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ON MEASURABLE SPACES AND SEMI–TOPOGENOUS SPACES

BYUNG SIK IN

We are to present some properties of binary relations by means of categorical method. The concept of semi-topogenous structures is due to Császár.

Using this, we give a new definition of a σ-topogenous structure as a particular type of semi-topogenous structure.

In this paper, we concern with the relationships between $\underline{Mes}$ (the category of measurable spaces) and subcategories of $\underline{ST}$ (the category of semi-topogeous spaces), and study some properties of these categories.

Further, it turn out that $\underline{Mes}$, $\underline{\sigma TS}$ (the category of σ-topologeous spaces) resp. are coreflective subcategories of $\underline{IST}$ (the category of interpolation semi-topogenous spaces), $\underline{ST}$ resp..

Also, we construct the category $\underline{\sigma ISTG}$ (the category of interpolation symmetrical σ-topogeous spaces with generating sets) as a coreflective subcategory of $\underline{IST}$, which is isomorphic to $\underline{Mes}$.

Finally we obtain that the functor $G : \underline{\sigma Latt} \longrightarrow \underline{\sigma TS}$ is a full embedding, where $\underline{\sigma Latt}$ is the category of σ-lattices and isotone maps.

1. Preliminaries

In this section we introduce categorical properties of the category $\underline{Mes}$ of measurable spaces and measurable maps.

DEFINITION 1.1. Let X be a set. A collection $\mathcal{A}$ of subsets of X is called a σ-algebra on X if the following conditions are satisfied:

(i) $X \in \mathcal{A}$,

(ii) for each $A \in \mathcal{A}$, $A^c \in \mathcal{A}$,

(iii) for each infinite sequence (A_i) $(i \in I)$ of sets such that $A_i \in \mathcal{A}$, $A_i \in \mathcal{A}$ $(i \in I)$, where I is any countable set of indices.

In this case $(X, \mathcal{A})$ is called a measurable space.

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DEFINITION 1.2. Let $(X,\mathcal{A})$ and $(Y,\mathcal{A}')$ be measurable spaces. A map $f : X \longrightarrow Y$ is said to be measurable if $f^{-1}(A') \in \mathcal{A}$ for each $A' \in \mathcal{A}'$.

NOTATION. *The class of all measurable spaces and measurable maps forms a category, which is denoted by* $\underline{Mes}$.

THEOREM 1.3. *The category* $\underline{Mes}$ *is topological.*

Proof. Let X be a set, $((X_\alpha,\mathcal{A}_\alpha))_{\alpha\in\Lambda}$ a family of measurable spaces and $(f_\alpha : X \longrightarrow X_\alpha)_{\alpha\in\Lambda}$ a source. Let $\mathcal{A}$ be the σ-algebra generated by $\{f_\alpha^{-1}(A_\alpha) | A_\alpha \in \mathcal{A}_\alpha, \alpha \in \Lambda\}$. Then $(X,\mathcal{A})$ is a measurable space and $f_\alpha : (X,\mathcal{A}) \longrightarrow (X_\alpha,\mathcal{A}_\alpha)$ is measurable for all $\alpha \in \Lambda$. For any measurable space $(Y,\mathcal{B})$, let $g : (Y,\mathcal{B}) \longrightarrow (X,\mathcal{A})$ be a map such that for all $\alpha \in \Lambda$, $f_\alpha \circ g : (Y,\mathcal{B}) \longrightarrow (X_\alpha,\mathcal{A}_\alpha)$ is measurable. For any $\alpha \in \Lambda$ and $A_\alpha \in \mathcal{A}_\alpha$, $(f_\alpha \circ g)^{-1}(A_\alpha) = g^{-1}(f_\alpha^{-1}(A_\alpha)) \in \mathcal{B}$. Hence $g^{-1}(\mathcal{A}) \subseteq \mathcal{B}$, because $\mathcal{A}$ is generated by $\{f_\alpha^{-1}(A_\alpha) | A_\alpha \in \mathcal{A}_\alpha, \alpha \in \Lambda\}$; so g is measurable. This completes the proof.

COROLLARY 1.4. *The category* $\underline{Mes}$ *is cotopological, complete and cocomplete.*

COROLLARY 1.5. *The forgetful functor* $U : \underline{Mes} \longrightarrow \underline{Set}$ *has a left adjoint.*

Proof. Since $U : \underline{Mes} \longrightarrow \underline{Set}$ is topological, it has a left adjoint.

2. Semi-topogenous spaces

The following definitions are due to Császár [7].

DEFINITION 2.1. Let X be a set. Then a relation $<$ on $\mathcal{P}(X)$ is called a semi-topogenous structure on X if it satisfies the following conditions:

(i) $\phi < \phi$, $X < X$;
(ii) $A < B$ implies $A \subset B$;
(iii) $A \subset A' < B' \subset B$ implies $A < B$.

The ordered pair $(X,<)$ is called a semi-topogenous space.

DEFINITION 2.2. Let X be a set and $\mathcal{F}$ a family of subsets of X with $\phi, X \in \mathcal{F}$. Then a semi-topogenous structure on X is said to be generated by $\mathcal{F}$ (or $\mathcal{F}$ is the generating system of sets) if it satisfies the following condition;

$A < B$ iff there is a set $S \in \mathcal{F}$ such that $A \subset S \subset B$.

EXAMPLE 2.3. For any measurable space $(X, \mathcal{A})$, define $<_{\mathcal{A}}$ on $\mathcal{P}(X)$ as follows:

$A <_{\mathcal{A}} B$ iff there is $S \in \mathcal{A}$ such that $A \subset S \subset B$. Then $<_{\mathcal{A}}$ is a semi-topogenous structure on X.

DEFINITION 2.4. Let $(X, <)$ and $(Y, <')$ be semi-topogenous spaces and $f : X \longrightarrow Y$ a map.

Then f is said to be continuous if for $A' <' B'$, $f^{-1}(A') < f^{-1}(B')$.

PROPOSITION 2.5. 1) *If $(X, <)$ is any semi-topogenous space, then the identity map $1_X : (X, <) \longrightarrow (X, <)$ is continuous.*

2) *If $f : (X, <) \longrightarrow (Y, <')$ continuous and $g : (Y, <') \longrightarrow (Z, <'')$ continuous, then $g \circ f : (X, <) \longrightarrow (Z, <'')$ is also continuous.*

Proof. It follows immediately from the definition.

We can easily obtain the following category from Proposition 2.5.

NOTATION. The class of all semi-topogenous spaces and continuous maps between them forms a category, which will be denoted by $\underline{ST}$.

PROPOSITION 2.6. *Let $(X, \mathcal{A})$ and $(Y, \mathcal{A}')$ be measurable spaces. Then $f : (X, \mathcal{A}) \longrightarrow (Y, \mathcal{A}')$ be measurable iff $f : (X, <_{\mathcal{A}}) \longrightarrow (Y, <_{\mathcal{A}'})$ is continuous.*

Proof. ($\Rightarrow$) Let $A' <_{\mathcal{A}'} B'$. Then there is $S' \in \mathcal{A}'$ such that $A' \subset S' \subset B'$. Since f is measurable, $f^{-1}(S') \in \mathcal{A}$ and $f^{-1}(A') \subset f^{-1}(S') \subset f^{-1}(B')$. Thus $f^{-1}(A') <_{\mathcal{A}} f^{-1}(B')$ and hence f is continuous.

($\Leftarrow$) Let $A' \in \mathcal{A}'$. Then $A' <_{\mathcal{A}'} A'$.

Since f is continuous, $f^{-1}(A') <_{\mathcal{A}} f^{-1}(A')$ and hence $f^{-1}(A') \in \mathcal{A}$. Thus f is measurable.

COROLLARY 2.7. *The functor $F : \underline{Mes} \longrightarrow \underline{ST}$ defined by $F(X, \mathcal{A}) = (X, <_{\mathcal{A}})$ and $F(f) = f$, is a full embedding.*

Proof. It is immediate from Proposition 2.6.

DEFINITION 2.8. Let $(X,<)$ be a semi-topogenous space and define a relation $<^c$ on $P(X)$ by $A <^c B$ iff $X-B < X-A$. Then $<^c$ is called the complement of $<$. If $<=<^c$, then $<$ is said to be symmetrical. In this case $(X,<)$ is called a symmetrical semi-topogenous space.

DEFINITION 2.9. A semi-topogenous structure $<$ on X is called a σ-topogenous structure if for $A_i < B_i (i \in I)$, $\bigcup_{i\in I} A_i < \bigcup_{i\in I} B_i$ and $\bigcap_{i\in I} A_i < \bigcap_{i\in I} B_i$, where I denotes any countable set of indices. In this case, $(X,<)$ is called a σ-topogenous space.

REMARK 2.10. Let $(X,\mathcal{A})$ be any measurable space. Then $(X,<_{\mathcal{A}})$ is a symmetrical σ-topogenous space.

Proof. For any $(X,\mathcal{A}) \in \underline{Mes}$, we define a relation $<_{\mathcal{A}}$ on $P(X)$ by $A <_{\mathcal{A}} B$ iff there is $S \in \mathcal{A}$ such that $A \subset S \subset B$. Then $X-B \subset X-S \subset X-A$ for some $X-S \in \mathcal{A}$, so that $(X,<_{\mathcal{A}})$ is symmetrical. In order to show that $(X,<_{\mathcal{A}})$ is a σ-topogenous space, let $A_i <_{\mathcal{A}} B_i (i \in I)$, where I is any countable index set. Then there is $S_i \in \mathcal{A}(i \in I)$ such that $A_i \subset S_i \subset B_i (i \in I)$, so we have $\bigcup_{i\in I} A_i \subset \bigcup_{i\in I} S_i \subset \bigcup_{i\in I} B_i$ and $\bigcap_{i\in I} A_i \subset \bigcap_{i\in I} S_i \subset \bigcap_{i\in I} B_i$. Thus $\bigcup_{i\in I} A_i <_{\mathcal{A}} \bigcup_{i\in I} B_i$ and $\bigcap_{i\in I} A_i <_{\mathcal{A}} \bigcap_{i\in I} B_i$ since $\bigcup_{i\in I} S_i, \bigcap_{i\in I} S_i \in \mathcal{A}$.

NOTATION. The full subcategory of $\underline{ST}$ determined by σ-topogenous spaces will be denoted by $\underline{\sigma TS}$.

THEOREM 2.11. *The category $\underline{\sigma TS}$ is coreflective in $\underline{ST}$.*

Proof. Let $(X,<) \in \underline{ST}$ and define a relation $<^\sigma$ on $P(X)$ as follows: $A <^\sigma B$ iff there are sequence $(A_i)_{i\in I}$ and $(B_j)_{j\in J}$ such that $A = \bigcup_{i\in I} A_i, B = \bigcap_{j\in J} B_j$ and $A_i < B_j (i \in I, j \in J)$, where I and J are countable sets of indices. Then we can easily get that $<^\sigma$ is a semi-topogenous structure.

Let us prove that $<^\sigma$ is a σ -topogenous structure. Suppose $A_k <^\sigma B_k (k \in K)$ for any countable index set K.

Then $A_k = \bigcup_{i\in I} A_{ki}, B_k = \bigcap_{j\in J} B_{kj}$ and $A_{ki} < B_{kj} (k \in K,$ $i \in I, j \in J)$; so $\bigcup_{k\in K} A_k = \bigcup_{k\in K}(\bigcup_{i\in I} A_{ki}) = \bigcup_{i\in I}(\bigcup_{k\in K} A_{ki})$, $\bigcup_{k\in K} B_k = \bigcup_{k\in K}(\bigcap_{j\in J} B_{kj}) = \bigcap_{\{j_k\}\in J^K}(\bigcup_{k\in K} B_{kj_k})$ and $\bigcup_{k\in K} A_{ki}$ $< \bigcup_{k\in K} B_{kj_k}$. It follows that $\bigcup_{k\in K} A_k <^\sigma \bigcup_{k\in K} B_k$.

Similarly, we obtain that $\bigcap_{k\in K} A_k <^\sigma \bigcap_{k\in K} B_k$. Thus $<^\sigma$ is a σ-topogenous structure, evidently finer than $<$, consequently $(X,<^\sigma) \in$

$\underline{\sigma TS}$ and the identity map $1_X : (X, <^\sigma) \longrightarrow (X, <)$ is continuous. Take any $(Y, <') \in \underline{\sigma TS}$ and any continuous $f : (Y, <') \longrightarrow (X, <)$. Since $(Y, <') \in \underline{\sigma TS}, (Y, <') = (Y, <'^\sigma)$, and so $f : (Y, <') \longrightarrow (X, <^\sigma)$ is continuous. Moreover, such an f is unique because 1_X is bijective. Thus $1_X : (X, <^\sigma) \longrightarrow (X, <)$ is the $\underline{\sigma TS}$-coreflection of $(X, <)$.

DEFINITION 2.12. Let $(X, <)$ be a semi-topogenous space. Then $<$ is called an interpolation semi-topogenous structure if for $A < B$, there is a subset S of X such that $A < S < B$.

REMARK 2.13. For any $(X, \mathcal{A}) \in \underline{Mes}$, $<_\mathcal{A}$ is an interpolation semi-topogenous structure on X.

Proof. Suppose $A <_\mathcal{A} B$. Since $<_\mathcal{A}$ is a semi-topogenous structure, there is $S \in \mathcal{A}$ such that $A \subset S \subset B$. Hence $A \subset S \subset S$ and $S \subset S \subset B$ imply $A <_\mathcal{A} S$ and $S <_\mathcal{A} B$. Thus the proposition is proved.

NOTATION. The full subcategory of $\underline{ST}$ determined by interpolation semi-topogenous spaces (interpolation σ-topogenous spaces, interpolation symmetrical σ-topogenous spaces, interpolation symmetrical σ-topogenous spaces with generating sets) will be denoted by $\underline{IST}$($\underline{\sigma IT}$, $\underline{\sigma IST}$, $\underline{\sigma ISTG}$).

THEOREM 2.14. *The category $\underline{\sigma IT}$ is coreflective in $\underline{IST}$.*

Proof. Let $(X, <) \in \underline{IST}$ and define a relation $<^\sigma$ on $P(X)$ as follows: $A <^\sigma B$ iff there are sequences $(A_i)_{i\in I}$, $(B_i)_{i\in I}$, $(A_{ij})_{j\in J}$, $(B_{ij})_{j\in J}$ and $(C_{ij})_{j\in J}$ such that

$$A = \cup_{i\in I} A_i, B = \cup_{i\in I} B_i, C = \cup_{i\in I} C_i,$$

$$A_i = \cap_{j\in J} A_{ij}, B_i = \cap_{j\in J} B_{ij}, C_i = \cap_{j\in J} C_{ij},$$

$$A_{ij} < C_{ij} < B_{ij} (i \in I, j \in J)$$

where I and J are countable sets of indices.

By the definition of $<^\sigma$, we can easily show that $<^\sigma$ is an interpolation semi-topogenous structure.

Moreover, as in proof of Theorem 2.11, $<^\sigma$ is a σ-topogenous structure. Hence $(X, <^\sigma) \in \underline{\sigma IT}$ and the identity map $1_X : (X, <^\sigma) \longrightarrow (X, <)$ is continuous.

Take any $(Y,<')\in \underline{\sigma IT}$ and any continuous $f:(Y,<')\longrightarrow (X,<)$. Since$(Y,<')\in \underline{\sigma IT}$, $(Y,<')=(Y,<'^{\sigma})$, so that $f:(Y,<')\longrightarrow (X,<^{\sigma})$ is continuous. Furthermore, such an f is unique.

Thus $1_X:(X,<^{\sigma})\longrightarrow (X,<)$ is the $\underline{\sigma IT}$-coreflection of $(X,<)$.

COROLLARY 2.15. (1) *The category* $\underline{\sigma IST}$ *is coreflective in* $\underline{\sigma IT}$.
(2) *The category* $\underline{\sigma ISTG}$ *is coreflective in* $\underline{\sigma IST}$.

THEOREM 2.16. *The categories* $\underline{\sigma ISTG}$ *and* $\underline{Mes}$ *are isomorphic.*

Proof. For any $(X,<)\in \underline{\sigma ISTG}$, let $\mathcal{A}_<=\{A\subset X\mid A<A\}$. Then it is clear that $(X,\mathcal{A}_<)\in \underline{Mes}$. If $f:(X,<)\longrightarrow (Y,<')$ is continuous ,then $f:(X,\mathcal{A}_<)\longrightarrow (Y,\mathcal{A}_{<'})$ is measurable. Thus $G:\underline{\sigma ISTG}\longrightarrow \underline{Mes}$ is a functor defined by $G(X,<)=(X,\mathcal{A}_<)$ and $G(f)=f$. Conversely, for any $(X,\mathcal{A})\in \underline{Mes}$, we define a relation $<_{\mathcal{A}}$ on $P(X)$ as follows: $A<_{\mathcal{A}}B$ iff there is $S\in\mathcal{A}$ such that $A\subset S\subset B$. By Remark 2.11 and Theorem 2.14, we have $(X,<_{\mathcal{A}})\in \underline{\sigma ISTG}$. Suppose $g:(X,\mathcal{A})\longrightarrow (Y,\mathcal{A}')$ is measurable, then $g:(X,<_{\mathcal{A}})\longrightarrow (Y,<_{\mathcal{A}'})$ is continuous. Thus $F:\underline{Mes}\longrightarrow \underline{\sigma ISTG}$ is a functor defined by $F(X,\mathcal{A})=(X,<_{\mathcal{A}})$ and $F(g)=g$. Moreover, for any $(X,<)\in \underline{\sigma ISTG}$, $F(G(X,<))=F(X,\mathcal{A}_<)=(X,<_{\mathcal{A}_<})$. Since $A<_{\mathcal{A}_<}B$ iff there is $S\in\mathcal{A}_<$ such that $A\subset S\subset B$ iff $A\subset S\subset B$ and $S<S$ iff $A<B$, $(X,<_{\mathcal{A}_<})=(X,<)$. Hence $F\circ G=1_{\underline{\sigma ISTG}}$. For any $(X,\mathcal{A})\in \underline{Mes}$, $G(F(X,\mathcal{A}))=F(X,<_{\mathcal{A}})=(X,\mathcal{A}_{<_{\mathcal{A}}})=(X,\mathcal{A})$, so that $G\circ F=1_{\underline{Mes}}$. Therefore $\underline{\sigma ISTG}$ and $\underline{Mes}$ are isomorphic.

THEOREM 2.17. *The category* $\underline{Mes}$ *is coreflective in* $\underline{IST}$

Proof. It follows from Theorem 2.14, Corollary 2.15 and Theorem 2.16.

We have the following by the above theorem and Theorem 1.3.

THEOREM 2.18. *The category* $\underline{\sigma ISTG}$ *has the following :*

(1) $\underline{\sigma ISTG}$ *is topological and cotopological.*
(2) $\underline{\sigma ISTG}$ *is complete and cocomplete.*
(3) *The forgetful functor* $U:\underline{\sigma ISTG}\longrightarrow \underline{Set}$ *has a left adjoint.*

PROPOSITION 2.19. *Let $\mathcal{L}$ be a lattice of subsets of a set X with $\phi, X \in \mathcal{L}$ and let $<$ be a semi-topogenous structure generated by $\mathcal{L}$. Then $<$ is a σ-topogenous structure iff $\mathcal{L}$ is closed under countable unions and intersections.*

Proof. ($\Rightarrow$) If $<$ is a σ-topogenous structure and $S_i \in \mathcal{L}(i \in I)$ for any countable index set I, then $\bigcup_{i\in I} S_i < \bigcup_{i\in I} S_i$ and $\bigcap_{i\in I} S_i < \bigcap_{i\in I} S_i$. Hence $\bigcup_{i\in I} S_i, \bigcap_{i\in I} S_i \in \mathcal{L}$.

($\Leftarrow$) If $A_i < B_i(i \in I)$, then there is $S_i \in \mathcal{L}(i \in I)$ such that $A_i \subset S_i \subset B_i, (i \in I)$. Hence $\bigcup_{i\in I} A_i < \bigcup_{i\in I} B_i$ and $\bigcap_{i\in I} A_i < \bigcap_{i\in I} B_i$, because $\bigcup_{i\in I} S_i, \bigcap_{i\in I} S_i \in \mathcal{L}$.

DEFINITION 2.20. Let X be a lattice. Then X is called a σ-lattice if every countable subset D of X has a join $\bigvee D$ and a meet $\bigwedge D$.

PROPOSITION 2.21. *Let $(X, \leq)$ be any σ-lattice and let $<_\leq$ be a relation on $P(X)$ defined by $A <_\leq B$ iff $\uparrow A \subset B$. Then $<_\leq$ is a σ-topogenous structure on X.*

Proof. Since $\uparrow \phi = \phi$ and $\uparrow X = X, \phi <_\leq \phi$ and $X <_\leq X$. If $A <_\leq B$, then $\uparrow A \subset B$. Since $A \subset \uparrow A, A \subset B$. Suppose $A' \subset A <_\leq B \subset B'$, then $\uparrow A \subset B$. Since $\uparrow A' \subset \uparrow A, \uparrow A' \subset B'$, so we have $A' <_\leq B'$. Thus $<_\leq$ is a semi-topogenous structure. We claim that $<_\leq$ is a σ-topogenous structure. Suppose $A_i <_\leq B_i(i \in I)$ for any countable index set I, then $\uparrow A_i \subset B_i(i \in I)$, so that $\bigcup_{i\in I}(\uparrow A_i) \subset \bigcup_{i\in I} B_i$ and $\bigcap_{i\in I}(\uparrow A_i) \subset \bigcap_{i\in I} B_i$ i.e., $\uparrow (\bigcup_{i\in I} A_i) \subset \bigcup_{i\in I} B_i$ and $\uparrow \bigcap_{i\in I} A_i) \subset \bigcap_{i\in I} B_i$ because $\uparrow (\bigcup_{i\in I} A_i) = \bigcup_{i\in I}(\uparrow A_i)$ and $\uparrow (\bigcap_{i\in I} A_i) \subset \bigcap_{i\in I}(\uparrow A_i)$. Hence $\bigcup_{i\in I} A_i <_\leq \bigcup_{i\in I} B_i$ and $\bigcap_{i\in I} A_i <_\leq \bigcap_{i\in I} B_i$. This completes the proof.

THEOREM 2.22. *Let $(X, \leq)$ and $(Y, \leq')$ be σ-lattices. Then $f:(X, \leq) \longrightarrow (Y, \leq')$ is an isotone map iff $f : (X, <_\leq) \longrightarrow (Y, <_{\leq'})$ is a continuous map.*

Proof. ($\Rightarrow$) Let $A' <_{\leq'} B'$ and $x \in \uparrow f^{-1}(A')$. Then there is $a \in f^{-1}(A')$ such that $a \leq x$, so $f(a) \in A'$ and $f(a) \leq f(x)$ because f is an isotone map. Hence $f(x) \in \uparrow A'$. Since $\uparrow A' \subset B', f(x) \in B'$, so that $x \in f^{-1}(B')$. Consequently, $\uparrow f^{-1}(A') \subset f^{-1}(B')$, which implies $f^{-1}(A') <_\leq f^{-1}(B')$.

($\Leftarrow$) Suppose $x \leq y$ in X. Since $\uparrow \{f(x)\} \subset \uparrow \{f(x)\}$, $\{f(x)\} <_{\leq'} \uparrow f(x)$; therefore $f^{-1}(f(x)) <_\leq f^{-1}(\uparrow f(x))$ because f is continuous, then by definition of $<_{\leq}$ we obtain $\uparrow f^{-1}(f(x)) \subset f^{-1}(\uparrow f(x))$. Since $x \in f^{-1}(f(x))$ and $x \leq y$, $y \in f^{-1}(\uparrow f(x))$. Thus $f(x) \leq f(y)$. This completes the proof.

Using the above theorem, one has the following:

COROLLARY 2.23. *The functor* $G : \underline{\sigma Latt} \longrightarrow \underline{\sigma TS}$ *is a full embedding, where* $\underline{\sigma Latt}$ *is the category of* σ*-lattices and isotone maps.*

References

1. J. Adamek and H. Herrich and G. E. Strecker, *Abstract and concrete categories*, John Wiley and Sons, 1990.
2. N. Bourbaki, *General topology Part I*, Addision-Wesley, 1966.
3. T. Bromek and M. Moszyńska, *Various categorical approaches to statiscal spaces*, Polska Akademia, Instytut Mathematyczny, Disser. Math., 1983.
4. D. C. J. Burgess and M. Fitzpatrick, *Syntopogenous preordered spaces*, Math. Proc. Comb. phil. Soc. **80** (1978), 19-24.
5. S. H. Chung, *Categories of syntopogenous spaces*, Sogang Univer., Disser. Math., 1988.
6. D. L. Cohn, *Measure theory*, Birkhäuser, 1980.
7. Á. Császár, *Foundations of General topology*, The Macmillan Co., New York, 1963.
8. A. W. Hager, *Measurable uniform spaces*, Fund. Math. **77** (1972), 51-73.
9. M. Herrlich and G. E. Strecker, *Coreflective subcategories in general topology*, Fund. Math. **73** (1972), 199-213.
10. C. Y. Kim, S. S. Hong, Y. H. Hong, P. U. Park, *Algebras in Cartesian closed topological categories*, Lecture Notes Series in Seminars Math. 1, Yonsei Univ., Seoul, 1972.
11. G. Preuss, *Theory of topological structures*, D. Reidel Publishing Company, 1988.

Department of Mathematics
Sungshin Women's University
Seoul 136-742, Korea