研究論文

A Modification in the Analysis of the Growth Rate of Short Fatigue Cracks in S45C Carbon Steel under Reversed Loading

A. J. McEvily* and Yong Seung Shin**

반복 하중조건하에서의 S45C 탄소강에 대한 미소피로균열 성장속도 해석의 수정

A. J. McEvily* · 신 용 승**

Key Words : Cyclic Yield Stree(반복 항복응력), Effective Stress Intensity Factor(유효응력 강도계수), Linear Elastic Fracture Mechanics(선형탄성파괴역학), Opening Stress Intensity Factor(개구응력강도계수), Threshold Stress Intensity Factor(하한계응력강도계수).

초 록

본 연구에서는 종래의 미소피로균열 성장속도 해석방법에 대한 수정안을 제시하고, 수정 후의 방법에 의해서 계산한 값들과 S45C 탄소강에 대한 Nisitani와 Goto의 실험결과를 비교하여 계산한 값과 실험데이터 사이에 양호한 일치가 있음을 보였다. 이미 제시된 피로균열성장속도 식에는 하한계수준과 피로한도를 연관시키는 재료상수와 탄소성 거동에 대한 수정 및 균열닫힘효과를 나타내는 방법이 포함되어 있다.

본 연구에서 행한 수정중의 하나는 기하학적인 상수대신에 퍼만(Forman)의 탄성응력 강도계수 범위식을 이용하는 것이고, 다른 하나는 균열이 성장함에 따라 편심형단면으로 되면서 모멘트에 기인해 발생되는 굽힘효과를 고려하는 것이다. 이 방법을 수명예측에 사용하면 용접구조물은 물론 기계구조물의 보다 정확한 수명예측이 가능할 것이다.

Abstract

A modified method for the analysis of short fatigue crack growth has been presented, and calculations based upon the modified method are compared with experimental results for S45C carbon steel.

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It is also shown that the modified method is in good agreement with experimental data. The proposed equation for the fatigue crack growth rates includes a material constant which relates the threshold level to the endurance limit, a correction for elastic-plastic behaviour and a means for dealing with the effects of crack closure.

In this study one of the modifications is to substitute the Forman's elastic expression of the stress intensituy factor range into the geometrical factor. The other is a consideration of the bending effect which is developed from the moment caused by the eccentric cross sectional geometry as the crack grows. Thus, this method is useful for residual life prediction of the mechanical structures as well as the welding structures.

1. Introduction

The problem of fatigue has been approached by determining the nominal stress range which could be applied to either a smooth or a notched specimen without occurring failure for an indefinite number of cycles[1]. In smooth specimens, this stress range is termed the endurance limit, $\Delta\sigma_{end}$, and is typically defined in terms of a given number of applied cycles ranging from 10^7 to 10^9 cycles.

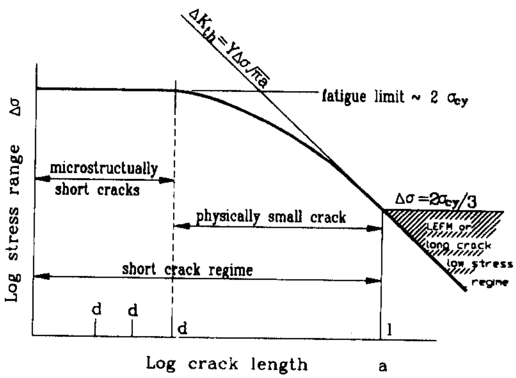


Fig. 1 Schematic of the Kitagawa-Takahashi curve showing the thresholds between propagating and non-propagating cracks.[2]

In Fig. 1, the Kitagawa-Takahashi plot[2], the line given by ΔK_{th} represents the threshold condition which a crack should not grow under linear elastic fracture mechanics(LEFM) assumptions. Obviously they are invalid when small scale yielding conditions are exceeded, and this occurs to a greater or less extent when the term $\Delta\sigma$ exceeds about two thirds of the cyclic yield stress, σ_{cy} in a reversed stress test. A second line on the plot is the fatigue limit itself, which can be approximated to the cyclic yield stress range. Obviously LEFM is not applicable at these levels of stress. An examination of the figure reveals why cracks can grow at levels less than ΔK_{th}, but it should also be appreciated that cracks growing on surface of plane specimens at stress levels below the fatigue limit have been reported.

An analytical method for predicting of the fatigue crack growth rate of short and long fatigue cracks has recently been proposed[3]. The method involves three modifications which include a material constant, concept of the plastic zone size to crack length ratio and the development of crack closure. Comparisons were made between calculations based upon the modified method and experimental data for 7075-T6 and 2024-T3 aluminum alloys[4]. By comparisons it has found to be in good agreement with each other.

The expressions or related versions thereof, have been used in the analysis of fatigue cracks growing from sharp notches in Armco iron[5], as a function of R, as well as in the study of the growth of short surface cracks in steels[6]. And the applicability of the approach to short fatigue crack growth in a S45C plain carbon steel was already examined[7]. The value of K_{min} used in those papers at $R=-1$ was 0 but McEvily insists it should be a value lower than zero. And at the higher growth rates a large deviation between calculations from experimental data and the line obtained for the analytically calculated values[7]. Therefore it is necessary to make further modifications in the expression.

2. Method of analysis

2.1 Overall approach

The following equation is the basic constitutive equation used to relate the rate of fatigue crack growth, da/dN, to the effective stress intensity factor range, ΔK_{eff}, and the effective stress intensity factor range at threshold, ΔK_{effth}.

$$\frac{da}{dN} = A(\Delta K_{eff} - \Delta K_{effth})^2 \quad (1)$$

where A is a material constant. Experimental results for the aluminum alloy 6061-T6[4] and for the alloy steel 9Cr-1Mo[8] have been found to be in good agreement with Eq. (1). In order to deal with short cracks this expression has been generalized to form three modifications along with the basic equation.

The first of these modifications is to include a material constant to relate the endurance limit and ΔK_{effth}. The second modification accounts for the fact that the plastic zone size associated with a newly formed crack is large in comparison with the crack length in violation of a restriction imposed upon the LEFM. The third modification accounts for the development of crack closure in the wake of a newly formed crack.

After these modifications have been made the resulting equation, the equivalent of Eq.(1) is as follows ;

$$\frac{da}{dN} = A(M)^2 \quad (2)$$

where

$$M=\left\{\sqrt{2\pi r_e}+Y\sqrt{\frac{\pi}{2}a}\left(\sec\frac{\pi\sigma_{max}}{2\sigma_y}+1\right)\right\}(\sigma_{max}-\sigma_{min}) - (1-e^{-k\lambda})(K_{opmax}-K_{min})-\Delta K_{effth} \quad (3)$$

where r_e is a material constant of the order of 1μm ;

Y is a geometrical factor, which for surface flaws is equal to 0.65 ;

λ is the length of a newly formed crack measured from the initiation sites ;

σ_{max} and σ_{min} are the maximum and minimum applied gross stresses ;

σ is the yield stress ;

k is a materrial constant of dimensions mm^{-1} which reflects the rate of crack closure development as a new crack grows ;

K_{opmax} is the closure level associated with a long crack.

However in this study the value of Y is modified to a variable as the crack length is increased.

To show that this expression is equivalent to Eq. (1), it is noted that when crack closure is fully developed, i.e., when $(1-e^{-k\lambda})$ approaches unity, the above equation can be written as

$$\frac{da}{dN}=A\left\{Y\sqrt{\frac{\pi}{2}}a\left(\sec\frac{\pi\sigma_{max}}{2\sigma_y}+1\right)\sigma_{max}-K_{opmax}-\Delta K_{effth}\right\}^2 \quad (4)$$

since $\sqrt{2\pi r_e}$ is order of 10^{-3} and for the small value of σ in comparison with σ_y, Eq. (4) can be written as

$$\frac{da}{dN}=A(Y\sqrt{\pi a}\sigma_{max}-K_{opmax}-\Delta K_{effth})^2 \quad (5)$$

This equation in turn can be written as

$$\frac{da}{dN}=A(K_{max}-K_{opmax}-\Delta K_{effth})^2=A(\Delta K_{eff}-\Delta K_{effth})^2 \quad (6)$$

which is equivalent to Eq. (1).

The nature of each of the three modifications will next be discussed.

2.2 Modifications

We begin by considering the first term, the quantity $\sqrt{2\pi r_e}(\sigma_{max}-\sigma_{min})$, in the expression for M. In the linear elastic range, the stress concentration factor K_T of such as elliptical and circular holes[8] is

$$K_T = 1 + 2\sqrt{\frac{a}{\rho}} \tag{7}$$

And the maximum stress at the stress raiser[8] is

$$\sigma_{peak} = K_T \sigma \tag{8}$$

If $\rho \to 0$, especially, for the ellipse of the major axis, $2c$ and the minor axis, $2b$, then the ellipse is identical to a crack of $2a=2c$ and the stress concentration factor K_T increases to infinity. When a crack of the crack length $2a$ is deformed by the Mode-I loading condition, the stress intensity factor[9] of the crack is

$$K = \sigma\sqrt{\pi a} \tag{9}$$

And from the stress concentration factor, K_T, of Eq.(7) the factor of $\sqrt{\pi a}$ of Eq. (9) is

$$\lim_{\rho\to 0} K_T = \lim_{\rho\to 0}\left(1 + 2\sqrt{\frac{a}{\rho}}\right) \cong 2\lim_{\rho\to 0}\sqrt{\frac{a}{\rho}} \tag{10}$$

$$\sqrt{\pi a} = \lim_{\rho\to 0}\sqrt{\frac{\pi\rho}{4}}K_T \tag{11}$$

Therefore Eq. (9) is

$$K = \sigma\sqrt{\pi a} = \lim_{\rho\to 0}\sqrt{\frac{\pi\rho}{4}}K_T\sigma \tag{12}$$

Howerer, if we consider that the radius of a fatigue crack tip is of a finite size, ρ_e, rather than zero, Eq. (12) can be written as

$$K = \lim_{\rho\to\rho_e}\sqrt{\frac{\pi\rho}{4}}K_T\sigma = \sqrt{\frac{\pi\rho_e}{4}}\sigma_{peak} \tag{13}$$

where α_{peak} is the maximum stress at the stress raiser, equal to $K_T\sigma$. Eq. (13) can also be written as

$$K = \lim_{\rho\to\rho_e}[1 + (K_T - 1)]\ \sigma\sqrt{\frac{\pi\rho}{4}} \tag{14}$$

The above also be written as

$$K = \left(\sqrt{\frac{\pi\rho_e}{4}} + Y\sqrt{\pi a}\right)\sigma \tag{15}$$

where $Y\sqrt{\pi a}$ is equal to $(K_T - 1)\sqrt{\frac{\pi\rho_e}{4}}$.

Ther stress immediately ahead of the crack tip, σ_{yy}, is expressible in terms of the stress intensity factor and the distance ahead of the crack tip,[9] r, i.e.,

$$\sigma_{yy} = \frac{K}{\sqrt{2\pi r}} \tag{16}$$

By setting σ_{yy} equal to σ_{peak}, an effective distance r_e, can be defined in terms of ρ_e as follows :

$$\sigma_{peak}\sqrt{2\pi r_e} = \sigma_{peak}\sqrt{\frac{\pi\rho_e}{4}} \tag{17}$$

so that

$$r_e = \frac{\rho_e}{8} \tag{18}$$

and Eq. (15), can then be written for cyclic loading in the linear elastic range as

$$\Delta K = (\sqrt{2\pi r_e} + Y\sqrt{\pi a})\Delta\sigma \tag{19}$$

In order to evaluate the constant r_e we could let the crack length, α, in the above equation go to zero and determine r_e by setting ΔK equals to ΔK_{effth} and $\Delta\sigma$ equals to the range of the endurance limit.

However, for the second step, since in the case of such a short crack the size of the plastic zone would be large to the crack size, the Irwin's correction(see Fig. 2) incorporated in the second term of the expression for M must be used. Irwin suggested[10] that if the plastic zone size is large with respect to the crack length, the crack length be increased by an amount equal to one-half of the plastic zone size in order to maintain a linear-elastic formation. In this expression the plastic zone size has been computed on the basis of Dugdale's analysis[11]. From Fig. 2, the enlarged crack length a^* is

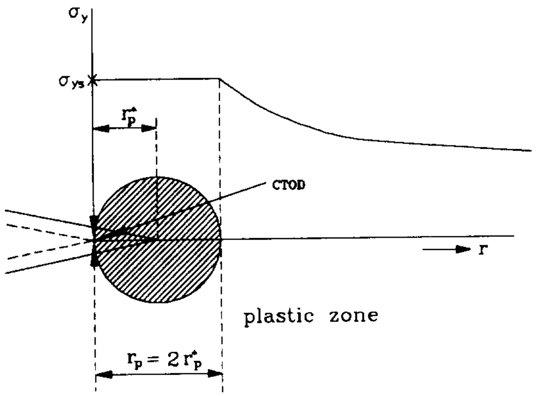


Fig. 2 Irwin's plastic zone correction.[10]

$$a^*=a+\frac{1}{2}r_p \tag{20}$$

And the plastic zone size is

$$r_p=a\left(\sec\frac{\pi\sigma_{max}}{2\sigma_y}-1\right) \tag{21}$$

So Eq. (20) is

$$a^*=\frac{a}{2}\left(1+\sec\frac{\pi\sigma_{max}}{2\sigma_y}\right) \tag{22}$$

In addition it has been observed that microcracks below a certain size are ineffective in reducing the fatigue strength[12]. This finding has been interpreted to indicate that the microstructure becomes the controlling factor as the size of the fatigue crack approaches to zero. In the present analysis, the material constant r_e will be taken to represent this distance below which the microstructure exerts a more significant effect on resistance to fatigue than a microcrack does. Therefore A.J. McEvily and Z. Yang[5] substituted r_e for a in Eq. (2), and Y equals to 0.65 for a semi-circular surface flaw, the following equation is obtained for r_e :

$$\left\{\sqrt{2\pi r_e}+Y\sqrt{\frac{\pi}{2}r_e\left(\sec\frac{\pi\sigma_{maxend}}{2\sigma_y}+1\right)}\right\}\Delta\sigma_{end}=\Delta K_{effth} \tag{23}$$

Eq. (23) can be solved for r_e iteratively, $Y=0.65$.

However, instead of $\Phi=Y\sqrt{\frac{\pi}{2}a\left(\sec\frac{\pi\sigma_{max}}{2\sigma_y}+1\right)}$, the author modified it as follows :

$$\Phi=F_0(\xi)(1+\textit{bending effect})\sqrt{\frac{\pi}{2}a\left(\sec\frac{\pi\sigma_{max}}{2\sigma_y}+1\right)} \tag{24}$$

where $F_0(\xi)$ is a function of $\xi=\frac{a}{D}$ in the Forman's formula[15] for stress intensity factors. The formula is as follows :

$$K_I=[\sigma_0F_0(\xi)+\sigma_0F_0(\xi)]\sqrt{\pi a} \tag{25}$$

and where $F_0(\xi)=g(\xi)\left\{0.752+2.02\xi+0.37\left(1-\sin\frac{\pi\xi}{2}\right)^3\right\}$ is for the tensile stress, and $F_B(\xi)=g(\xi)\left\{0.923+0.199\left(1-\sin\frac{\pi\xi}{2}\right)^4\right\}$ is for the bending stress where

$$g(\xi)=0.92\left(\frac{2}{\pi}\right)\left\{\frac{\tan\frac{\pi\xi}{2}}{\frac{\pi\xi}{2}}\right\}^{1/2}\sec\frac{\pi\xi}{2}$$

In this case there is no bending stress caused by a bending load. The bending effect comes from the fact that the specimen becomes eccentric as the crack grows. The bending effect is defined as the ratio of the bending stress, σ_{ben}, occurred at the deepest point by the bending moment of the eccentric specimen under the tensile stress, σ_{ten}.

$$\textit{bending effect}=\frac{\sigma_{ben}}{\sigma_{ten}}=\frac{\pi D^2hh'}{4I_{net}} \tag{26}$$

where $\sigma_{ben} = \frac{Phh'}{I_{net}}$, $\sigma_{ben} = \frac{4P}{D^2 - 2ca}$, $h = \frac{ca\left(D - \frac{8a}{3\pi}\right)}{D^2 - 2ca}$,

$h' = \frac{D}{2} - \frac{4a}{3\pi} + h$, $I_{net} = I_{cg} - I_{cig}$, $I_{cg} = \frac{\pi D^4}{64} + \frac{\pi D^2}{4} h^2$,

and $I_{cig} = \frac{\pi a^3 c}{8}\left(1 - \frac{64}{9\pi^2}\right) + \frac{\pi ac}{2} h^2$

So Eq. (24) is

$$\Phi' = 0.92\left(\frac{2}{\pi}\right)\left(\frac{\tan\frac{\pi\xi}{2}}{\frac{\pi\xi}{2}}\right)^{1/2} \sec\frac{\pi\xi}{2}\left\{0.75 + 2.02\xi + 0.37\left(1 - \sin\frac{\pi\xi}{2}\right)^3\right\}\left(1 + \frac{\pi D^2 hh'}{4I_{net}}\right)\sqrt{\frac{\pi}{2}a\left(\sec\frac{\pi\sigma_{max}}{2\sigma_y} + 1\right)} \qquad (24)'$$

If the crack length is small, then the ratio of crack length to the diameter of the specimen, $\xi = \frac{a}{D}$ and the bending effect go to zero. Especially in case of $\sigma_{max} \ll \sigma_y$ the value of Y approaches to 0.657. Therefore Eq. (23) is

$$(\sqrt{2\pi r_e} + \Phi')\Delta\sigma_{end} = \Delta K_{effth} \qquad (23)'$$

and we can get the value of the constant r_e should not depend upon the mean stress under which the endurance limit was determined. However, if in practice there should be a difference in r_e for different mean stresses then an average value of r_e would be used.

The third term in the expression for M accounts for development of closure as a crack grows from an embryonic state to a length beyond which there is no further increase in crack closure[13]. Fig. 3 shows how crack closure develops over a distance, λ, as a function of the material parameter k[14]. λ is the length of the newly formed crack. It has been proposed that the transient development of closure with crack extension can be expressed in the following manner[14] :

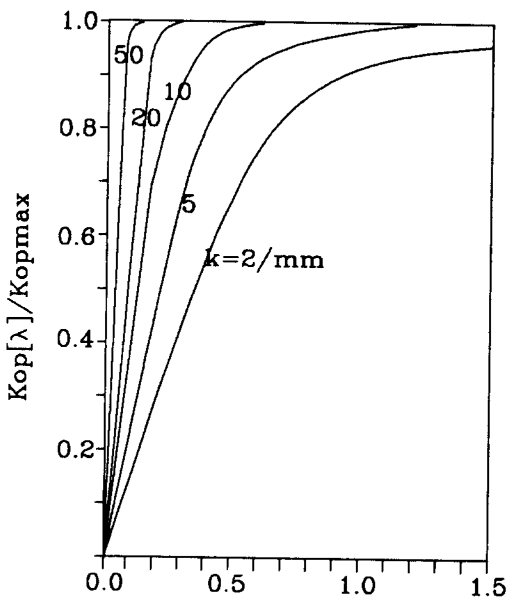
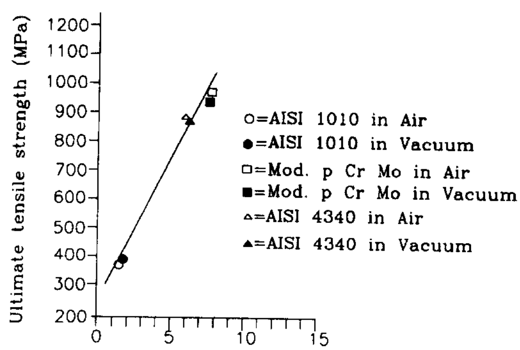


Fig. 3 The ratio of the crack opening level for a short crack to that of a long crack as a function of crack length and the parameter k, where k has the units of mm^{-1}[13]

$$K_{op} = (1 - e^{k\lambda})(K_{opmax} - K_{min}) \qquad (27)$$

The material parameter k can be determined experimentally, but in Fig. 4 it could be determined with increasing ultimate tensile strength in ferritic



Material constant k (mm^{-1})

Fig. 4 Dependence of k on strength level for steels tested in air (open symbols) and in vacuum (filled symbols).[14]

materials. Therefore the authors calculated the value of k from Eq. (28).

$$k(\mathrm{mm}^{-1}) = 0.01\sigma_{uts}(\mathrm{MPa}) - 2.2 \tag{28}$$

The fourth term in the expression for M; simply reflects the fact that the threshold level must be exceeded in order that the crack grows.

3. Application

Fig. 5 shows the rate of growth of short, semi-circular cracks in a S45C carbon steel as a function of crack length at $R=-1$(R is the ratio of the minimum to the maximum stress in a cycle) for the plane specimens. These data were obtained by Nisitani and Goto[15] for a steel that had been heat treated to a yield strength of 364 MPa. It is noted that the shortest cracks are of the order of 10μm in length. These cracks were grown in cylindrical specimens of 5mm diameter. Fig. 6 shows the data of Nisitani and Goto plotted as a function of the crack lengths for the drilled specimens at $R=-1$.

For the macroscopic crack, ΔK_{th} was determined

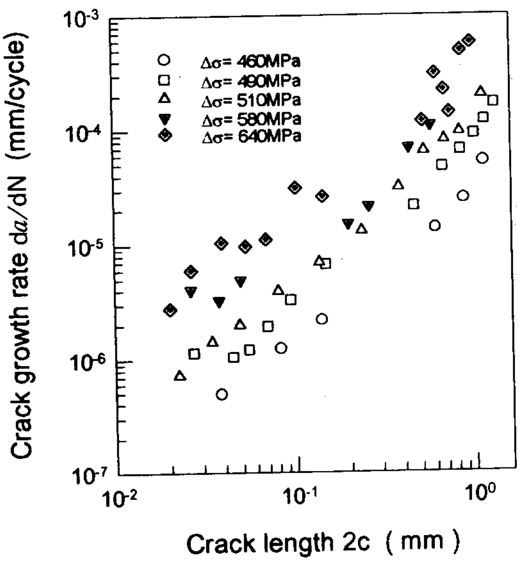


Fig. 5 Rate of fatigue crack growth at $R=-1$ for short cracks in the plane specimens as a function of the crack length[15].

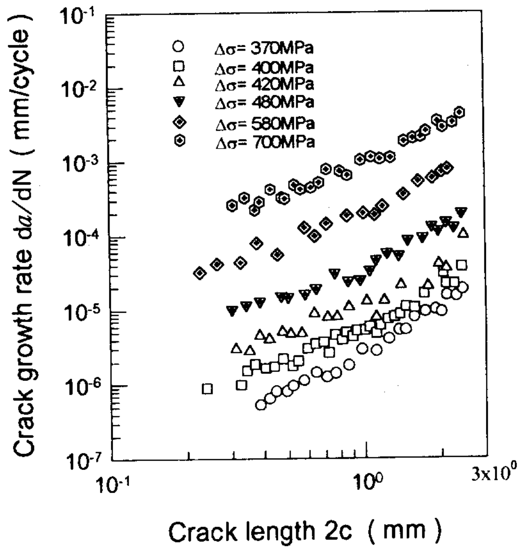


Fig. 6 Rate of fatigue crack growth at $R=-1$ for short cracks in the drilled specimens as a function of the crack length[15].

to be 6.0 MPa · $m^{1/2}$. Crack closure measurements indicated that the crack opening level at threshold was 3.3 MPa · $m^{1/2}$, so that ΔK_{effth} was 2.7 MPa · $m^{1/2}$. Nisitani and Goto reported that the value of $\Delta\sigma_{end}$ was 440 MPa. After substitution of the appropriate value in Eq. (23), a value for r_e of 2.54 μm was determined. Since in the range where the crack is small, the value r_e calculated by Eq. (23)′ is almost same as that of Eq. (23), the authors used the value of r_e=2.54μm calculated by Eq. (23).

Fig. 7 shows the data of Nisitani and Goto plotted as a function of the parameter M in Eq. (2) for the plane specimens. In calculationg the value of M, Eq. (3) was used. For the constants used, r_e=2.5 4μm, $\sigma_{max}=1/2\ \Delta\sigma$ and $R==-1$, so $\sigma_{min}=-\sigma_{max}$, and K_{opmax}=3.3MPa · $m^{1/2}$, at the threshold level. And instead of constant Y=0.65, $Y=F_0(\xi)$ (1+*bending effect*) in Eq. (24) was used. Where $F_0=g(\xi)\{0.75+2.02\xi+0.37(1-\sin\pi\xi/2)^3\}$ and $g(\xi)=0.92(2/\pi)\ \{(\tan\pi\xi/2)/(\pi\xi/2)\}^{1/2}\cdot\sec\pi\xi/2$ and *bending effect*$=\sigma_{ben}/\sigma_{ten}=4D^2hh'\ /\ (4\ Inet)$ in Eq. (26) where $\xi=a/D$. $K_{min}=\sigma_{min}\ F_0(\pi a)^{1/2}$, $\Delta K_{th}=\{(2\pi\ r_e)^{1/2}+F_0\}\Delta\sigma_{end}$ in Eq. (23′). $\Delta\ \sigma_{end}$=440MPa $m^{1/2}$. The value of k was taken to be 4 mm^{-1}, a value which seemed to be appropriate

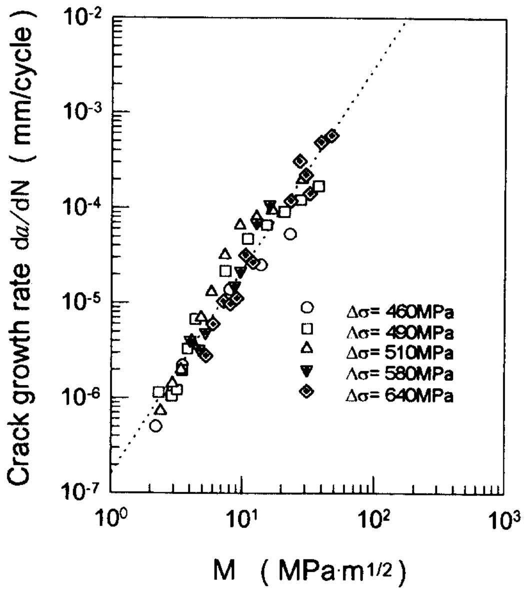


Fig. 7 Rate of fatigue crack growth at $R=-1$ for short cracks in the plane specimens as a function of the calculated parameter M with the regression (dotted straight line).

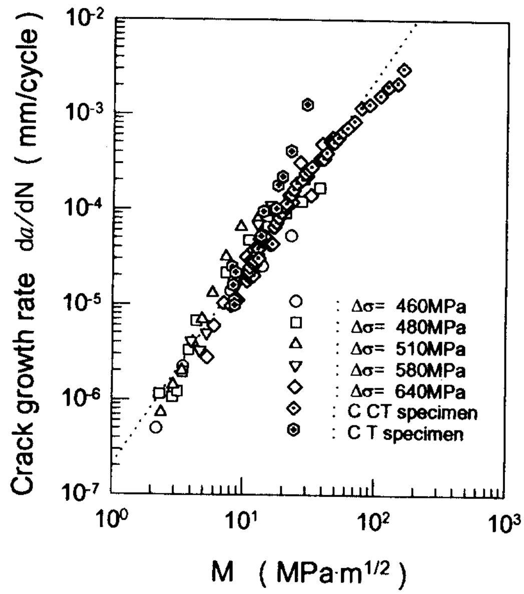


Fig. 8 Rate of fatigue crack growth at $R=-1$ for short cracks in the plane specimens and for long cracks in CT and CCT specimen as a function of the calculated parameter M with the regression (dotted straight line).

to the strength level of the S45C steel from Eq. (28).

The dotted straight line in Fig. 7 represents the regression of all data. The slope of the dotted straight line is 2.14, and a value for the constant A in Eq. (2) which means the intersection with the y-axis is 1.62×10^{-7} mm/cycle. As in Fig. 7 the data obtained at low growth rates agree with Eq. (2). And at the higher growth rates the deviation is also decreased.

In Fig. 8 the data optained from CT and CCT specimens of S45C steel plotted with the data from plane specimens. The slope of the straight regression line is 2.04 and a value for the constant A in Eq. (2) is 2.04×10^{-7} mm/cycle.

Fig. 9 also shows the data of Nisitani and Goto plotted as a function of the parameter M for the drilled specimens. In calçulationg the value of M for the drilled specimens the same equation and the same constants as used in calculation for the plane specimens were used. But instead of the newly formed crack length λ, $\lambda=a\text{-}a_0$ is used, where a_0 is the initial drilled depth of the drilled specimens.

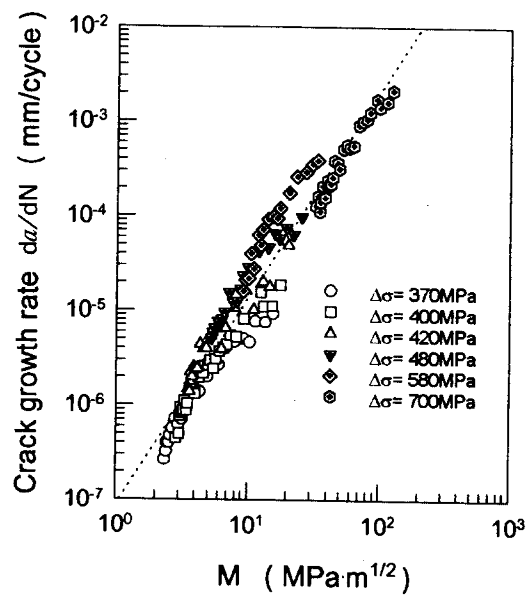


Fig. 9 Rate of fatigue crack growth at $R=-1$ for short cracks in the drilled specimens as a function of the calculated parameter M with the regression (dotted straight line).

The dotted straight line in Fig. 9 represents the regression of all data. The slope of the dotted straight line is 2.19 and the line intersects the y-axis at 0.87×10^{-7} mm/cycle.

From Fig. 9 we can see the data obtained at low growth rates agree with Eq. (2). But at the higher rates of the stress range of 370 MPa, and 420 MPa, the values of M look like to reach at saturation points for each range. These observed abnormal behavior possibly caused from the tangent and the sine factors in Eq. (24)'.

A modified LEFM approach to the analysis of the short caracks has been presented. The modified method is in good agreement with the experimental crack growth rate data of the short and long cracks in the plane specimens. In addition the agreement is good for the cracks started from small drilled holes in the specimens. However the agreement is not as good for the crack whose lengths are longer than 2 mm. Further works would be needed to resolve this problem.

4. Conclusions

1) The further modifications to the basic linear elastic approach to the analysis of the rate of fatigue crack propagation for short cracks was presented. Basically these modifications include a material constant relationg the threshold level to the endurance limit, a correction for elastic-plastic behavior and means for dealing with the effects of crack closure.

2) By substituting the Forman's elastic expression into the geometrical factor and considering the bending effect, good agreements were found between calculations and experimental results.

3) This method is useful for residual life prediction of the mechanical structures as well as of the welding structures.

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