On the Improvement of the Stability Robustness in the Discrete-time LQ Regulator

이산시간 LQ 조절기의 안정도 강인성 향상에 관한 연구

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요 약 : 본 논문에서는 이산시간 LQ 조절기의 안정도 강인성을 주파수영역 및 시간영역에서 고찰하고 그 향상책을 제시한다. 주파수영역에서 강인성 척도인 궤환차행렬(return difference matrix)의 최소특이치가 상태가중치 행렬과 제어가중치 행렬의 비와 반비례함을 보이고, 시간영역에서 매개변수의 변화에 대한 안정도 강인성 범위들을 얻는다. 이 범위들의 점근적 성질을 밝히기 위하여 LQ 궤환이득의 특이치들이 상태가중치 행렬과 제어가중치 행렬의 비의 증가함수임을 보인다. 몇가지 조건하에서 시스템행렬(입력행렬)에 대한 안정도 강인성 범위가 상태가중치 행렬과 제어가중치 행렬의 비가 증가(감소)함에 따라서 증가함을 보이고, 이러한 사실들을 예제를 통하여 검증한다.

Keywords: discrete-time LQ regulator, stability robustness, return difference matrix, parameter variation

I. Introduction

We consider the linear discrete-time controllable system

$$x_{k+1} = Ax_k + Bu_k \tag{1}$$

where $x_k \in R^n$, $u_k \in R^m$, $x_0 \neq 0$, A and B are constant matrices with the appropriate dimensions, and (A, B) is a controllable pair. The LQ control law which minimizes the following LQ performance :

$$J(x_i, u_i) = \sum_{i=0}^{\infty}[\ x_i^TQx_i + u_i^TRu_i] \tag{2}$$

is given by

$$u_k = -(R+B^TPB)^{-1}B^TPAx_k = -Kx_k \tag{3}$$

where $Q \in R^{n\times n}$ is positive semi-definite, $(Q^{1/2}, A)$ is an observable pair, $R \in R^{m\times m}$ is positive definite and diagonal, and P is the positive definite solution of the following algebraic Riccati equation (ARE) :

$$P = A^TPA + Q - A^TPB(R+B^TPB)^{-1}B^TPA \tag{4}$$

In general, the model uncertainty is represented by two types :

$$G_p(z) = L(z)G(z) \tag{5}$$

and

$$x_{k+1} = (A+\delta A)x_k + (B+\delta B)u_k \tag{6}$$

in the frequency and time domains, respectively, where $L(z)$ is the multiplicative model uncertainty in the frequency domain, and $\{\delta A, \delta B\}$ modeling errors in the time domain. Since the relationship between $L(z)$ and $\{\delta A, \delta B\}$ is not clear, the robustness against each type of uncertainty has been investigated by different methods.

The stability robustness in the frequency domain can be determined by the minimum singular value of the return difference matrix. The larger the minimum singular value of the return difference matrix is, the more uncertainty of $L(z)$ can be allowed [1,2]. It is well known that the continuous-time LQR (CLQR) with a diagonal control weighting matrix is considered to be robust since the minimum singular value of the return difference matrix is always greater than or equal to 1 for multi-input systems [1,3]. For the discrete-time LQR (DLQR), however, the minimum singular value of the return difference matrix is not greater than 1 even for single input systems [4]. It is usually less than 1 and can be equal to 1 for the trivial case of zero feedback. Thus the general method to increase the minimum singular value of the return difference matrix of DLQR is important for the stability robustness. But this has not been discussed in the literature. This paper suggests the general method to increase the minimum singular value of the return difference matrix of DLQR in terms of weighting matrices.

The stability robustness in the time domain depends on how large parameter variations $\{\delta A, \delta B\}$ can be allowed. The maximum singular value of allowed modeling errors of $\{\delta A, \delta B\}$ are often called stability robustness bounds in the time domain [5,6]. It is known that CLQR with cheap control and the continuous-time LQG/LTR (loop transfer recovery) regulator may not be robust in the time domain even though they are known to be robust in the frequency domain [7,8]. Therefore the stability robustness bounds against parameter variations are important and these bounds were obtained for CLQR [9] and the continuous-time LQG regulator [10]. In the discrete-time case, however, the bounds have not been investigated. In this paper, the stability robustness bounds in the time domain for DLQR are obtained and their properties are investigated. The relationship between the stability robustness bounds in the time domain and the minimum singular value of the return difference matrix is not known for CLQR, but will be discussed for DLQR in this paper.

This paper is organized as follows. In Section II, the method to increase the minimum singular value of the return difference matrix for the stability robustness in the frequency domain is suggested. In Section III, the stability robustness bounds in the time domain are

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obtained and their properties are investigated. Conclusions are given in Section IV.

II. Stability Robustness of DLQR in the Frequency Domain

The stability robustness in the frequency domain is closely related to the return difference matrix. The return difference matrix $F(z)$ of the closed loop DLQR is given by

$$F(z) = I_m + K(zI_n - A)^{-1}B \tag{1}$$

where K is a constant feedback gain of the DLQR which is defined as

$$K = (R + B^TPB)^{-1}B^TPA. \tag{2}$$

From I(4) and II(1) we can obtain the following equation.

$$\begin{aligned} &F^*(z)(R + B^TPB)F(z) \\ &= R + B^T(z^{-1}I_n - A^T)^{-1}Q(zI_n - A)^{-1}B \end{aligned} \tag{3}$$

where $[\ \cdot\ ]^*$ denotes the conjugate transpose of $[\ \cdot\ ]$. From this equation we can also obtain the following inequality where $\sigma_1[\ \cdot\ ]$ and $\sigma_m[\ \cdot\ ]$ denote the maximum and minimum singular values of $[\ \cdot\ ]$, respectively.

$$\begin{aligned} \sigma_m[F^*(z)F(z)] \geq \sigma_m[&R + B^T(z^{-1}I_n - A^T)^{-1}Q(zI_n \\ &- A)^{-1}B] / \sigma_1[R + B^TPB] \end{aligned} \tag{4}$$

We define $\mu_F^2 = \min_{|z|=1} \{ \sigma_m[R + B^T(z^{-1}I_n - A^T)^{-1} Q(zI_n - A)^{-1}B] / \sigma_1[R + B^TPB] \}$. It is known that with respect to model uncertainty L(z) of I(5), the closed loop system is stable under some conditions if $\sigma_1[L^{-1}(z) - I] < \mu_F$ [1] and it is guaranteed that the discrete time LQR has at least the gain margin of $[1/(1+\mu_F), 1/(1-\mu_F)]$ and the phase margin over $\cos^{-1}(1-0.5\mu_F^2)$ [2]. The larger μ_F is, the larger L(z) can be allowed. However it is known that in the single input case, $\sigma_m[F(z)]$ is less than or equal to 1 and the equality holds for zero feedback only [4] and this fact can be extended to the multi input case as follows.

Lemma 1 : $\min_{|z|=1} \sigma_m[F(e^{j\omega})] \leq 1$ and the equality holds for zero feedback only.

The proof of this lemma is omitted for brevity. From this lemma, we can know that the discrete time LQR has the poor stability robustness in the frequency domain. But we can improve the stability robustness by making μ_F as large as possible. To make μ_F large, we must know the property of the solution P of the ARE I(4) in terms of weighting matrices.

Assume that $Q = \alpha Q_0$ and $(Q_0^{1/2}, A)$ is an observable pair. Then P is a monotonic increasing function of α [11,15] and the increasing rate $dP/d\alpha$ is less than or equal to P [11]. By using the above properties of P, we can obtain the following theorem.

Theorem 1 : Assume that $Q = \alpha Q_0$ and $(Q_0^{1/2}, A)$ is an observable pair where $\alpha > 0$. Then the following statements hold.

(1) The smaller α is, the larger μ_F.

(2) When the system matrix A is stable, $\sigma_m[F(z)]$ goes to 1 as α tends to zero.

Proof : (1) Without loss of generality, we can assume that $R = I_m$. From the definition of μ_F,

$$\begin{aligned} \mu_F^2 = \min \sigma_m &[I_m + B^T(z^{-1}I_n - A^T)^{-1}Q(zI_n - A)^{-1}B] \\ &/ \sigma_1[I_m + B^TPB], \quad |z| = 1. \end{aligned}$$

It is well known that $P \geq Q$. This fact implies that $dP/d\alpha \geq Q_0$. So, for any normalized $x \in R^m$,

$$\begin{aligned}
&\frac{d}{d\alpha}\sigma_1[I_m + B^TPB] - \frac{d}{d\alpha}\{\min_{|z|=1}\sigma_m[I_m + B^T(z^{-1}I_n - A^T)^{-1}Q(zI_n - A)^{-1}B]\} \\
&\geq \frac{d}{d\alpha}\sigma_1[B^TQB] - \frac{d}{d\alpha}\{\min_{|z|=1}\sigma_m[B^T(z^{-1}I_n - A^T)^{-1}Q(zI_n - A)^{-1}B]\} \\
&= \sigma_1[B^TQ_0B] - \min_{|z|=1}\sigma_m[B^T(z^{-1}I_n - A^T)^{-1}Q_0(zI_n - A)^{-1}B] \\
&\geq \sigma_1[B^TQ_0B] - \sigma_m[B^T(-jI_n - A)^{-1}Q_0(jI_n - A)^{-1}B] \\
&\geq x^T[B^TQ_0B - B^T(-jI_n - A^T)^{-1}Q_0(jI_n - A)^{-1}B]x \\
&= x^TB^T(-jI_n - A^T)^{-1}[(-jI_n - A^T)Q_0(jI_n - A) - Q_0](jI_n - A)^{-1}Bx \\
&= x^TB^T(-jI_n - A^T)^{-1}A^TQ_0A(jI_n - A)^{-1}Bx \geq 0.
\end{aligned}$$

This implies that

$$\frac{d}{d\alpha}\sigma_1[I_m + B^TPB] \geq \frac{d}{d\alpha}\{\min_{|z|=1}\sigma_m[I_m + B^T(z^{-1}I_n - A^T)^{-1}Q(zI_n - A)^{-1}B]\}$$

Thus μ_F increases as α decreases.

(2) It is known that P monotonically tends to zero as α goes to zero when the system matrix A is stable. Thus both B^TPB and $B^T(z^{-1}I_n - A^T)^{-1}Q(zI_n - A)^{-1}B$ tend to zero as α goes to zero. From the equation II(3), $\sigma_m[F(z)]$ tends to 1 as α tends to zero because R is diagonal. ■

From Theorem 1, the minimum singular value of the return difference matrix of the discrete time LQR can be made large if the ratio of Q to R becomes small. Thus the stability robustness with respect to model uncertainty L(z) of I(5) can be improved by decreasing the ratio of Q to R. This fact is illustrated by the following example.

Example 1: We consider a system and weighting matrices as follows.

$$A = \begin{bmatrix} 9.512\times10^{-1} & -2.529\times10^{-2} & -1.624\times10^{-2} \\ 9.385\times10^{-2} & 8.673\times10^{-1} & -2.393\times10^{-1} \\ 1.264\times10^{-2} & 2.309\times10^{-1} & 6.363\times10^{-1} \end{bmatrix},$$

$$B = \begin{bmatrix} 4.803\times10^{-3} & 9.385\times10^{-2} \\ -1.294\times10^{-2} & 5.244\times10^{-3} \\ 1.065\times10^{-1} & 6.825\times10^{-2} \end{bmatrix},$$

$Q = \alpha \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, and $R = \beta\ diag(1,1)$, where A is sta ble. When α/β is $10^8, 10^4, 1$ and 10^{-4} the minimum singular value of the return difference matrix is 0.16806, 0.50335, 0.96951, and 1, respectively. From each minimum singular value, we obtain the gain margin of [0.8561, 1.2020], [0.6652, 2.0135], [0.5077, 32.796], and [0.5, 212370], respectively, and the phase margin of 23.656°, 41.555°, 58.986°, and 60°, respectively.

From the above example, it is noted that the minimum singular value of the return difference matrix and the gain and phase margins increase monotonically as the ratio of Q to R decreases and converge to 1, $[0.5, \infty)$ and 60°, respectively.

Ⅲ. Stability Robustness Bounds of LQR in the time domain

Now we consider the robustness against modeling errors in the time domain. We assume modeling errors δA and δB in A and B, respectively. Then the real plant is represented by the equation I(6). We can state the following theorem.

Theorem 2 : Assume that the control I(3) is applied to the real plant I(6). Then the resulting closed loop system is asymptotically stable if the following inequalities hold.

$$\sigma_1(\delta A) < \mu_A \quad when\ \delta B = 0 \qquad (1)$$

$$\sigma_1(\delta B) < \mu_B \quad when\ \delta A = 0 \qquad (2)$$

$$\sigma_1(\delta A - \delta BK) < \mu_A \qquad (3)$$

where

$$\mu_A = 1/[\sigma_1(P_L A_C) + \{\sigma_1^2(P_L A_C) + \sigma_1(P_L)\}^{1/2}],$$

$\mu_B = \mu_A/\sigma_1(K)$, $A_C = A - BK$, K is given by Ⅱ(2), and P_L is the positive definite solution of a Lyapunov equation :

$$A_C^T P_L A_C - P_L = -I_n. \qquad (4)$$

Proof : Let's choose a candidate of the Lyapunov function as $V_k(X_k) = x_k^T P_L x_k$. Then

$$\begin{aligned}\delta V_k(x_k) &= V_{k+1}(x_{k+1}) - V_k(x_k)\\ &= x_k^T(A_C + \delta A - \delta BK)^T P_L (A_C + \delta A - \delta BK)x_k - x_k^T P_L x_k\\ &\le [\sigma_1(P_L)\sigma_1^2(\delta A - \delta BK) + 2\sigma_1(P_L A_C)\sigma_1(\delta A - \delta BK) - 1]\ x_k^T x_k\end{aligned}$$

Thus $\delta V_k(x_k)$ is negative if the inequality Ⅲ(3) holds. When $\delta B = 0$,

$$\begin{aligned}\delta V_k(x_k) &= x_k^T(\delta A^T P_L \delta A + \delta A^T P_L A_C - I_B)x_k\\ &\le [\sigma_1(P_L)\sigma_1^2(\delta A) + 2\sigma_1(P_L A_C)\sigma_1(\delta A) - 1]\ x_k^T x_k.\end{aligned}$$

The right hand side of this inequality is negative if the inequality Ⅲ(1) holds. When δA=0,

$$\begin{aligned}\delta V_k(x_k) &= x_k^T(K^T \delta B^T P_L \delta BK + 2K^T \delta B^T P_L A_C - I_n)x_k\\ &\le [\sigma_1(P_L)\ \sigma_1^2(K)\ \sigma_1^2(\delta B) + 2\sigma_1(P_L A_C)\sigma_1(K)\sigma_1(\delta B) - 1]\ x_k^T x_k.\end{aligned}$$

The right hand side of this inequality is negative if the inequality Ⅲ(2) holds. ■

μ_A and μ_B are functions of the feedback gain K and K is a function of the ratio of Q to R. In order to explain properties of μ_A and μ_B, we must know the relationship between K and the ratio of Q to R. It is well-known that K is finite even though the ratio of Q to R tends to infinity. The following lemma states a monotonic property of K.

Lemma 2 : Assume that $Q = \alpha Q_0$ and $(Q_0^{1/2}, A)$ is a observable pair where $\alpha > 0$. Then the non zero singular values of K are nondecreasing functions of α when the system matrix A is nonsingular.

Proof : Since the non-zero singular values of K are the square roots of eigenvalues of $Y = KK^T$, we only show that $dY(\alpha)/d\alpha \ge 0$. From the equation Ⅱ(2)

$$\begin{aligned}\frac{dY(\alpha)}{d\alpha} &= (R + B^T PB)^{-1} B^T [\frac{dP}{d\alpha}\{I_n - B(R + B^T PB)^{-1} AA^T P\} + PAA^T\{I_n - PB(R + B^T PB)^{-1} B^T\}\frac{dP}{d\alpha}]\ B(R + B^T PB)^{-1}\\ &= (R + B^T PB)^{-1} B^T [\frac{dP}{d\alpha}(I_n + BR^{-1}B^T P)^{-1} B^T)\} AA^T P + PAA^T (I_n + PBR^{-1}B^T)^{-1} \frac{dP}{d\alpha}]\ B(R + B^T PB)^{-1}.\end{aligned}$$

Let

$$T = P^{1/2},\ U = (I_n + P^{1/2}BR^{-1}B^T P^{1/2})^{-1/2},$$

$$S = UT^{-1}(dP/d\alpha)T^{-1}U, \text{ and } W = UTAA^T TU,$$

then

$$\begin{aligned}\frac{dY(\alpha)}{d\alpha} &= (R + B^T PB)^{-1} B^T TU^{-1} [UT^{-1}\frac{dP}{d\alpha} T^{-1}UUTAA^T TU + UTAA^T TUUT^{-1}\frac{dP}{d\alpha} T^{-1}U]\ U^{-1}TB(R + B^T PB)^{-1}\\ &= (R + B^T PB)^{-1} B^T TU^{-1} [SW + WS]\ U^{-1}TB(RB^T PB)^{-1}\\ &= (R + B^T PB)^{-1} B^T TU^{-1} [SWS + W - (I_n - S)W(I_n - S)]\ U^{-1}TB(R + B^T PB)^{-1}.\end{aligned}$$

Since $S \ge 0$ and $I_n - S = UT^{-1}(PBR^{-1}B^T P + P - dP/d\alpha)T^{-1}U \ge 0$, $0 \le I_n - S \le I_n$. W is positive-definite because A is nonsingular and both U and T are positive definite. Both $W > 0$ and $0 \le I_n - S \le I_n$ imply that $W - (I_n - S)W(I_n - S) \ge 0$. Therefore $\frac{dY(\alpha)}{d\alpha}$ is positive semi-definite. ■

From Theorem 2 and Lemma 2, it is noted that 1) unlike the continuous-time case, both μ_A and μ_B are not necessarily small even though the ratio of Q to R tends to infinity, 2) the smaller $\sigma_1(P_L)$ and $\sigma_1(A_C)$ are, the larger μ_A is, and 3) when the system matrix A is stable, the smaller $\sigma_1(K)$ is, the larger μ_B is.

It is known that the closed loop poles of the discrete-time LQR tend to the inside the unit circle images of the open loop zeros or the origin as the ratio Q to R approaches infinity [12]. If there is no finite open loop zero, the closed loop poles go to the origin as α tends to infinity. Since $\sigma_1(A_c) = \{\lambda_1(A_c^T A_c)\}^{1/2}$, it decreases as the eigenvalues of A_c go to zero. It is also known that the solution of the Lyapunov equation III(5) can be written as [13]

$$P_L = \sum_{k=0}^{\infty} (A_c^T)^k A_c^k \tag{5}$$

So $\sigma_1(P_L)$ decreases as the eigenvaluese of A_c go to zero and it is finite even for very small K if the system matrix A stable. From these facts and Lemma 2, we can obtain the following corollary.

Corollary 1 : 1) μ_A increases as the ratio of Q to R increases if there is no finite open loop zero.
2) μ_B increases as the ratio of Q to R decreases if the system matrix A is stable.

The results in Corollary 1 can be utilized to increase robustness against parameter variations. It is noted that both μ_L defined in section II(4) and μ_B have the same property with respect to weighting matrices, i.e. those increase as the ratio of Q to R decreases. Therefore the minimum singular value of the return difference matrix is used as a measure for robustness when the modeling error exists in the input matrix only. When both δA and δB are present simultaneously, it is not clear whether the robustness increases or not since both K and μ_A increase as the ratio of Q to R increases. But the robustness when both modeling errors exist at the same time is guaranteed to some extent since K is finite even when the ratio of Q to R is very large. The properties of Corollary 1 are illustrated by the following example.

Example 2 : We consider a system and weighting matrices as follows :

$$A = \begin{bmatrix} 0 & 1 \\ 0.56 & 0.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$C = [\ 1\ 0\ ], \quad Q = \alpha C^T C, \text{ and } R = 1.$$

Then open loop poles are 0.8 and −0.7 and there is no open loop zero. When α = 0.01, 1, 100 and 10000, μ_A = 0.19435, 0.33270, 0.36601, and 0.36603, μ_B = 21.5140, 1.06800, 0.64971, and 0.64350, respectively.

From this example, we know that μ_A increases and μ_B decreases as α increases. But unlike the continuous time case, μ_B is not necessarily small even for large α.

IV. Conclusions

In this paper, the general methods to improve the stability robustness of the discrete time LQR in both the frequency domain and the time domain were suggested. By using several properties of the ARE solution, it was shown that the minimum singular value of the return difference matrix can be increased by decreasing the ratio of Q to R. By using this method, the gain margin of [0.5, ∞) and phase margins of 60o can be achieved in the discrete time LQR with specific weighting matrices if the system matrix A is stable.

For the robustness against modeling errors, it was shown that unlike the continuous time LQR, the robustness for both δA and δB is not necessarily poor even for large Q or small R. It was also shown that the stability robustness bound for δA increases as the ratio of Q to R goes to infinity if the open loop system has no finite zero. The stability robustness bound for δB increases are the ratio of Q to R decreases if the system matrix A stable. It is noted that under this condition, the method to improve the stability robustness bound for δB is the same as the method to increase the minimum singular value of the return difference matrix.

The methods to improve the stability robustness in the frequency and time domain of this paper are believed to be useful for the robust design of the discrete time LQR. The discrete time LQR/LTR method [14] is rarely used because of the poor robustness of the discrete time LQR, but it can be useful if the LQR is designed by the methods suggested in this paper. The properties of the stability robustness bounds when δA and δB exist at the same time need to be investigated in detail.

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