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무작위 오류가 있는 선형 어레이의 최적 광대역 영점 형성
Synthesis of Optimum Broadband Null in Randomly Perturbed Linear Arrays

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요약/ABSTRACT

광대역 방해신호가 수신되는 센서 또는 안테나 어레이에는 어레이 성능을 향상시키기 위하여 각 방해신호 방향에 광대역 영점(null)을 형성하는 것이 바람직하다. 광대역 영점을 효과적으로 형성하는 한 방법은 미분 영점을 사용하는 것이다. 어레이 계수와 감지기 위치의 무작위 오류가 있는 어떤 조건이 주어진 선형 어레이에서의 미분영점의 성능을 논의하며 컴퓨터 실험 결과를 제시한다.

If broadband interference signals are incident at an array of sensor/antenna elements, it is desirable to form a set of broadband nulls around the directions of the interference signals to improve the array performance. An efficient way of generating a broadband null is to synthesize a derivative null. The performance of a derivative null in a constrained linear array which is subjected to random variations with respect to array weight and element position is discussed. Computer simulation results are presented.

INTRODUCTION

An array of antenna/sensor elements yields an improved directivity compared to a single element and is efficiently used to estimate a desired signal corrupted by undesired interference signals. Array processing techniques have been applied in many areas which include seismology [1], radar [2], and sonar [3]. When random errors with respect to array weight, element position, or input signal wave exist, their effects on the array performance should be considered to obtain a more practical solution. Maximization of the exepected directivity or signal to noise ratio, and minimization of output power with/without constraints have been widely discussed [4-7]. It was shown that the array performance was degraded as the variance of random errors increases. Also, It was shown that a derivative null constraint in the sidelobe yielded a broadband null appropriate for eliminating a broadband interference signal [8,9].

In this paper, the concept of a pattern derivative is introduced in an optimal sense in a randomly perturbed linear array with equispaced isotropic elements. The effects of the random variations of array weight and element position on the expectation of the first-order derivative of array factor and its power response are analyzed and employed in finding an optimum weight solution for a broadband nulling problem. Without loss of generality, it is assumed that the average positions of elements are confined to a one-dimensional space, the directions of incoming signals are confined to a two-dimensional space, and the positions vary randomly in a three-dimensional space.

In a narrowband linear array with N equispaced isotropic elements on the x-axis in the three-dimensional space, the random variations of array weight and element position can be expressed as

$$w_n = c_n + x_n \tag{1}$$

and

$$d_n = ndu_x + \rho_n, \qquad \text{for } 1 \le n \le N, \tag{2}$$

where the c_n are the average values of randomly perturbed complex weights w_n, ndu_x are the average element positions, d_n are randomly perturbed element positions, d is the inter-element spacing with no random error, u_x is a unit vector on the x-axis, and x_n are independent random complex variables for weight errors with mean zero, and ρ_n are independent random vectors for position errors with all having the same statistical distributions. If it is assumed that the cartesian components of ρ_n are of independent normal distribution with mean zero and variances of $\sigma^2/3$, the marginal probability density function for each component is given by

$$p(\rho_c) = \frac{\sqrt{3}}{\sqrt{2\pi}\,\sigma} e^{\frac{-3\rho_c^2}{2\sigma^2}}, \qquad \text{for}$$

$$c = xn,\ yn,\ \text{and } zn. \tag{3}$$

Also it is assumed that all the array elements are identical and distortionless and the incoming signals are plane waves. If random variations are not present in the array, the array factor is given by

$$H_n(u) = \sum_{n=1}^{N} c_n e^{-jkndu}, \tag{4}$$

where $u = \cos\theta$, θ is an angle from the array axis $k=\frac{2\pi}{\lambda}$, λ is the wavelength of the incoming signals, and $j=\sqrt{-1}$. It can be shown that the power of the nominal (i.e., with no random errors) array factor is expressed as

$$|H_n(u)|^2 = c^H P c, \quad (5)$$

where P is an autocorrelation matrix of a steering vector p given by

$$p = [\ 1 \ \ e^{jkdu} \cdots e^{jk(N-1)du}\ ]^T, \quad (6)$$

$$c = [\ c_1 \ c_2 \cdots \ c_N\ ]^T \quad (7)$$

and T and H denote transpose and complex conjugate transpose, respectively. If random variations with respect to array weight and element position are present, the array factor is given by

$$H(u) = \sum_{n=1}^{N} w_n e^{-jk(ndu + \rho_{xn} u + \rho_{yn}\sqrt{1-u^2})} \quad (8)$$

It can be shown that the average array output power is given by

$$E[\ |H(u)|^2] = c^H \hat{P} c, \quad (9)$$

where

$$\hat{P} = \frac{1}{1+\delta^2}(rI + P), \quad (10)$$

$$r = \delta^2 + \gamma^2 + \delta^2\gamma^2, \quad (11)$$

$$\delta^2 = e^{-\frac{k^2\sigma^2}{3}} - 1, \quad (12)$$

$$E[\ |x_n|^2] = \gamma^2 |c_n|^2, \ \text{for } 1 \le n \le N, \quad (13)$$

I is an $N \times N$ identity matrix. In (13), it is assumed that the variance of x_n is proportional to the power of c_n with the ratio of γ^2 for all n. It is to be noted that the r in (11) comprehensively represents the power of relevant errors.

A simple way of synthesizing a broadband null is to put multiple point nulls within a spatial region of interest. To form a L point nulls at $u_1, u_2, \cdots, u_L$ with a unit gain constraint at u_c, we solve the following constrained optimization problem.

$$\min_{c} \quad E[\ |H(u)|^2]$$
$$\text{subject to} \quad E[\ p_{rc}^H w\ ] = 1, \quad (14)$$

where p_{rc} is a perturbed steering vector for the look direction (i.e., the direction of a desired signal), whose nth component is given by

$$[\ p_{rc}\ ]_n = e^{jk_c(ndu_c + \rho_{xn} + \rho_{yn}\sqrt{1-u_c^2})} \quad (15)$$

and

$$w = [\ w_1 \ w_2 \cdots \ w_N\ ]^T. \quad (16)$$

It can be shown that the constraint in (14) is evaluated as

$$p_c^H c = \sqrt{1+\delta^2}, \quad (17)$$

where p_c is a nominal steering vector for the look direction, i.e.,

$$[p_c]_n = e^{jk_c ndu_c}, \quad (18)$$

c is a nominal weight vector given by $u_c = \cos\theta_c$, and θ_c is the incident angle of a desired signal.

Using the method of Lagrange multipliers, it can be shown that the optimum weight vector is given by

$$c_{opt} = \frac{\sqrt{1+\delta^2}\,\underline{P}^{-1}\,p_c}{p_c^{H}\,\underline{P}^{-1}\,p_c}, \tag{19}$$

where

$$\underline{P} = \hat{P} + Lr\,I, \tag{20}$$

$$\hat{P} = \tilde{p}\,\tilde{p}^{H}, \tag{21}$$

$$\tilde{p} = [\ p_1\ \ p_2\ \cdots\ p_L\ ]\ , \tag{22}$$

and

$$p_l = [\ 1\ e^{jkdu_l} \cdots e^{jk(N-1)du_l}\ ]^{T} \tag{23}$$

It is assumed that the inverse of $\underline{P}$ exists. Multiple point nulls are experimented with a 16-element linear array with $r = 0.001$. The beam pattern with 11 point nulls between 40 ° and 45 ° is shown in Fig. 1. It is observed that the width and depth of the broadband null are 6 ° and -66 db, respectively. Synthesis of a broadband null by the point null requires more point nulls

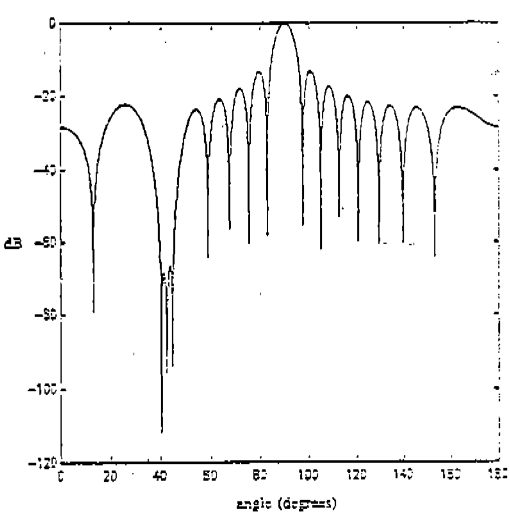

Fig. 1 Beam pattern for a 16-element linear array with 11 point nulls for 40 ° and 45 °.

to achieve a better performance, which results in more computaional load. A simple way of forming a broadband null is to use a pattern derivative.

AVERAGE PATTERN DERIVATIVE

The first-order derivative of the nominal array factor with respect to the directional parameter u is given by

$$\frac{dH_n(u)}{du} = p_d^{H}\,c, \tag{24}$$

where

$$p_d = [\ jkde^{jkdu}\ \ jk2de^{jk2du}\ \cdot\cdot\ jkNde^{jkNdu}]^{T} \tag{25}$$

Also, the nominal squared derivative is expressed as

$$\left|\frac{dH_n(u)}{du}\right|^2 = c^{H}P_d\,c, \tag{26}$$

where the nth row and mth column of P_d matrix is given by

$$[\ P_d]_{nm} = (kd)^2\,nm\,e^{k(n-m)du}. \tag{27}$$

When the random variations are present, the first-order derivative of a perturbed array factor is

$$\frac{dH(u)}{du} = \sum_{n=1}^{N} w_n(-jk)\left(nd + \rho_{xn} - \rho_{zn}\frac{u}{\sqrt{1-u^2}}\right) e^{-jk(ndu + \rho_{xn}u + \rho_{zn}\sqrt{1-u^2})} \tag{28}$$

Taking the expectation of (28) and comparing the result with (24), we get the following relationship.

$$E\left[\frac{\partial H(u)}{\partial u}\right] = \frac{1}{\sqrt{1+\delta^2}}\frac{\partial H_n(u)}{\partial u}. \qquad (29)$$

To find the average squared derivative of the array factor, we first evaluate the squared derivative as

$$\left|\frac{dH(u)}{du}\right|^2 = \sum_{n=1}^{N}\sum_{m=1}^{N} w_n w_m^* k^2\left[nd+\rho_{nx}-\frac{u}{\sqrt{1-u^2}}\rho_{nz}\right]\left[md+\rho_{mx}-\frac{u}{\sqrt{1-u^2}}\rho_{mz}\right] e^{-jk[(n-m)du+(\rho_{nx}-\rho_{mx})u+(\rho_{nz}-\rho_{mz})\sqrt{1-u^2}]} \qquad (30)$$

Taking the expectation of (30), we get

$$E\left[\left|\frac{\partial H(u)}{\partial u}\right|^2\right] = c^H \hat{P} c, \qquad (31)$$

where the nth row and mth column of $\hat{P}$ is given by

$$[\hat{P}]_{nm} = \begin{cases} (kd)^2 n^2 (1+\gamma^2) & n = m \\ (kd)^2 nm \dfrac{e^{jk(n-m)du}}{1+\delta^2} & n \neq m, \\ 1 \leq n, m \leq N. \end{cases} \qquad (32)$$

Rearranging (32), $\hat{P}$ can be expressed as

$$\hat{P} = \frac{1}{1+\delta^2}\{P_d + (kd)^2\gamma \qquad (33)$$

$$diag[1, 2^2, 3^2, \cdots, N^2]\}$$

and $diag[\cdots]$ denotes a diagonal matrix whose diagonal elements are the contents of the brackets. If multiple broadband interference signals are coming from different directions, we need to form a set of derivative nulls at the angular regions of the interference signals.

OPTIMUM WEIGHT VECTOR FOR MULTIPLE DERIVATIVE NULLS

If L pattern derivatives with gains of $\alpha_1, \alpha_2, \cdots, \alpha_L$ are synthesized at $u_1, u_2, \cdots, u_L$ with a constraint gain β at the look direction, we need to solve the problem of minimizing the squared norm of the error vector between the derivative gains and the derivatives of relevent array factors at the specified locations with a constraint gain at the look direction, which is formulated as

$$\min_{c} E[\,|\varepsilon|^2\,]$$

$$\text{subject to } E[\,p_{rc}^H w\,] = \beta \qquad (34)$$

where

$$\varepsilon = \alpha - p_o^H w, \qquad (35)$$

$$[p_{rc}]_n = e^{jk_c(ndu_c+\rho_{nx}+\rho_{nz}\sqrt{1-u_c^2})}, \qquad (36)$$

$$\alpha = [\alpha_1 \ \alpha_2 \cdots \alpha_L]^T, \qquad (37)$$

$$p_o = [p_{o1} \ p_{o2} \ldots p_{oL}], \qquad (38)$$

the nth component of the column vectors

p_{ol} is given by

$$[p_{ol}]_n = jk_{ol}\left(nd + \rho_{xn} - \rho_{zn}\frac{u_{ol}}{\sqrt{1-u_{ol}^2}}\right)e^{jk_{ol}(ndu_{ol}+\rho_{xn}u_{ol}+\rho_{zn}\sqrt{1-u_{ol}^2})} \tag{39}$$

$u_{ok} = \cos\theta_{ok}$, and k_c and k_{ol} are k for the constrained and the lth pattern derivative directions, respectively. Assuming that c and its complex conjugate c^* are independent each other [10] and using the method of Lagrange multipliers, the optimum weight vector is given by

$$c_{opt} = \frac{1}{\sqrt{1+\delta^2}}\hat{P}^{-1}\left\{ p_o a + \frac{[(1+\delta^2)\beta - \hat{p}_o^H \hat{P}^{-1} p_o a]\ \hat{p}_c}{\hat{p}_c^H \hat{P}^{-1}\hat{p}_c} \right\} \tag{40}$$

It is assumed that $\hat{P}$ is nonsingular in (40). For derivative nulls to be synthesized, a need to be set to a null vector.

OPTIMUM WEIGHT VECTOR FOR A SINGLE DERIVATIVE NULL

If a broadband interference signal is coming from a non-look direction (i.e., a direction different from that of a desired signal), we need to form a single derivative null. Thus the a and p_o in (37) and (38) become a scalar and an $N\times 1$ steering vector, respectively. Using a matrix inversion lemma [11], the inverse of $\hat{P}$ is evaluated as

$$\hat{P}^{-1} = \frac{1+\delta^2}{(kd)^2 r}\, diag\left[1, \frac{1}{2}, \cdots, \frac{1}{N}\right] J\, diag\left[1, \frac{1}{2}, \cdots, \frac{1}{N}\right] \tag{41}$$

where

$$J = I - \frac{p_o p_o^H}{N + r} \tag{42}$$

and the nth component of p_o is given by

$$[p_o]_n = e^{jk_o ndu_o}, \quad \text{for } 1 \le n \le N. \tag{43}$$

Substituting (41) into (40), with α=0 and β=1, the optimum weight vector which yields a minimum mean squared error between a zero response and the first-order derivative of the array factor at u_o with a unit gain constraint at u_c is given by

$$c_{c,opt} = \frac{\sqrt{1+\delta^2}\, diag\left[1, \frac{1}{2}, \cdots, \frac{1}{N}\right] J\, diag\left[1, \frac{1}{2}, \cdots, \frac{1}{N}\right] p_c}{\sum_{n=1}^{N}\frac{1}{n^2} - \frac{|\hat{p}_o^H \hat{p}_c|^2}{N + r}} \tag{44}$$

where

$$[\hat{p}_o]_n = \frac{p_o}{\sqrt{n}}, \tag{45}$$

$$[\hat{p}_c]_n = \frac{p_c}{\sqrt{n}}, \tag{46}$$

and

$$[p_c]_n = e^{jk_c ndu_c}, \quad \text{for } 1 \le n \le N. \tag{47}$$

From (44), it is shown that the optimum weight vector is obtained by scaling operations with the diagonal matricies and a nonothogonal projection with the J matrix. Each component of the steering vector p_c is scaled by $1/n$, $1 \le n \le N$ and nonothogonally projected onto the p_0, and thus the resulting vector is a slanted version of the complement of p_0. Then the components of the resulting vector are again scaled by $1/n$ factor and a relevant constant such that the unit gain constraint is satisfied.

SIMULATION RESULTS

A 3-sensor linear array is used to evaluate the performance of the derivative null constraint. It is assumed that the spacing between neighboring elements d is half the wave length corresponding to the array center frequency and the frequencies of the desired and interference signals f_0 and f_c are the same as the array center frequency. The derivative gain α at the interference direction is set equal to zero and the constraint gain β is assumed to be $1/\sqrt{1+\delta^2}$ such that a unit gain is maintained at the look direction. Then the matrix $\hat{P}$ is given by

$$\hat{P} = \begin{pmatrix} 1+r & 2e^{-\pi u_o} & 3e^{-2\pi u_o} \\ 2e^{\pi u_o} & 4(1+r) & 6e^{-\pi u_o} \\ 3e^{2\pi u_o} & 6e^{\pi u_o} & 9(1+r) \end{pmatrix} \quad (48)$$

When the incident angles of the desired and broadband interference signals are 90° and 60° from the array axis, respectively, the beam patterns for r = 0.1, 0.01, 0.001 are plotted in Fig. 2. It is observed that each beam pattern is almost the same, i.e., the derivative null is not much sensitive to random variations. The reason for this phenomenon is due to the fact that the two diagonal matrices rotate the steering vector p_c such that the nonorthogonal projection does not affect the overall performance significantly as indicated in (44). One way to improve the broadband nulling performance is to add a point null to the derivative null. This can be done by modifying the correlation matrix in such a way that it is added by the correlation matrix related to a point null. Then the resulting correlation matrix is given by

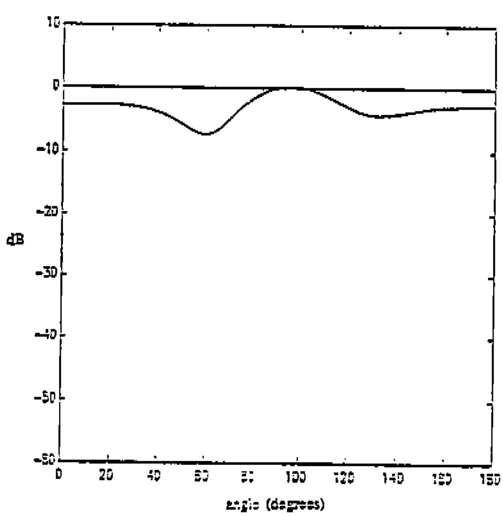

Fig. 2 Overlapped beam patterns: a 3-element linear array by a derivative null at 60° for r = 0.1, 0.01, 0.001.

$$P = \hat{P} + P_o + rI, \tag{49}$$

where

$$P_o = pp^H, \tag{50}$$

and p is the same as in (6). The resulting beam patterns are shown in Fig. 3. It is shown that the broadband nulling performance improves as the random error decreases. It is to be noted that if a higher-order derivative null is added, the null width becomes broader.

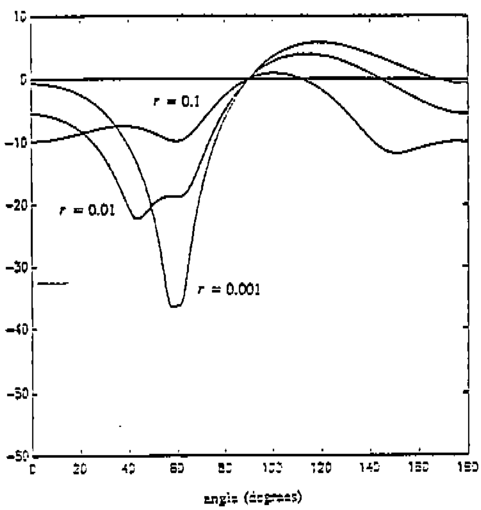

Fig. 3 Overlapped beam patterns: a 3-element linear array by a derivative null plus a point null at 60 ° for r = 0.1, 0.01, 0.001.

CONCLUSIONS

The performance of the pattern derivative is discussed in a linear array which is subjected to random variations of array weight and element position. A signle and multiple derivative nulls are synthesized with a constraint gain at the look direction to counteract broadband interference signals. It was shown that the derivative null was not sensitive to the random errors. The nulling performance improved by employing the derivative null with a point null such that the derivative null becomes deeper as the random variations decrease.

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