### 복합 적응 어레이 처리기의 성능 Performance of a Modified Composite Array Processor

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### 요약/ABSTRACT

이 논문은 적응 복합 어레이 처리기에 공간과 주파수 영역에 미분 영점 (mull)조건을 사용한 개조복합 어레이 처리기에 대하여 서술한다. 방해신호와 원하는 신호의 주파수는 같으나 방향은 서로 다르며, 방해신호의 방향과 주파수는 알고 있다고 가정하였다. 컴퓨터 실험결과, 고계의 미분영점은 영점 폭을 넓게 하여 광대역 방해신호를 제거하는데 적합하며, 주파수 영역에서의 영계의 미분영점 (단순점 영점)은 복합어레이 처리기에 내재하는 여분의 오류신호를 줄여 어레이의 성능을 향상시킬 수 있음이 판명되었다.

This paper concerns the use of derivative null constraint in an adaptive array processor in the spatial and frequency domains with respect to a composite array processor to obtain a modified composite array processor. It is assumed that the frequency of interference signals is the same as that of a desired signal, interference directions are different from the desired signal, and interference directions and frequencies are known. Simulation results demonstrate that a higher-order derivative null broadens the null width which is appropriate for eleminating a broadband interference and a zero-order derivative null (i.e., a simple point null) with respect to frequency reduces the residual error inherent in the composite array processor.

### INTRODUCTION

An adaptive array processor consists of an array of elements (i.e., antennas/sensors) followed by an adaptive multichannel filter. The array of elements is steered by delaying element output signals to yield maximum gain at the look direction (i.e., the direction of a desired signal), while the coefficients of the multichannel filter are updated recursively such that nulls are created at the non-

look directions (i.e., the directions of interference signals). The application areas of adaptive array processing are seismology [1], radar [2], and sonar [3]. Power minimization subject to derivative constraints in the spatial domain was discussed in the litereature It was shown that a derivative null constraint in the main beam resulted in a flat beam in the look direction [4]. The derivative null constraint applied in the sidelobe was shown to broaden the null width [5]. If the interference directions are known, the constrained nulling technique provides a powerful tool for rejecting interference signals. When the interference directions are not known, the Frost beamformer [6] may be used assuming that the direction and frequency of the desired signal is known. However, the Frost beamformer has a signal cancelation problem caused by signal and interference interaction during To seperate the signal from the process. interferences in the adaptive process, a masterslave type composite array processor was proposed by Duvall [7]. In this method, the signal components are eleminated by subtractive preprocessing and the resulting interferences are processed by the constrained LMS algorithm [6]. The weights independent of the effect of the desired signal in the master processor are copied to a slave processor and processed with time-adjusted input singuals to produce an array output. Even though signal cancelation is significantly reduced, some residual power variations from non-look direction interference signals in the master processor are present in the array output since the weights of the slave processor are the same as those of the master processor. An alternative way to rejecting non-look direction interferences more effectively is to create explicit nulls in the interference directions, as well as at interference frequencies. This is possible by combining pertinent derivative null constraints into the constrained LMS algorithm.

In this paper, assuming known interference directions and frequencies, derivative null constraints at frequencies as well as directions are formulated and implemented in the composite array processor to improve the nulling performance. Interference directions may be obtained using some estimation methods described in [8.9]. To find the performance of the proposed processor, a single and multiple derivative null constraints are simulated and their performance is compared with that of the composite array processor in the spatial and frequency domains. It is shown that the nulling capability is improved by adding higher-order derivative null constraints in the spatial as well as frequency domains.

## DERIVATIVE NULL CONSTRAINT

An M-element linear array with equally spaced elements in a two-dimensional space is considered. Each element is followed by a time delay and a multichannel tapped delay line (TDL) filter with N taps. The element spacing is expressed as

$$d = \alpha \lambda_c \tag{1}$$

where  $\alpha$  is a real number,  $\lambda_c (= c/f_c)$  is the wave length corresponding to the

array center frequency  $f_c$ , and c is the incident wave velocity. Let the array be illuminated by P plane waves at angles  $\theta_1, \theta_2, \cdots, \theta_P$  from the array axis. Each plane wave consists of Q frequency components,  $f_1, f_2, \cdots, f_Q$ 

The array factor in terms of incident angle and frequency is given by

$$H(\theta, f) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_{m+nM} e^{-i\frac{2\pi f}{f_c}(ma\cos\theta + \frac{nf_c}{f_t})},$$
(2)

where the  $a_{m+nM}$  are array weights,  $f_s$  is a sampling frequency, and  $j=\sqrt{-1}$ . Then the lth partial derivative of  $H(\theta,f)$  with respect to  $\theta$  for  $f=f_q$  is given by

$$\frac{a^{i}H(\theta, f_{0})}{a^{i}\theta} = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \left[ j \frac{2\pi f_{0}}{f_{c}} m\alpha \sin \theta \right]^{i} a_{m+nM}$$

$$e^{j\frac{2\pi f_{0}}{f_{c}}(m\alpha \cos \theta + \frac{nf_{c}}{f_{c}})}$$

$$q = 1, 2, \dots, Q.$$
(3)

Also the *l*th-order partial derivative of  $H(\theta, f)$  with respect to f for  $\theta = \theta_p$  is given by

$$\frac{\partial^{l}H(\theta_{b},f)}{\partial^{l}f} = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \left[ -j\frac{2\pi}{f_{c}}(m\alpha\cos\theta_{b} + \frac{nf_{c}}{f_{s}}) \right]^{l}$$

$$a_{m+nM} \cdot e^{-j\frac{2\pi}{f_{c}}(m\alpha\cos\theta_{b} + \frac{nf_{c}}{f_{s}})}.$$

$$p = 1, 2, \dots, P. \tag{4}$$

The lth-order derivative null constraint for l

= 1, 2,  $\cdot \cdot \cdot$ , L at  $\theta$ , and  $f_q$  is obtained by setting the lth-order partial derivatives of the array factor in (3) and (4) to zero at  $\theta = \theta$ , and  $f = f_q$ , which are given by

$$\frac{\partial^{l} H(\theta, f_{q})}{\partial^{l} \theta} |_{\theta = \theta} = 0$$
 (5)

and

$$|l=0,1,\cdots,L,$$

$$\frac{\partial^{l}H(\theta,p,f)}{\partial^{l}f}|_{f=f_{q}}=0 \quad \text{for } p=1,2,\cdots,P,$$

$$q=1,2,\cdots,Q.$$
(6)

Using (5) and (6), the derivative null constrains are combined into the constraint matrix and vector in the constrained LMS algorithm which will be discussed in the following section.

# MODIFIED COMPOSITE ARRAY PROCESSOR

It is assumed that a desired signal is incident from a known direction and its spectral characteristics are known while interference signals which are uncorrelated with the desired signal are coming from different directions from the desired signal and their spectra are unknown a priori. The adaptive array processor consists of an *M*-element linear array followed by a multichannel TDL filter with *N* taps per element. Each element output signal is time-delayed by some amount such that the desired signal

appears in phase one another in the time-delayed output. It is assumed that incident signals are plane waves and the elements are identical and distortionless. The problem is to extract the desired signal by filtering out the interference signals in both the spatial and frequency domains. To this end, the constrained LMS algorithm updates the multichannel TDL filter coefficients by minimizing the mean squared output under the unit gain constraint at the frequency band of the desired signal. Frost used the method of Lagrange multipliers and derived the following constrained LMS algorithm.

$$A(k+1) = D[A(k) - \mu y(k)X(k)] + G,$$
 (7)

where

$$D = I - C(C^{T}C)^{-1}C^{T}, (8)$$

$$G = C(C^TC)^{-1}F, (9)$$

$$A(0) = G, (10)$$

and A(k) is an  $MIN \times 1$  weight vector, an  $MN \times MN$  matrix D is a projection matrix, X(k) is the input signal vector, y(k) is  $MN \times MN$ I is an the output signal, indentity matrix, C is  $MN \times N$  constraint matrix, F is an  $N\times 1$  constraint vector,  $\mu$ Tis the convergence parameter, denotes matrix transpose. It is to be noted X(k) does not include the look that direction signal in the composite array processor. The ith column of C matrix is expressed as

and F is given by

$$F^T = [f_1 f_2 \cdot \cdot \cdot f_N], \qquad (12)$$

where  $f_n$ ,  $1 \le n \le N$  is the impulse response of a single TDL filter equivalent to that of the multichannel TDL filter which forms the look direction unit gain constraint. It is to be noted that the Frost beamformer based on the above constrained LMS algorithm have a signal cancellation phenomenon which is successfully avoided by the composite array processor.

The modified composite array processor is shown in Fig. 1 where a direction estimator for interference signals is added to the master processor in the composite array processor to estimate the interference arrival angle [8,9]. The estimated angles are used in the formulation of derivative null constraints in the master processor. The constraint matrix and vectors C and F are modified to include the derivative null constraints in the composite array processor. For a single

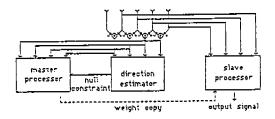


Fig. 1 Block diagram of modified composite array processor.

derivative null constraint, two columns are added to the constraint matrix C, each of which comes from the real and imaginary parts of (3) or (4), and two rows of zeros are added to the constraint vector F. Thus, if U derivative null constraints are added to the look direction unit gain constraint, C and F become an  $MN \times (N+2U)$  matrix and  $(N+2U) \times 1$  vector respectively. For the uth derivative null constraint with respect to  $\theta$ , the vth row component of the 1st and 2nd columns to be added is given by

$$C_{N+2u-1,v}^{T} = \left[ (v-m_1)^t \cos \left\{ -\frac{2\pi f_g}{f_a} \left( (v-m_1) a \cos \theta + \frac{nf_e}{f_e} \right) \right\} \right]$$

$$\tag{13}$$

and

$$C_{N+2u,v}^{T} = \left[ (v-m_1)^l \sin \left\{ -\frac{2\pi f_q}{f_c} \left( (v-m_1) a \cos \theta + \frac{nf_c}{f_i} \right) \right\} \right]$$

(14)

where n = mod(v-1,M) and  $m_1 = nM+1$ . For the uth derivative null constraint with respect to f, the vth row component of the 1st and 2nd columns to be added is given by

$$C_{N+2n-1,v}^{T} = \left[ \left[ (v-m_1) \alpha \cos \theta_{,i} + \frac{nf_c}{f_i} \right]^{i} \right]$$

$$\cos \left\{ \frac{2\pi f_c}{f_c} \left( (v-m_1) \alpha \cos \theta_{,i} + \frac{nf_c}{f_i} \right) \right\}$$
(15)

and

$$C_{N+2u,v}^{T} = \left[ \left[ (v-m_1) \alpha \cos \theta_{j} + \frac{nf_c}{f_i} \right]^{t} \\ \sin \left\{ \frac{2 \pi f_g}{f_c} \left( (v-m_1) \alpha \cos \theta_{j} + \frac{nf_c}{f_i} \right) \right\} \right].$$
 (16)

The constraint vector F for U constraints is given by

$$F^{T} = \begin{bmatrix} f_1 f_2 \cdot \cdot \cdot f_N & 00 \cdot \cdot \cdot 0 \\ \hline 2U & 0s \end{bmatrix}$$
 (17)

The maximum number of null constraints is limited by  $u < \frac{N(M-1)}{2}$ .

The remaining (N(M-1)-2u) weights are used to minimize the array output power. The filter weights in the master processor are updated iteratively with the preprocessed interferences via the constrained LMS algorithm in (7) with the modified C and F and then copied to the slave processor. The input signals which contain the time-aligned look-direction signal as well as the interferences are processed by the copied weights to produce the array output signal.

### SIMULATION RESULTS

The look direction signal consists of two sinusoids of 10 and 12 Hz and arrives at a direction normal to the array axis. Αп interference signal identical to the desired signal is incident at 51.32° from the array The array center frequency and axis. sampling frequency are 10 Hz and 160Hz, respectively. An equispaced 5-element (i.e., M = 4) linear array with 5 taps at each element is employed. The element spacing is half the wave length of the array center frequency (i.e.,  $\alpha = 0.5$  in (1)). It is assumed that the array elements are identical

and distortionless, and no steering errors are present. Also, it is assumed that the interference directions are estimated using an available bearing estimation technique. An all-pass filter is used for the constraint vector F, i.e.,

$$F^T = [10 \cdot \cdot \cdot 00], \tag{18}$$

which means that no interference signal is coming from the look direction frequency band is different from that of the The convergence parameter desired signal. is 0.001, and is normalized by a time-varying estamate of the input signal power. 1st, and 2nd-order derivative null constraints for frequency as well as angle Also the 0th order derivative were tested. null constraints at one and two frequencies were implemented in the interference directi-Fig. 2 shows the beam patterns at the 3001th sample for the composite array processor and modified one with the 0th, 0th plus 1st, and 0th plus 1st plus 2nd-order derivative null constraints in the interference direction. It is observed that as higher-order derivative null constraints are added, both the null width and depth increase. phenomenon is consistent with the result in [5], where the 0th, 1st, and 2nd order derivative null constraints are tested separately. The null with the largest width and depth is shown in (d) and occurs when the constraints up to the 2nd-order are employed. It is shown that the null widths at the gain of about -50 db are 0.4°, 0.4°, 4.2°, and 11.5° in (a), (b), (c), and (d). Also, the null depths in (a), (b), (c), and (d) are -51.1 db, -70.9 db, -135.6 db, and -144.5db, respectively, in the interference direction.

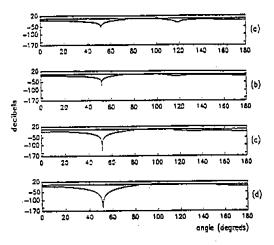


Fig. 2 Beam pattern for (a) composite array processor and modified processor with (b) the 0th, (c) 0th plus 1st, and (d) 0th plus 1st plus 2nd-order derivative null constraints at 51.32°.

To examine the effects of the derivative constraint in frequency domain, the 0th, 0th plus 1st, and 0th plus 1st plus 2nd-order derivative null constraints at the 10 Hz frequency component were implemented in the interference direction. The resulting frequency responses, in the interference direction are shown in Fig. 3. The null width as higher-order derivative null increases constrains are added, while the null depth is greater in (c) compared to (b), and less in (d) compared to (b) and (c). The null widths at the gains of about -50 db are 0.2 Hz, 0.8 Hz, and 1.7 Hz in (b), (c), and (d). Also, the null depths at 10 Hz in (a), (b), (c) and (d) are -10.2 db, -138 db, -142.7 db, and -118.9 db, respectively.

The nulling capability at multiple frequencies was experimented by placing a single

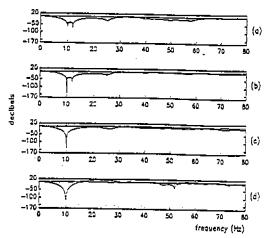


Fig. 3 Frequency responses in the interference dirction for (a) composite array processor and modified processor with (b) the 0th, (c) 0th plus 1st, and (d) 0th plus 1st plus 2nd-order derivative null constraints at 10 Hz.

0th-order derivative null constraint at 10 Hz, and two 0th order derivative null constraints at 10 and 12 Hz in the interference direction. Fig. 4 shows the corresponding frequency responses in the interference direction. The look direction signal and output signals are displayed in Fig. 5. Even though the output signals seem to be almost same as the look

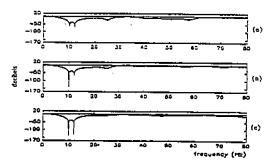


Fig. 4 Frequency responses in the interference direction for (a) composite array processor and modified processor with (b) one constraint at 10 Hz and (c) two constraints at 10 and 12 Hz.

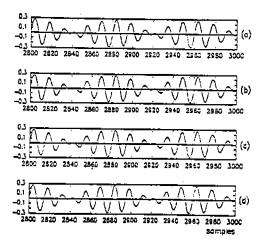


Fig. 5 (a) Look direction signal and output signals: (b) composite array processor (c), (d) modified processor with one constraint at 10 Hz and two constraints at 10 and 12 Hz, respectively.

direction signal, it is observed that the output signal corresponding to two null constraints is slightly less distorted than that obtained from the composite array processor or a single null constraint. This fact is evident in the error signals in Fig. 6, which are obtained by subtracting each output signal from the desired signal. The amount

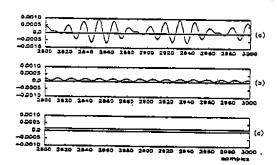


Fig. 6 Error signals: (a) composite array processor (b), (c) modified processor with one constraint at 10 Hz and two constraints at 10 and 12 Hz, respectively.

of the error signal variations for the modified composite array processor is much smaller than that for the composite array processor. In addition, the amount of variation for the case of two null constraints is reduced more than that for a single null constraint case. This is also apparent from the power spectra for the error signals as shown in Fig. 7 in which the 10 Hz component is eliminated and the 12 Hz component is greatly reduced in (b), while both frequency components are eliminated in (c).

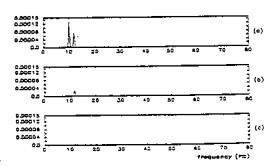


Fig. 7 Power spectra for the error signals shown in Fig. 6

### CONCLUSIONS

Derivative null constraint was introduced to improve the performance of the composite array processor in the spatial and frequency domains whose weights are updated by the constrained LMS algorithm. It was assumed that the incident angle and frequency of the interference signals are known. It was shown that the proposed method was efficient in eleminating broadband interference signals with respect to space and frequency. The power of the residual interference signals

was successfully reduced by placing null constraints at multiple frequencies in the interference direction. The simulation results demonstrate that the proposed processor performs better than the composite array processor.

### REFERENCES

- [1] R. T. Lacoss, "Adaptive combining of wideband array data for optimal reception," IEEE Trans. Geosci. Electron., vol. GE-6, no. 2, pp. 78-86, May 1968.
- [2] S. P. Applebaum and D. J. Champman, "Adaptive array with main beam constraints," IEEE Trans. Antennas and Propagat., vol. AP-24, no. 5, pp. 650-662, September 1976.
- [3] J. H. Chang and F. B. Tuteur, "A new class of adaptive array processors," J. Acoust. Soc. Am., vol. 49, по. 3, pp. 639-649, March 1971.
  - [4] M. H. Er and A. Cantoni, "Derivative constraints for broadband element space antenna array processors." IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-31, no. 6, pp. 1378-1393, December 1983.
  - [5] H. Steykal, "Synthesis of antenna patterns with prescribed nulls," IEEE Trans. Antenna Propagat., vol. AP-30, no. 2,pp 273-279, March 1982.
  - [6] O. L. Frost, III, "An algorithm for linearly constrained adaptive array processing," Proc. IEEE, vol. 60, no. 8, pp. 926-935, August 1972.
  - [7] B. Widrow, K. M. Duvall, R. P., Gooch, and W. C. Newman, "Signal cancellation

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- phenomena in adaptive antennas: causes and cures," IEEE Trans. Antennas Propagat., vol. AP-30, no. 3,pp. 467-478, May 1982.
- [8] D. H. Johnson, "The application of spectral estimation methods to bearing estimation problems," Proc. IEEE, vol. 70, no. 9, pp. 1088-1028, September 1982.
- [9] T. J. Shan, M Wax, and T. Kailath, "On spatial smoothing for direction-of-arrival estimation of coherent signals," IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-33, no. 4, pp. 806-811, August 1985.

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