is devoted to the numerical example. The computational experience is summarized in section 5. Conclusions are presented in section 6.

2. Scheduling model

2.1 Problem formulation

Consider an n-job, single-stage scheduling problem with the following assumptions :

(1) Jobs are all ready to be processed at time $t=0$.

(2) Job processing times are independent random variables following normal distributions with known means and variances.

(3) Setup times are included in processing times and are independent of the job sequence.

(4) Job processing is completed without interruption.

(5) The machine is always available.

The following notations are used throughout this paper :

n : the number of jobs

Π : the set of permutation schedules

π : a permutation schedule

$J_{[i]}$: the job assigned to the ith position in a schedule($i=1, 2, \cdots, n$).

$X_{[i]}$: the processing times of job $J_{[i]}$

$\mu_{[i]}$: the mean of processing times $X_{[i]}$

$\sigma_{[i]}$: the standard deviation of processing times $X_{[i]}$

$d_{[i]}$: the due date of job $J_{[i]}$

$C_{[i]}$: the completion times of job $J_{[i]}$

$$C_{[i]} = \sum_{j=1}^{i} X_{[j]} \sim N(\sum_{j=1}^{i}\mu_{[j]}, \sum_{j=1}^{i}\sigma^2_{[j]})$$

$E[C_{[i]}]$: the expected value of completion times of job $J_{[i]}$

$V[C_{[i]}]$: the variance of completion times of job $J_{[i]}$

$$E[C_{[i]}] = \sum_{j=1}^{i}\mu_{[j]}, \quad V[C_{[i]}] = \sum_{j=1}^{i}\sigma^2_{[j]}$$

$F(\pi)$: the total flowtime of schedule π

$$F(\pi) = \sum_{i=1}^{n} C_{[i]} = \sum_{i=1}^{n}(n-i+1)X_{[i]}$$

$\overline{P}_{tard}(\pi)$: the mean probaility of tardiness of schedule π

$$\overline{P}_{tard}(\pi) = \frac{1}{n}\sum_{i=1}^{n}\Pr.\{C_{[i]} \geq d_{[i]}\}$$

where "$[i]$" denotes the ith position in a permutation schedule.

2.2. Performance measures

We consider the multi-objective scheduling problem to simultaneously minimize the following three performance measures : the expected value $E[F(\pi)]$ and the variance $V[F(\pi)]$ of the total flowtime, and the mean probability of tardiness $\bar{P}_{tard}(\pi)$ defined by

$$E[F(\pi)] = \sum_{i=1}^{n}(n-i+1)\mu_{[i]}, \tag{1}$$

$$V[F(\pi)] = \sum_{i=1}^{n}(n-i+1)^2\sigma_{[i]}^2, \tag{2}$$

$$\bar{P}_{tard.}(\pi) = \frac{1}{n}\sum_{i=1}^{n}\{1-\Phi(\alpha_{[i]})\}$$

$$= 1-\sum_{i=1}^{n}\Phi(\alpha_{[i]})/n, \tag{3}$$

where $\Phi(\alpha_{[i]})$ denotes a standard normal *c. d. f.* defined by

$$\Phi(\alpha_{[i]}) = \int_{-\infty}^{\alpha_{[i]}} \frac{1}{\sqrt{2\pi}}e^{-t^2/2}dt$$

$$\alpha_{[i]} \equiv \frac{d_{[i]}-E[C_{[i]}]}{\sqrt{V[C_{[i]}]}},\ i=1,\ 2,\ \cdots,\ n. \tag{4}$$

Our purpose is to find the set of efficient schedules for this three-objective scheduling problem. Letting $z_1(\pi)$, $z_2(\pi)$ and $z_3(\pi)$ denote the values of $E[F(\pi)]$, $V[F(\pi)]$ and $\bar{P}_{tard.}(\pi)$ associated with schedule π, respectively, we define the efficient schedule as follows :

Definition 1. A schedule π is efficient if and only if there exists no schedule π' such that $z_l(\pi') \leq z_l(\pi)$ for all $l\in\{1, 2, 3\}$ and $z_l(\pi)$ for some $l\in\{1, 2, 3\}$.

A branch-and-bound algorithm can be developed for finding the set of efficient schedules but requires too much computation time to use in practice. Since the development of efficient algorithm to generate all of the efficient schedules is very hard, an efficient heuristic algorithm is proposed in this paper.

First, we construct the following three schedules, i. e., SPT_μ, SPT_σ and ***EDD***, and analyze their characteristics to develop the heuristic algorithm :

SPT_μ is a shedule obtained by sequencing jobs in nondecreasing order of μ_i(break ties in nondecreasing order of σ_i, and if ties still exist, break them in nondecreasing order of d_i).

SPT_σ is a schedule obtained by sequencing jobs in nondecreasing order of σ_i(break ties in nondecreasing order of μ_i, and if ties still exist, break them in nondecreasing order of d_i).

EDD is a schedule obtained by sequencing jobs in nondecreasing order of d_i(break ties in nondecreasing order of μ_i, and if ties still exist, break them in nondecreasing order of σ_i).

Proposition 1. SPT_μ and SPT_σ are efficient schedules to the above three-objective problem.

Prool. (a) If SPT_μ is unique, then there is obviously no schedule $\pi \neq SPT_\mu$ such that

$$E[F(\pi)] \leq E[F(SPT_\mu)],\ V[F(\pi)] \leq V[F(SPT_\mu)],\ \bar{P}_{tard}(\pi) \leq \bar{P}_{tard}(SPT_\mu) \quad (5a)$$

$$\text{and } E[F(\pi)] \neq E[F(SPT_\mu)],\ V(F(\pi)] \neq V[F(SPT_\mu)],\ \bar{P}_{tard}(\pi) \leq \bar{P}_{tard}(SPT_\mu) \quad (5b)$$

(b) If SPT_μ is not unique, let Π_μ be a set of SPT_μ. From the process of generating SPT_μ, we cannot find any $\pi \neq \Pi_\mu$ satisfying equations (5a) and (5b). Therefore, any schedule in Π_μ, SPT_μ is efficient schedule. We can prove the case of SPT_σ in a similar argument. □

EDD is not always effieient schedule because **EDD** can not always minimize $\bar{P}_{tard}(\pi)$. However, in the following special cases, SPT_μ, SPT_σ and **EDD** are all the same schedule, and become a superior schedule which simultaneously minimizes the three-objective.

(1) When $\mu_i = \mu$ and $\sigma_i = \sigma$ for $i = 1, 2, \cdots, n$, **EDD** minimizes $\bar{P}_{tard}(\pi)$ because $E[C_{[i]}] = i\mu$ and $V[C_{[i]}] = i\sigma^2$ are independent of schedule. Therefore, **EDD** is a superior schedule that simultaneously minimizes the three-objective.

(2) When $\mu_i = \mu$ and $d_i = d$ for $i = 1, 2, \cdots, n$, SPT_σ minimizes $\bar{P}_{tard}(\pi)$ because $E[F(\pi)]$ and the numerator in equation(4), $\sqrt{V[C_{[i]}]}$, is also minimized by SPT_σ. Therefore, SPT_σ is a superior schedule that simultaneously minimizes the three-objective.

(3) When $\sigma_i = \sigma$ and $d_i = d$ for $i = 1, 2, \cdots, n$, SPT_μ minimizes $\bar{P}_{tard}(\pi)$ because the denominator in equation(4) $\sqrt{V[C_{[i]}]}$ is independet of schedule and $E[C_{[i]}]$ is minized by SPT_μ. Therefor, SPT_μ is a superior schedule that simultaneously minimizes the three-objective.

(4) When $d_i = d$ and $c.v_i (= \sigma_i / \mu_i) = c.v$ for $i = 1, 2, \cdots, n$, $SPT_\mu = SPT_\sigma$ and SPT_μ obviously minimizes both of $E[F(\pi)]$ and $V[F(\pi)]$. $SPT_\mu (= SPT_\sigma)$ also minimizes $\bar{P}_{tard}(\pi)$ because SPT_μ minimizes both of $E[C_{[i]}]$ and $V[C_{[i]}]$ in equation(4), and then maximizes $\alpha_{[i]}$. Therefore, $SPT_\mu (= SPT_\sigma)$ is a wuperior schedule that simultaneously minimizes the three-objective.

(5) When $d_i = d$, $\mu_j \leq \mu_{j+1}$ and $\sigma_j \leq \sigma_{j+1}$ for $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, n-1$, we get $SPT_\mu = SPT_\sigma$, and $SPT_\mu (= SPT_\sigma)$ becomes a superior shedule that simultaneously minimizes the three-objective, in a similar argument to case(4).

However, most of multi-objective scheduling problems do not always have a superior schedule. It is necessary to find all of the efficient schedules when there is no superior schedule in a given multi-objective scheduling problem.

3. Generating the set of efficient schedules

In most cases, an efficient schedule is similar to some of the other efficient schedules. In other words, it is expected that an efficient schedule be drived by exchanging positions of only a few jobs in other efficient schedules. This observation leads to a method for successively generating the set of efficient schedules.

Let $\tilde{\pi}$ be a schedule obtained by interchanging $J_{[i]}$ and $J_{[m]}$, $j<m$, without disturbing any other job in schedule π. Defining $\Delta\mu$, $\Delta\sigma^2$ and Δd for schedule π as

$$\Delta\mu = \mu_{[m]}-\mu_{[i]},$$

$$\Delta\sigma^2 = \sigma^2_{[m]}-\sigma^2_{[i]},$$

$$\Delta d = d_{[m]}-d_{[i]},\ 1\leq i<m\leq n,$$

we get the difference of the expected value and the variance of total flowtime between $\tilde{\pi}$ and π as follows :

$$\begin{aligned} E[F(\tilde{\pi})]-E[F(\pi)] &= \{(n-i+1)\mu_{[m]}+(n-m+1)\mu_{[i]}\} \\ &\quad -\{(n-i+1)\mu_{[i]}+(n-m+1)\mu_{[m]}\} \\ &= (m-i)(\mu_{[m]}-\mu_{[i]}) \\ &= (m-i)\Delta\mu, \end{aligned} \tag{6}$$

$$\begin{aligned} V[F(\tilde{\pi})]-V[F(\pi)] &= \{(n-i+1)^2\sigma^2_{[m]}+(n-m+1)^2\sigma^2_{[i]}\} \\ &\quad -\{(n-i+1)^2\sigma^2_{[i]}+(n-m+1)^2\sigma^2_{[m]}\} \\ &= (2n-m-i+2)(m-i)(\sigma^2_{[m]}-\sigma^2_{[i]}) \\ &= (2n-m-i+2)(m-i)\Delta\sigma^2, \end{aligned} \tag{7}$$

From equation(3), the difference of the mean probability of tardiness between $\tilde{\pi}$ and π is obtained by

$$\begin{aligned} \bar{P}_{tard.}(\tilde{\pi})-\bar{P}_{tard}(\pi) &= [\sum_{j=1}^{n}\Phi(\alpha_{[j]}(\pi))-\sum_{j=1}^{n}\Phi(\alpha_{[j]}(\tilde{\pi}))]/n \\ &= [\{\Phi(\alpha_{[i]}(\pi))-\Phi(\alpha_{[i]}(\tilde{\pi}))\}+\{\Phi(\alpha_{[m]}(\pi))-\Phi(\alpha_{[m]}(\tilde{\pi}))\} \\ &\quad +\sum_{j\neq i,m}\{\Phi(\alpha_{[j]}(\pi))-\Phi(\alpha_{[j]}(\tilde{\pi}))\}]/n \\ &= (\beta_{[i]}+\beta_{[m]}+\sum_{j\neq i,m}\beta_{[j]})/n, \end{aligned} \tag{8}$$

where

$$\beta_{[i]} = \Phi(\alpha_{[i]}(\pi))-\Phi(\alpha_{[i]}(\tilde{\pi})),$$

$$\beta_{[m]} = \Phi(\alpha_{[m]}(\pi))-\Phi(\alpha_{[m]}(\tilde{\pi})),$$

$$\beta_{[j]} = \Phi(\alpha_{[j]}(\pi))-\Phi(\alpha_{[j]}(\tilde{\pi}))\ \text{for any}\ j\neq i,\ m.$$

From equations(6), (7) and (8), we have the following relationships between schedules $\tilde{\pi}$ and π :

$$\Delta\mu \geqslant 0 \Leftrightarrow E[F(\tilde{\pi})] \geqslant E[F(\pi)],$$
$$\Delta\sigma^2 \leqslant 0 \Leftrightarrow V[F(\tilde{\pi})] \leqslant V[F(\pi)],$$
$$\beta_{[i]} + \beta_{[m]} + \sum_{j \neq i,\, m} \beta_{[j]} \geqslant 0 \Leftrightarrow \bar{P}_{tard.}(\tilde{\pi}) \geqslant \bar{P}_{tard.}(\pi)$$

From equation(4), $\alpha_{[j]}(\pi)$ and $d_{[j]}(\tilde{\pi})$ are represented as

$$\alpha_{[j]}(\pi) = \frac{d_{[j]} - E[C_{[j]}]}{\sqrt{V[C_{[j]}]}},\ j=1,\ 2,\ \cdots,\ n, \tag{9}$$

$$\alpha_{[j]}(\tilde{\pi}) = \begin{cases} \dfrac{d_{[m]} - (E[C_{[j]}] + \Delta\mu)}{\sqrt{V[C_{[j]}] + \Delta\sigma^2}}, & j=i\ ; \\ \dfrac{d_{[j]} - (E[C_{[j]}] + \Delta\mu}{\sqrt{V[C_{[j]}] + \Delta\sigma^2}}, & i<j<m\ ; \\ \dfrac{d_{[j]} - (E[C_{[m]}] + \Delta\mu)}{\sqrt{V[C_{[m]}]}}, & j=m\ ; \\ \alpha_{[j]}(\pi), & j<i \text{ or } m<j. \end{cases} \tag{10}$$

If $\Delta\mu \geq 0$, $\Delta\sigma^2 \geq 0$, and $\Delta d \geq 0$, then we get $\alpha_{[j]}(\pi) \geq \alpha_{[j]}(\tilde{\pi})$, for any $j \neq i$, equivalently, $\beta_{[j]} \geq 0$, for any $j \neq i$, applying the following inequalities to equations(9) and (10) :

$$E[C_{[i]}] < E[C_{[m]}],\ V[C_{[i]}] < V[C_{[m]}],$$
$$E[C_{[j]}] \leq E[C_{[j]}] + \Delta\mu,\ V[C_{[j]}] \leq V[C_{[j]}] + \Delta\sigma^2 \text{ for any } j.$$

Unfortunately, we can not say whether or not $\alpha_{[i]}(\pi) \geq \alpha_{[i]}(\tilde{\pi})$, equivalently $\beta_{[i]} \geq 0$, because the difference between $\alpha_{[i]}(\pi)$ and $\alpha_{[i]}(\tilde{\pi})$ depends on the sizes of $\Delta\mu$, $\Delta\sigma^2$ and Δd. However, in most cases, it is expected that $\beta_{[i]} + \beta_{[m]} + \sum_{j \neq i,\, m} \beta_{[j]} > 0$, and that $\bar{P}_{tard.}(\tilde{\pi})$ is not smaller than $\bar{P}_{tard.}(\pi)$. We can use this relationship for generating the set of efficient schedules.

Generating method

Schedule $\tilde{\pi}$ obtained by interchanging $J_{[i]}$ with $J_{[m]}$, $1 \leq i < m \leq n$, in an efficient schedule, π, is not always an efficient schedule. If efficient schedules exist among all of the schedules obtained by such interchanges of two jobs in the efficient for generating a set of efficient schedules, since the maximum cardinality of the set of schedule $\tilde{\pi}$ is less than or equal to ${}_nC_2 = n(n-1)/2$ which is much smaller than $|\Pi| = n!$ Therefore, we propose a method using pairwise interchange of any two jobs(**PITJ**) in schedule π for generating a set of efficient schedules. The proposed method creates an efficient schedule from an initial efficient schedule and repeatedly generates the next efficient schedule from the current efficient schedules.

In our scheduling problem, we get the initial set of efficient schedules as $\Pi_1 = \{\boldsymbol{SPT}_\mu\}$. The second efficient schedule is obtained by **PITJ** in schedule $\boldsymbol{SPT}_\mu$. Let Π_q be a current set of efficient schedules obtained at iteration q and π be a schedule in Π_q. From the current set of efficient schedules, Π_q, we generate the next set of efficient schedules, Π_{q+1}, by **PITJ** in any schedule π in Π_q. If there exist two jobs at positions i and m in schedule π such that

$$(i,\ m)\in I(\pi)\equiv\{(i,\ m)\,|\,\mu_{[i]}<\mu_{[m]} \text{ and } \sigma_{[i]}>\sigma_{[m]}, \text{ or } \mu_{[i]}<\mu_{[m]} \text{ and } d_{[i]}>d_{[m]}, \text{ or}$$

$$\mu_{[i]}=\mu_{[m]},\ \sigma_{[i]}=\sigma_{[m]} \text{ and } d_{[i]}=d_{[m]},\ 1\le i<m\le n\},$$

then create new schedules by interchanging $J_{[i]}$ with $J_{[m]}$ for any $(i,\ m)$ in $I(\pi)$. Interchanges of the other two jobs not in $I(\pi)$ do not probably generate the next efficient schedule because both the variance of total flowtime and the mean probability of tardiness are likely to increase according to relationships of equations(7) and (8).

Defining $\Pi(\Pi_q)$ as

$$\Pi(\Pi_q)\equiv\{\tilde{\pi}\,|\,\text{schedules generated by interchanging } J_{[i]} \text{ with } J_{[m]} \text{ in schedule } \pi$$
$$\text{for } \forall(i,\ m)\in I(\pi) \text{ and } \forall\pi\in\Pi_q\}.$$

We have

$$\Pi_F(\Pi_q)\equiv\{\tilde{\pi}^*\,|\,\tilde{\pi}^*\in\Pi(\Pi_q),\ E[F(\tilde{\pi}^*)]\le E[F(\tilde{\pi})] \text{ for } \forall\tilde{\pi}\in\Pi(\Pi_q)\},$$

where $\Pi_F(\Pi_q)$ is the set of candidates for efficient schedules that minimizes the expected value of total flowtime in $\Pi(\Pi_q)$. From $\Pi_F(\Pi_q)$, we get

$$\Pi_T(\Pi_q)\equiv\{\tilde{\pi}^*\,|\,\tilde{\pi}^*\in\Pi_F(\Pi_q),\ V[F(\tilde{\pi}^*)]\le V[F(\tilde{\pi})] \text{ for } \forall\tilde{\pi}\in\Pi_F(\Pi_q)\},$$

where $\Pi_T(\Pi_q)$ is the set of candidates for efficient schedules that minimizes the variance of total flowtime in $\Pi_F(\Pi_q)$. Finally, we obtain

$$\tilde{\Pi}(\Pi_q)\equiv\{\tilde{\pi}^*\,|\,\tilde{\pi}^*\in\Pi_T(\Pi_q),\ \bar{P}_{tard}(\tilde{\pi}^*)\le\bar{P}_{tard}(\tilde{\pi})] \text{ for } \forall\tilde{\pi}\in\Pi_T(\Pi_q)\},$$

where $\tilde{\Pi}(\Pi_q)$ is the set of candidates for efficient schedules that minimizes the mean probability of tardiness in $\Pi_T(\Pi_q)$. Uniting this set $\tilde{\Pi}(\Pi_q)$ and Π_q, we get the next set of approximate efficient schedules, Π_{q+1} generated from the current set Π_q. The procedure of the proposed **PITJ** algorithm is stated as follows :

Algorithm

Step 1. Determine the initial efficient schedule $\pi_1=\boldsymbol{SPT}_\mu$ and set $\Pi_1=\{\pi_1\}$ and $q=1$.

Step 2. Find $\Pi(\Pi_q)$. If $\Pi(\Pi_q)=\phi$, go to Setp 5.

Step 3. Find $\tilde{\Pi}(\Pi_q)$. If $\tilde{\Pi}(\Pi_q)=\phi$, go to Step 5.

Step 4. Set $\Pi_{q+1}=\Pi_q\cup\tilde{\Pi}(\Pi_q)$ and q=q+1. go to Step 2.

Step t. Stop.

Π_q is the set of efficient schedules obtained from **PITJ** algorithm. And the overall time complexity by ***PITJ*** algorithm is $O(N^2n^2)$, where N is the number of efficient schedules.

4. Numerical example

We illustrate an application of the proposed **PITJ** algorithm to a 5-job problem with data shown in Table 1.

Table 1. Data in an example problem

J_i	J_1	J_2	J_3	J_4	J_5
μ_i	30	28	22	45	15
σ_i	5	10	6	8	3
d_i	86	81	125	131	60

From Step 1, the initial efficient schedule is obtained as

$$\pi_1 = \boldsymbol{SPT}_\mu = (J_5,\ J_3,\ J_2,\ J_1,\ J_4) \text{ and then } \Pi_1 = \{\pi_1\},$$

where $E[F(\pi_1)]=352$, $V[F(\pi_1)]=1865$, and $\bar{P}_{tard.}(\pi_1)=0.3188$.

From Step 2. we get the following set of job pairs in π_1 for generating the next efficient schedule :

$$I(\pi_1) \equiv \{(J_3,\ J_2),\ (J_3,\ J_1),\ (J_2,\ J_1),\ (J_2,\ J_4)\}.$$

Interchanging any two jobs in $I(\pi_1)$, we obtain

$$\Pi(\Pi_1) \equiv \{\tilde{\pi}_{11} = (J_5,\ J_2,\ J_3,\ J_1,\ J_4),\ \tilde{\pi}_{12} = (J_5,\ J_1,\ J_2,\ J_3,\ J_4),$$

$$\tilde{\pi}_{13} = (J_5,\ J_3,\ J_1,\ J_2,\ J_4),\ \tilde{\pi}_{14} = (J_5,\ J_3,\ J_4,\ J_1,\ J_2)\},$$

where the value of three performance measures associated with each schedule are

$E[F(\tilde{\pi}_{11})]=358$, $V[F(\tilde{\pi}_{11})]=2313$, and $\bar{P}_{tard.}(\tilde{\pi}_{11})=0.2954$,

$E[F(\tilde{\pi}_{12})]=368$, $V[F(\tilde{\pi}_{12})]=1733$, and $\bar{P}_{tard.}(\tilde{\pi}_{12})=0.1955$,

$E[F(\tilde{\pi}_{13})]=354$, $V[F(\tilde{\pi}_{13})]=1490$, and $\bar{P}_{tard.}(\tilde{\pi}_{13})=0.3184$,

$E[F(\tilde{\pi}_{14})]=386$, $V[F(\tilde{\pi}_{14})]=1577$, and $\bar{P}_{tard.}(\tilde{\pi}_{14})=0.3975$,

From Steo 3, the minimal value of $E[F(\tilde{\pi})]$ is given by $\tilde{\pi}_{13}$ and therefore we get

$$\tilde{\Pi}(\Pi_1) \equiv \{\tilde{\pi}_{13}\} \text{ and } \pi_2 = \{\tilde{\pi}_{13}\}.$$

From Step 4, we obtain $\Pi_2 = \Pi_1 \cup \tilde{\Pi}(\Pi_1) = \{\pi_1,\ \pi_2\}$. Returning to Step 2, we have the two sets of job pairs, $I(\pi_1)$ and $I(\pi_2)$, as follows :

$$I(\pi_1) \equiv \{(J_3,\ J_2),\ (J_3,\ J_1),\ (J_2,\ J_4)\},$$

$$I(\pi_2) \equiv \{(J_3,\ J_1),\ (J_3,\ J_2),\ (J_2,\ J_4)\},$$

where pair (J_2, J_1) in $I(\pi_1)$ is eliminated because the interchange of this pair produces the second efficient schedule π_2. Interchanging any two jobs in $I(\pi_1) \cup I(\pi_2)$ yields

$$\Pi(\Pi_q) \equiv \{\tilde{\pi}_{11} = (J_5, J_2, J_3, J_1, J_4),\ \tilde{\pi}_{12} = (J_5, J_1, J_2, J_3, J_4),$$
$$\tilde{\pi}_{14} = (J_5, J_3, J_4, J_1, J_2),\ \tilde{\pi}_{21} = (J_5, J_1, J_3, J_2, J_4),$$
$$\tilde{\pi}_{22} = (J_5, J_2, J_1, J_3, J_4),\ \tilde{\pi}_{23} = (J_5, J_3, J_1, J_4, J_2)\},$$

where the values of three performance measure associated with each schedule are

$$E[F(\tilde{\pi}_{11})] = 358,\ V[F(\tilde{\pi}_{11})] = 2313,\ \bar{P}_{tard.}(\tilde{\pi}_{11}) = 0.2954,$$
$$E[F(\tilde{\pi}_{12})] = 368,\ V[F(\tilde{\pi}_{12})] = 1733,\ \bar{P}_{tard.}(\tilde{\pi}_{12}) = 0.1955,$$
$$E[F(\tilde{\pi}_{14})] = 386,\ V[F(\tilde{\pi}_{14})] = 1577,\ \bar{P}_{tard.}(\tilde{\pi}_{11}) = 0.3975,$$
$$E[F(\tilde{\pi}_{21})] = 362,\ V[F(\tilde{\pi}_{21})] = 1413,\ \bar{P}_{tard.}(\tilde{\pi}_{21}) = 0.3161,$$
$$E[F(\tilde{\pi}_{22})] = 366,\ V[F(\tilde{\pi}_{22})] = 2258,\ \bar{P}_{tard.}(\tilde{\pi}_{22}) = 0.1727,$$
$$E[F(\tilde{\pi}_{23})] = 371,\ V[F(\tilde{\pi}_{23})] = 1382,\ \bar{P}_{tard.}(\tilde{\pi}_{23}) = 0.2124.$$

From Step 3, the minimal value of $E[F(\pi)]$ is given by $\tilde{\pi}_{11}$ and therefore we get

$$\tilde{\Pi}(\Pi_2) \equiv \{\tilde{\pi}_{11}\} \text{ and } \pi_3 = \{\tilde{\pi}_{11}\}.$$

From Step 4, we get $\Pi_3 = \Pi_2 \cup \tilde{\Pi}(\Pi_2) = \{\pi_1, \pi_2, \pi_3\}$. Repeating the procedure until either $\Pi(\Pi_q)$ or $\tilde{\Pi}(\Pi_q)$ becomes empty, we obtain the results shown in Talbe 2.

Table 2. The set of efficient schedules obtained from **PITJ**

the efficient schedules Π_q	$E[F(\pi_i)]$	$V[F(\pi_i)]$	$\bar{P}_{tard}(\pi_i)$
$\pi_1 : J_5, J_3, J_2, J_1, J_4$	352	1865	0.3138
$\pi_2 : J_5, J_3, J_1, J_2, J_4$	354	1490	0.3184
$\pi_3 : J_5, J_2, J_3, J_1, J_4$	358	2313	0.2954
$\pi_4 : J_5, J_1, J_3, J_2, J_4$	362	1413	0.3161
$\pi_5 : J_5, J_2, J_1, J_3, J_4$	366	2258	0.1727
$\pi_6 : J_5, J_1, J_2, J_3, J_4$	368	1733	0.1955
$\pi_7 : J_5, J_3, J_1, J_4, J_2$	371	1382	0.2124
$\pi_8 : J_5, J_1, J_3, J_4, J_2$	379	1305	0.2101

5. Computational experience

This section present the computational experience of the proposed **PITJ** algorithm. Our purpose is to compare the computation time and the number of efficient schedules obtained from **PITJ** algorithm with those obtained from a branch and-bound(**BAB**) algorithm for generating all of efficient schedules. To evaluate the accuracy of an approximate set of efficient schedules obtained from **PITJ** algorithm, we use two scales, named the "degree of set approximation" and the "degree of point approximation"(see Appendix for detail).

5.1 Scheduling problems

Scheduling problems are generated by the following procedure : The number of jobs is specified as $n=5$ and $n=10$. The value of μ_i, $i=1, 2, \cdots, n$, are given as integers distributed over three kinds of range such as [10, 50), [50, 100) and [10, 100). The values of σ_i, $i=1, 2, \cdots, n$, are given by $\mu_i\ c.v_i$, where $c.v_i(=\sigma_i/\mu_i)$, $i=1, 2, \cdots, n$, are generated from the three different types of a uniform distribution on (0, 0.5], (0.5, 1] and (0, 1], independent of μ_i. We construct 18 kinds of problems by combining two kinds of n, three kinds of μ and three kinds of $c.v$, and the values of d_i, $i=1, 2, \cdots, n$, are generated from another uniform distribution on $[T(1-r), T(1+r)]$, where $T=\sum_{i=1}^{n}\mu_i$ and r denotes the coefficient of the range of due dates ; $r=0.5$ in our problems. Twenty simulation runs are performed for each problem. The proposed **PITJ** and **BAB** algorithms are programmed in BASIC language and tested on an EPSON PC-386VS (CPU i80386) personal computer.

5.2 Results

The computational results are summarized in Table 3, where the number of efficient schedules, compuation time, the degree of set approximation and the mean of the degree of point approximation are the averages of 20 simulation runs. The Min and Max of the degree of point approximation are the minimum and the maximum values among the values of the degree of point approximation obtained from the 20 simulation runs, respectively.

Table 3. Computational results

Number of Jobs	Processing times		Number of efficient schedules		Computation time(sec)		Degree of set approximation	Degree of point approximation		
n	μ	$c.v$	PITJ	BAB	PITJ	BAB		Min	Max	Mean
5	[10, 50)	(0, 0.5]	10.5	10.6	1.3	5.1	0.0082	0	0.0946	0.0165
		(0.5, 1]	7.3	7.4	0.6	3.0	0.0004	0	0.0109	0.0033
		(0, 1]	13.7	14.1	2.9	8.0	0.0018	0	0.0320	0.0050
	[50, 100)	(0, 0.5]	15.5	15.7	2.8	8.6	0.0011	0	0.0406	0.0061
		(0.5, 1]	16.3	17.2	3.8	10.9	0.0018	0	0.0292	0.0056
		(0, 1]	17.1	17.4	3.2	11.0	0.0001	0	0.0091	0.0012
	[10, 100)	(0, 0.5]	12.6	13.0	2.0	7.0	0.0015	0	0.0776	0.017
		(0.5, 1]	8.9	9.0	1.2	4.0	0.0001	0	0.0056	0.0012
		(0, 1]	13.3	13.8	2.4	7.2	0.0014	0	0.0270	0.0045
10	[10, 50)	(0, 0.5]	19.9	29.5	17.8	3349	0.0006	0	0.1406	0.0330
		(0.5, 1]	23.1	39.8	28.9	2832	0.0009	0	0.1519	0.0333
		(0, 1]	27.4	36.1	26.4	3181	0.0003	0	0.1030	0.0186
	[50, 100)	(0, 0.5]	21.2	23.7	24.3	2496	0.0007	0	0.0704	0.0144
		(0.5, 1]	58.3	67.0	66.2	9842	0.0010	0	0.0792	0.0164
		(0, 1]	33.4	40.2	31.5	3302	0.0005	0	0.0777	0.0132
	[10, 100)	(0, 0.5]	14.1	22.5	10.0	2910	0.0004	0	0.1302	0.0335
		(0.5, 1]	22.4	32.3	27.2	1335	0.0014	0	0.1124	0.0226
		(0, 1]	16.5	27.0	11.5	1208	0.0006	0	0.0908	0.0209

In case of $n=5$, the number of efficient schedules obtained from **PITJ** is almost equal to that obtained from **BAB**. In case of $n=10$, the number of efficient schedules obtained from **PITJ** is smaller by 2~13(10~40%) than that obtained from **BAB**. However, the set of efficient schedules obtained from **PITJ** is very close to that of efficient schedules obtained from **BAB** because the degree of set approximation is sufficiently small, that is 0.01~0.82% and 0.03~0.14% for $n=5$ and $n=10$, respectively. **PITJ** also provides an efficient schedule very close to each efficient schedule obtained from **BAB** since the Min, Max and Mean degree of point approximation are 0%, 0.56~9.46% and 0.12~1.7% for $n=5$, and 0%, 7.04~15.19% and 1.32~3.35% for $n=10$. respectively. Furthermore, computation time for generating a set of efficient schedules using **PITJ** is much faster than that using **BAB**.

6. Conclusions

A multi-objective single-stage scheduling problem was constructed to minimize the expected value and the variance of total flowtime, and the mean probability of tardiness when processing times is normally distributed. **PITJ** algorithm was proposed to generate all of the approximate efficient schedules to this scheduling problem. The results of the computational experience made it clear that **PITJ** algorithm efficiently generates an approximate set of efficient schedules with high accuracy. Furthermore, **PITJ** algorithm is applicable to large-sized problems because computation time is $O(N^2n^2)$.

Appendix : two scales of the degree of approximation

We use two scales for evaluating the accuracy of an approximate set of efficient schedules obtained from **PITJ** algorithm. The "degree of set approximation" is used to evaluate the global accuracy of the set of approximate efficient schedules obtainded from **PITJ**. The "degree of point approximation" is used to evaluate the individual accuracy of approximate efficient schedules obtained from **PITJ**.

Notation

m : the number of efficient schedules

k : the number of approximate efficient schedules

π : a permutation schedule

π_e : an efficient schedule($e=1, 2, \cdots, m$)

π_q : an approximate efficient schedule($q=1, 2, \cdots, k$)

$Z(\pi)$: a criterion vector for schedule π

$Z(\pi)=(z_1(\pi), z_2(\pi), z_3(\pi))$

$\mathbf{I}_{point}$: the ideal point in the three-objective scheduling problem

$\mathbf{I}_{point}=(\min\{z_1(\pi)\}, \min\{z_2(\pi)\}, \min\{z_3(\pi)\})$,

$\mathbf{W}_{point}$: the worst point the in three-objective scheduling problem

$\mathbf{W}_{point}=(\max\{z_1(\pi)\}, \max\{z_2(\pi)\}, \max\{z_3(\pi)\})$,

Let $\mathbf{V}$ be the volume of the reference region defined by

$$\mathbf{V}=\prod_{j=1}^{3}(max\ z_j(\pi)-\min\ z_j(\pi)).$$

Let **A** and **B** be the volume of the region dominated by at least one of $\mathbf{Z}(\pi_e)$ $e=1, 2, \cdots, m$, and at least of $\mathbf{Z}(\pi_q)$, $q=1, 2, \cdots, k$, respectively, we have

$$\mathbf{A}=\mathbf{V}-\sum_{k=1}^{m-1}(z_1(\pi_{i+1})-z_1(\pi_i))(z_2(\pi_i)-\min\{z_2(\pi)\})(z_3(\pi_i)-\min\{z_3(\pi)\}),$$

$$\mathbf{B}=\mathbf{V}-\sum_{j=1}^{k-1}(z_1(\pi_{j+1})-z_1(\pi_j))(z_2(\pi_j)-\min\{z_2(\pi)\})(z_3(\pi_j)-\min\{z_3(\pi)\}),$$

The degree of set approximation(DSA) is given by

$$\text{DSA}=1-\frac{\mathbf{B}}{\mathbf{A}}.$$

Let $\mathbf{Z}(\pi'(\pi_e))$ be the nearest approximate efficient point to efficientj point $\mathbf{Z}(\pi_e)$. The degree of point approximation(DPA) with an efficient point $\mathbf{Z}(\pi_e)$ is given by

$$\text{DPA}_e=\frac{\mathbf{d}(\mathbf{Z}(\pi'\pi_e)),\ \mathbf{Z}(\pi_e))}{\mathbf{D}(\mathbf{W}_{\text{point}},\ \mathbf{Z}(\pi_e))},\quad e=1,\ 2,\ \cdots,\ m,$$

where $d(\mathbf{Z}(\pi'(\pi_e)),\ \mathbf{Z}(\pi_e))$ and $D(\mathbf{W}_{point},\ \mathbf{Z}(\pi_e))$ are the distances in Euclidean space between $\mathbf{Z}(\pi'(\pi_e))$ and $\mathbf{Z}(\pi_e)$ and between $\mathbf{W}_{point}$ and $\mathbf{Z}(\pi_e)$, respectively. Then, we get the Min, Max and Mean of the degree of point approximation as follows :

$$\text{DPA}_{\text{Min}}=\min\{\text{DPA}_e\},\ \text{DPA}_{\text{Max}}=\max\{\text{DPA}_e\},\ \text{DPA}_{\text{Mean}}=\sum_{e=1}^{m}\text{DPA}_e/m.$$

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