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FUZZY WEAKLY SEMICONTINUOUS MAPPINGS

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1. Introduction

The concept of a fuzzy set, which was introduced in [9], provides a natural framework for generalizing many of the concepts of general topology to what might be called fuzzy topological spaces. The idea of fuzzy topological spaces was introduced by Chang [2]. The idea is more or less a generalization of oridinary topological spaces.

In [3,4], A. Kal and P. Bhattacharyya have studied weakly semicontinuous mappings in topological spaces. In this paper, we generalize the concept of weakly semicontinuous mappings in fuzzy setting.

Also, we show that the class of fuzzy weakly semicontinuous mappings contains the class of fuzzy weakly continuous[1], fuzzy semicontinuous[1,6] and fuzzy weakly irresolute mappings[8]. And we characterize fuzzy weakly semicontinuous mappings and investigate relationships between fuzzy weakly semicontinuity and other weaker forms of continuity, and properties of the fuzzy weakly semicontinuity.

Throughout this paper, our notation and terminology will coincide that of [1], [8]; however, a brief review of basic terms will be given in here.

A fuzzy point p in X is a fuzzy set in X defined by

$$p(x) = \begin{cases} k \in (0,1) & \text{for } x = x_p, \\ 0 & \text{otherwise} \end{cases}$$

for each $x \in X$, where x_p and k are the support (written x_p=supp p) and the value of p, respectively. A fuzzy point p is said to belong to a fuzzy set λ in X, written $p \lessdot \lambda$, iff $p(x_p) < \lambda(x_p)$.

In what follows, $(X, \tau X)$ and $(Y, \tau Y)$ (or shortly X and Y) would mean fuzzy topological spaces unless otherwise specified.

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DEFINITION 1.1. ([1]) Let λ be a fuzzy set in X. The closure $\mathrm{Cl}\lambda$ and the interior $\mathrm{Int}\lambda$ of λ are defined by

$$\mathrm{Cl}\lambda = \inf\{\nu : \nu \geq \lambda, \quad \nu' \in \tau X\},$$

and

$$\mathrm{Int}\lambda = \sup\{\nu : \nu \leq \lambda, \quad \nu \in \tau X\}.$$

DEFINITION 1.2. ([1]) Let λ be a fuzzy set in X.

(a) λ is called a fuzzy semi-open set in X if there exists $\nu \in \tau X$ such that $\nu \leq \lambda \leq \mathrm{Cl}\nu$.

(b) λ is called a fuzzy semi-closed set in X if there exists $\nu' \in \tau X$ such that $\mathrm{Int}\nu \leq \lambda \leq \nu$.

LEMMA 1.3. ([1]) *Let λ be a fuzzy set in X. Then the following are equivalent:*

(a) *λ is a fuzzy semi-closed set.*

(b) *λ' is a fuzzy semi-open set.*

(c) $\mathit{IntCl}\lambda \leq \lambda$.

(d) $\mathit{ClInt}\lambda' \geq \lambda'$.

DEFINITION 1.4. ([7,8]) Let λ be a fuzzy set in X. The semi-closure $\mathrm{sCl}\lambda$ and the semi-interior $\mathrm{sInt}\lambda$ of λ are defined by

$$\mathrm{sCl}\lambda = \inf\{\mu : \lambda \leq \mu, \ \mu \text{ is fuzzy semi-closed}\}$$

and

$$\mathrm{sInt}\lambda = \sup\{\mu : \mu \leq \lambda, \ \mu \text{ is fuzzy semi-open}\}.$$

LEMMA 1.5. ([7,8]) *Let λ and μ be fuzzy sets in X satisfying $\lambda \leq \mu$. Then;*

(a) $\mathit{sCl}\lambda \leq \mathit{sCl}\mu$,

(b) $\mathit{sInt}\lambda \leq \mathit{sInt}\mu$,

(c) $\lambda \leq \mathit{sCl}\lambda \leq \mathit{Cl}\lambda$,

(d) $\lambda \geq \mathit{sInt}\lambda \geq \mathit{Int}\lambda$.

LEMMA 1.6. ([8]) *Let λ be a fuzzy set in X. Then;*

$$1 - \mathit{sInt}\lambda = \mathit{sCl}(1-\lambda) \text{ and } 1 - \mathit{sCl}\lambda = \mathit{sInt}(1-\lambda).$$

PROPOSITION 1.7. *Let λ be a fuzzy set in X. Then we have*

$$sInt\lambda = \lambda \cap ClInt\lambda.$$

Proof. Let λ be a fuzzy set in X. Then $\mathrm{sInt}\lambda$ is a fuzzy semi-open set in X. So, $\mathrm{sInt}\lambda \leq \mathrm{Cl}(\mathrm{Int}(\mathrm{sInt}\lambda)) \leq \mathrm{ClInt}\lambda$. Hence $\mathrm{sInt}\lambda \leq \lambda \cap \mathrm{ClInt}\lambda$.

Also, clearly $\mathrm{Int}\lambda \leq \lambda \cap \mathrm{ClInt}\lambda \leq \mathrm{ClInt}\lambda$. So, $\lambda \cap \mathrm{ClInt}\lambda$ is a fuzzy semi-open set and $\lambda \cap \mathrm{ClInt}\lambda \leq \lambda$. Hence $\lambda \cap \mathrm{ClInt}\lambda \leq \mathrm{sInt}\lambda$.

2. Fuzzy weakly semicontinuous mappings

In [3], A. Kal and P. Bhattacharyya defined the notion of a weakly semicontinuous mapping in topological spaces as follows;

DEFINITION 2.1. A mapping $f : X \to Y$ is said to be weakly semicontinuous if for each $x \in X$ and each open set V in Y containing $f(x)$, there exists a semi-open set U such that $x \in U$ and $f(U) \subset \mathrm{Cl}(V)$.

We generalize the above definition in fuzzy setting.

DEFINITION 2.2. A mapping $f : X \to Y$ is said to be fuzzy weakly semicontinuous if for each fuzzy point p in X and each fuzzy open set λ in Y satisfying $f(p) \prec \lambda$, there exists a fuzzy semi-open set μ in X such that $p \prec \mu$ and $f(\mu) \leq \mathrm{Cl}\lambda$

THEOREM 2.3. *Let $f : X \to Y$ be a mapping. Then the following are equivalent:*

(a) *f is fuzzy weakly semicontinuous.*

(b) *For any fuzzy open set λ in Y, $f^{-1}(\lambda) \leq sInt(f^{-1}(Cl\lambda))$.*

(c) *For any fuzzy open set λ in Y, $sClf^{-1}(\lambda) \leq f^{-1}(Cl\lambda)$.*

(d) *For any fuzzy open set λ in Y, $f^{-1}(\lambda) \leq ClIntf^{-1}(Cl\lambda)$.*

(e) *For any fuzzy open set λ in Y, $IntClf^{-1}(\lambda) \leq f^{-1}(Cl\lambda)$.*

Proof. (a)$\Longrightarrow$(b) Let λ be any fuzzy open set in Y and $p \prec f^{-1}(\lambda)$. Then $f(p) \prec f(f^{-1}(\lambda)) \leq \lambda$. Also, since f is fuzzy weakly semicontinuous, there exists a fuzzy semi-open set μ in X such that $p \prec \mu$ and $f(\mu) \leq \mathrm{Cl}\lambda$. This implies $p \prec \mu \leq f^{-1}(\mathrm{Cl}\lambda)$ and so by Lemma

1.5, $p \leq \mu = \text{sInt}\mu \leq \text{sInt}(f^{-1}(\text{Cl}\lambda))$. Thus, we have $f^{-1}(\lambda) \leq \text{sInt}(f^{-1}(\text{Cl}\lambda))$.

(b)$\Longrightarrow$(c) Let λ be fuzzy open in Y. Then $\text{Cl}\lambda$ is fuzzy closed set in Y and so $1 - \text{Cl}\lambda$ is fuzzy open in Y. It follows

$$\begin{aligned} 1 - f^{-1}(\text{Cl}\lambda) = f^{-1}(1 - \text{Cl}\lambda) &\leq \text{sInt}f^{-1}(\text{Cl}(1 - \text{Cl}\lambda)) \\ &= \text{sInt}(1 - f^{-1}(\text{IntCl}\lambda)) = 1 - \text{sCl}f^{-1}(\text{IntCl}\lambda). \end{aligned}$$

Now, we also have

$$\begin{aligned} f^{-1}(\text{Cl}\lambda) \geq \text{sCl}f^{-1}(\text{IntCl}\lambda) &\geq \text{sCl}f^{-1}(\text{Int}\lambda) \\ &= \text{sCl}f^{-1}(\lambda). \end{aligned}$$

(c)$\Longrightarrow$(d) Let λ be fuzzy open in Y. Then $1 - \text{Cl}\lambda$ is fuzzy open in Y. So, by (c), we have

$$1 - \text{sInt}f^{-1}(\text{Cl}\lambda) = \text{sCl}f^{-1}(1 - \text{Cl}\lambda) \leq f^{-1}(\text{Cl}(1 - \text{Cl}\lambda))$$

$$= f^{-1}(1 - \text{IntCl}\lambda) = 1 - f^{-1}(\text{IntCl}\lambda).$$

Hence by Propsition 1.7

$$\begin{aligned} f^{-1}(\lambda) = f^{-1}(\text{Int}\lambda) \leq f^{-1}(\text{IntCl}\lambda) &\leq \text{sInt}f^{-1}(\text{Cl}\lambda) \\ = f^{-1}(\text{Cl}\lambda) \cap \text{ClInt}f^{-1}(\text{Cl}\lambda) &\leq \text{ClInt}f^{-1}(\text{Cl}\lambda). \end{aligned}$$

(d)$\Longrightarrow$(e) Let λ be fuzzy open in Y. Then $1 - \text{Cl}\lambda$ is fuzzy open in Y. So,

$$\begin{aligned} 1 - f^{-1}(\text{Cl}\lambda) = f^{-1}(1 - \text{Cl}\lambda) &\leq \text{ClInt}f^{-1}(\text{Cl}(1 - \text{Cl}\lambda)) \\ &= \text{ClInt}f^{-1}(1 - \text{IntCl}\lambda) = \text{Cl}(1 - \text{Cl}f^{-1}(\text{IntCl}\lambda)) \\ &= 1 - \text{IntCl}f^{-1}(\text{IntCl}\lambda). \end{aligned}$$

Therefore we have

$$\text{IntCl}f^{-1}(\lambda) = \text{IntCl}f^{-1}(\text{Int}\lambda) \leq \text{IntCl}f^{-1}(\text{IntCl}\lambda) \leq f^{-1}(\text{Cl}\lambda).$$

(e) $\Longrightarrow$(a) Let p be a fuzzy point in X and λ fuzzy open in Y satisfying $f(p) \lessdot \lambda$. Then $p \lessdot f^{-1}(\lambda)$ and by (e), $\text{IntCl}f^{-1}(\lambda) \leq f^{-1}(\text{Cl}\lambda)$. Hence $\text{IntCl}f^{-1}(\lambda) \leq \text{sInt}f^{-1}(\text{Cl}\lambda) \leq f^{-1}(\text{Cl}\lambda)$ and by Proposition 1.7, $\text{sInt}f^{-1}(\text{Cl}\lambda) = f^{-1}(\text{Cl}\lambda) \cap \text{ClInt}f^{-1}(\text{Cl}\lambda)$. Also,

$$1 - \text{ClInt}f^{-1}(\text{Cl}\lambda) = \text{IntCl}f^{-1}(1 - \text{Cl}\lambda) \leq f^{-1}(\text{Cl}(1 - \text{Cl}\lambda))$$
$$= f^{-1}(1 - \text{IntCl}\lambda) = 1 - f^{-1}(\text{IntCl}\lambda) \leq 1 - f^{-1}(\lambda).$$

So, we get

$$f^{-1}(\lambda) \leq \text{ClInt}f^{-1}(\text{Cl}\lambda)$$

and

$$f^{-1}(\lambda) \leq f^{-1}(\text{Cl}\lambda).$$

Therefore

$$p \lessdot f^{-1}(\lambda) \leq \text{sInt}f^{-1}(\text{Cl}\lambda) \leq f^{-1}(\text{Cl}\lambda).$$

Putting $\mu = \text{sInt}f^{-1}(\text{Cl}\lambda)$, μ is fuzzy semi-open in X. Thus

$$f(\mu) \leq f(f^{-1}(\text{Cl}\lambda)) \leq \text{Cl}\lambda.$$

EXAMPLE 2.4. Let λ and μ be fuzzy sets in the unit interval I defined by

$$\lambda(x) = \begin{cases} 0 \text{ if } 0 \leq x \leq \frac{1}{3}, \\ 0.3 \text{ if } \frac{1}{3} < x \leq 1. \end{cases}$$

and

$$\mu(x) = \begin{cases} 0 \text{ if } 0 \leq x \leq \frac{1}{3}, \\ 0.4 \text{ if } \frac{1}{3} < x \leq \frac{2}{3}, \\ 0.2 \text{ if } \frac{2}{3} < x \leq 1. \end{cases}$$

Consider fuzzy topologies $\tau_1 I = \{0, 1, \lambda\}$ and $\tau_2 I = \{0, 1, \mu\}$ on I and a mapping $f : (I, \tau_1 I) \to (I, \tau_2 I)$ defined by $f(x) = x$ for each $x \in I$.

It is easy to show that $\text{sCl}f^{-1}(0) = f^{-1}(\text{Cl}0)$ and $\text{sCl}f^{-1}(1) = f^{-1}(\text{Cl}1)$. So, we will show that $\text{sCl}f^{-1}(\mu) \leq f^{-1}(\text{Cl}\mu)$.

$$(f^{-1}(\mu))(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq \frac{1}{3}, \\ 0.4 & \text{if } \frac{1}{3} < x \leq \frac{2}{3}, \\ 0.2 & \text{if } \frac{2}{3} < x \leq 1. \end{cases}$$

$$(\text{ sCl}f^{-1}(\mu))(x) = \begin{cases} 0 & \text{if } 0 \le x \le \frac{1}{3}, \\ 0.4 & \text{if } \frac{1}{3} < x \le \frac{2}{3}, \\ 0.3 & \text{if } \frac{2}{3} < x \le 1. \end{cases}$$

$$(\text{ Cl}\mu)(x) = \begin{cases} 1 & \text{if } 0 \le x \le \frac{1}{3}, \\ 0.6 & \text{if } \frac{1}{3} < x \le \frac{2}{3}, \\ 0.8 & \text{if } \frac{2}{3} < x \le 1. \end{cases}$$

$$(f^{-1}(\text{ Cl}\mu))(x) = \begin{cases} 1 & \text{if } 0 \le x \le \frac{1}{3}, \\ 0.6 & \text{if } \frac{1}{3} < x \le \frac{2}{3}, \\ 0.8 & \text{if } \frac{2}{3} < x \le 1. \end{cases}$$

Thus by Theorem 2.3, f is fuzzy weakly semicontinuous.

DEFINITION 2.5. ([1]) A fuzzy topological space X is product related to a fuzzy topological space Y if for any fuzzy set ν in X, η in Y, $\lambda \in \tau X$ and $\mu \in \tau Y$ satisfying $(\lambda \times \mu)' \ge \nu \times \eta$, there exist $\lambda_1 \in \tau X$ and $\mu_1 \in \tau Y$ such that $(\lambda_1' \ge \nu$ or $\nu_1' \ge \eta)$ and $\lambda_1 \times \mu_1 = \lambda \times \mu$.

LEMMA 2.6. ([1]) *Let X and Y be fuzzy topological spaces such that X is product related to Y. Let λ be a fuzzy set in X and μ a fuzzy set in Y. Then*

(a) $Cl(\lambda \times \mu) = Cl\lambda \times Cl\mu$,
(b) $Int(\lambda \times \mu) = Int\lambda \times Int\mu$.

THEOREM 2.7. *Let X_1, X_2, Y_1 and Y_2 be fuzzy topological spaces such that X_1 and Y_1 are product related to X_2 and Y_2, respectively. If $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ are fuzzy weakly semicontinuous, then the product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is fuzzy weakly semicontinuous.*

Proof. Let $\lambda = \cup(\lambda_\alpha \times \mu_\beta)$, where $\lambda_{\alpha's}$ and $\mu_{\beta's}$ are fuzzy open set in Y_1 and Y_2 respectively, be a fuzzy open set in $Y_1 \times Y_2$.

$$\begin{aligned} &(f_1 \times f_2)^{-1}(\lambda) \\ &= (f_1 \times f_2)^{-1}(\cup(\lambda_\alpha \times \mu_\beta)) = \cup(f_1 \times f_2)^{-1}(\lambda_\alpha \times \mu_\beta) \\ &= \cup[f_1^{-1}(\lambda_\alpha) \times f_2^{-1}(\mu_\beta)] \le \cup[\text{ ClInt}f_1^{-1}(\text{Cl}\lambda_\alpha) \times \text{ClInt}f_2^{-1}(\text{Cl}\mu_\beta)] \end{aligned}$$

$$= \cup \operatorname{ClInt}[f_1^{-1}(\operatorname{Cl}\lambda_\alpha) \times f_2^{-1}(\operatorname{Cl}\mu_\beta)] \leq \operatorname{ClInt} \cup [f_1^{-1}(\operatorname{Cl}\lambda_\alpha) \times f_2^{-1}(\operatorname{Cl}\mu_\beta)$$
$$= \operatorname{ClInt} \cup (f_1 \times f_2)^{-1}(\operatorname{Cl}\lambda_\alpha \times \operatorname{Cl}\mu_\beta)$$
$$= \operatorname{ClInt}(f_1 \times f_2)^{-1}(\cup(\operatorname{Cl}\lambda_\alpha \times \operatorname{Cl}\mu_\beta))$$
$$\leq \operatorname{ClInt}(f_1 \times f_2)^{-1}(\operatorname{Cl} \cup (\lambda_\alpha \times \mu_\beta)) = \operatorname{ClInt}(f_1 \times f_2)^{-1}(\operatorname{Cl}\lambda).$$

Thus by Theorem 2.3, $f_1 \times f_2$ is fuzzy weakly semicontinuous.

THEOREM 2.8. *Let $f : X \to Y$ be a mapping. If the graph $g : X \to X \times Y$ of f is fuzzy weakly semicontinuous, f is also fuzzy weakly semicontinuous.*

Proof. Let λ be a fuzzy open set in Y. Since g is fuzzy weakly semicontinuous,

$$\begin{aligned} f^{-1}(\lambda) = 1 \cap f^{-1}(\lambda) = g^{-1}(1 \times \lambda) &\leq \operatorname{ClInt} g^{-1}(\operatorname{Cl}(1 \times \lambda)) \\ &\leq \operatorname{ClInt} g^{-1}(1 \times \operatorname{Cl}\lambda) = \operatorname{ClInt}(1 \cap f^{-1}(\operatorname{Cl}\lambda)) \\ &= \operatorname{ClInt} f^{-1}(\operatorname{Cl}\lambda). \end{aligned}$$

The proof is complete.

3. Relationships between fuzzy weakly semicontinuity and weaker forms of continuity

DEFINITION 3.1. ([1]) A mapping $f : X \to Y$ is said to be fuzzy semicontinuous if for any fuzzy open set λ in Y, $f^{-1}(\lambda)$ is fuzzy semi-open in X.

LEMMA 3.2. ([6]) *Let $f : X \to Y$ be a mapping. Then the following are equivalent:*

(a) f is fuzzy semicontinuous.

(b) For each fuzzy point p in X and each fuzzy open λ in Y satisfying $f(p) \lessdot \lambda$, there exists a fuzzy semi-open set μ in X such that $p \lessdot \mu$ and $f(\mu) \leq \lambda$.

EXAMPLE 3.3. Let $(I, \tau_1 I)$ and $(I, \tau_2 I)$ be the same fuzzy topological spaces and f the fuzzy weakly semicontinuous map in Example 2.4. Also μ is the fuzzy open in $(I, \tau_2 I)$, but $f^{-1}(\mu)$ is not fuzzy semi-open in $(I, \tau_1 I)$. Hence f is not fuzzy semicontinuous.

DEFINITION 3.4. ([1]) A fuzzy topological space X is said to be fuzzy regular if each fuzzy open set λ in X is a union of fuzzy open sets $\lambda_{\alpha's}$ in X such that $\mathrm{Cl}\lambda_\alpha \leq \lambda$, for each α.

THEOREM 3.5. *Let $f : X \to Y$ be a mapping and Y a fuzzy regular space. Then f is fuzzy weakly semicontinuous if and only if f is fuzzy semicontinuous.*

Proof. It is clear that every fuzzy semicontinuous mapping is fuzzy weakly semicontinuous.

Conversely, let p be a fuzzy point in X and λ a fuzzy open in Y satisfying $f(p) \lessdot \lambda$. Since Y is a fuzzy regular space, $\lambda = \cup\lambda_\alpha,\ \lambda_\alpha \in \tau Y$ and $\mathrm{Cl}\lambda_\alpha \leq \lambda$ for each α. Now, $f(p) \lessdot \lambda$ implies that there exist a fuzzy open set λ_{α_1} in Y such that $f(p) \lessdot \lambda_{\alpha_1}$ and $\mathrm{Cl}\lambda_{\alpha_1} \leq \lambda$. Since f is fuzzy weakly semicontinuous, there exists a fuzzy semi-open set μ in X such that $p \lessdot \mu$ and $f(\mu) \leq \mathrm{Cl}\lambda_{\alpha 1}$. Hence $f(\mu) \leq \mathrm{Cl}\lambda_{\alpha_1} \leq \lambda$, so that f is fuzzy semicontinuous.

DEFINITION 3.6. ([8]) A mapping $f : X \to Y$ is said to be fuzzy weakly irresolute if for each fuzzy point p in X and each fuzzy semi-open set λ in Y satisfying $f(p) \lessdot \lambda$, there exists a fuzzy semi-open set μ in X such that $p \lessdot \mu$ and $f(\mu) \leq \mathrm{sCl}\lambda$.

THEOREM 3.7. ([8]) *$f : X \to Y$ is fuzzy weakly irresolute if and only if for any fuzzy semi-open set λ in Y, $sCl(f^{-1}(\lambda)) \leq f^{-1}(sCl\lambda)$.*

THEOREM 3.8. *If $f : X \to Y$ be fuzzy weakly irresolute, then f is fuzzy weakly semicontinuous.*

Proof. Let p be a fuzzy point in X and λ a fuzzy open in Y satsfying $f(p) \lessdot \lambda$. Then λ is fuzzy semi-open in Y. Since f is fuzzy weakly irresolute, there exists a fuzzy semi-open set μ in X such that $p \lessdot \mu$ and $f(\mu) \leq \mathrm{sCl}\lambda$. Also, Since $\mathrm{sCl}\lambda \leq \mathrm{Cl}\lambda$, we get $f(\mu) \leq \mathrm{sCl}\lambda \leq \mathrm{Cl}\lambda$. Therefore f is fuzzy weakly semicontinuous.

EXAMPLE 3.9. Let $(I,\ \tau_1 I)$ and $(I,\ \tau_2 I)$ be the same fuzzy topological spaces and f the fuzzy weakly semicontinuous map in Example 2.4.

Let ν be a fuzzy set in I defined by

$$\nu(x) = \begin{cases} 0.2 & \text{if } 0 \le x \le \frac{1}{3}, \\ 0.45 & \text{if } \frac{1}{3} < x \le \frac{2}{3}, \\ 0.25 & \text{if } \frac{2}{3} < x \le 1. \end{cases}$$

Then clearly, ν is fuzzy semi-open in $(I, \tau_2 I)$, $\text{sCl}\nu = \nu$ and $f^{-1}(\text{ sCl}\nu) = \nu$. Since $f^{-1}(\nu) = \nu$, we get

$$\text{sCl}f^{-1}(\nu)(x) = \begin{cases} 0.2 & \text{if } 0 \le x \le \frac{1}{3}, \\ 0.45 & \text{if } \frac{1}{3} < x \le \frac{2}{3}, \\ 0.3 & \text{if } \frac{2}{3} < x \le 1. \end{cases}$$

Thus, $\text{sCl}f^{-1}(\nu) \not\le f^{-1}(\text{ sCl}\nu)$. So f is not fuzzy weakly irresolute([8]).

REMARK 3.10. The composition of fuzzy weakly semicontinuous maps need not be fuzzy weakly semicontinuous as following example.

Let λ and μ be a fuzzy set in I defined by $\lambda(x) = 0.3$, and $\mu(x) = 0.4$. Clearly $\tau_1 I = \{0, 1, \lambda, \lambda'\}$, $\tau_2 I = \{0, 1, \lambda\}$ and $\tau_3 I = \{0, 1, \mu\}$ are fuzzy topologies on I. Let $f : (I, \tau_1 I) \to (I, \tau_2 I)$ and $g : (I, \tau_2 I) \to (I, \tau_3 I)$ be identity mappings and $h = g \circ f$. Then it is easy to show that f and g are fuzzy weakly semicontinuous. But, since $h^{-1}(\mu)(x) = 0.4$, $sClh^{-1}(\mu)(x) = \lambda'(x) = 0.7$ and $h^{-1}(Cl\mu)(x) = 0.6$ for any $x \in I$, we get $sClh^{-1}(\mu) \not\le h^{-1}(Cl\mu)$, and so h is not fuzzy weakly semicontinuous.

DEFINITION 3.11. ([5]) A fuzzy topological space X is said to be fuzzy extremely disconnected if the closure of each fuzzy open set in X is a fuzzy open set in X.

THEOREM 3.12. *Let Z be fuzzy extremely disconnected space. If $f : X \to Y$ is fuzzy weakly irresolute and $g : Y \to Z$ is fuzzy weakly semicontinuous, then $g \circ f : X \to Z$ is fuzzy weakly semicontinuous.*

Proof. Let p be a fuzzy point in X and λ a fuzzy open set in Z satisfying $(g \circ f)(p) = g(f(p)) \lessdot \lambda$. Since g is fuzzy weakly semicontinuous, there exists fuzzy semi-open ν in Y such that $f(p) \lessdot \nu$ and $g(\nu) \le \text{Cl}\lambda$.

So, $f(p) \in \nu \leq g^{-1}(\text{ Cl}\lambda)$ and $\text{sCl}\nu \leq \text{ sCl}(g^{-1}(\text{ Cl}\lambda))$. Since Z is extremely disconnected, $\text{Cl}\lambda$ is fuzzy open in Z. Hence by Theorem 2.3,

$$\text{sCl}g^{-1}(\text{ Cl}\lambda) \leq g^{-1}(\text{ ClCl}\lambda) = g^{-1}(\text{ Cl}\lambda).$$

Since f is fuzzy weakly irresolute, there exists fuzzy semi-open μ in X such that $p \in \mu \leq f^{-1}(\text{ sCl}\lambda)$. Thus, we have

$$\begin{aligned} p \in \mu &\leq f^{-1}(\text{ sCl}\lambda) \leq f^{-1}(\text{ sCl}(g^{-1}(\text{ Cl}\lambda)) \\ &\leq f^{-1}(g^{-1}(\text{ Cl}\lambda)) = (g \circ f)^{-1}(\text{ Cl}\lambda). \end{aligned}$$

DEFINITION 3.13. ([6]) A mapping $f : X \to Y$ is said to be fuzzy irresolute if for any fuzzy semi-open set λ in Y, $f^{-1}(\lambda)$ is fuzzy semi-open in X.

THEOREM 3.14. *Let $f : X \to Y$ be fuzzy irresolute and $g : Y \to Z$ be fuzzy weakly semicontinuous. Then $g \circ f : X \to Z$ is fuzzy weakly semicontinuous.*

Proof. Let p be a fuzzy point in X and λ a fuzzy open set in Z satisfying $(g \circ f)(p) = g(f(p)) \in \lambda$. Since g is fuzzy weakly semicontinuous, there exists fuzzy semi-open ν in Y such that $f(p) \in \nu \leq g^{-1}(\text{ Cl}\lambda)$. Since f is fuzzy irresolute, take a fuzzy semi-open set $\mu = f^{-1}(\nu)$ in X. Then $p \in f^{-1}(\nu) \leq f^{-1}(g^{-1}(\text{ Cl}\lambda)) = (g \circ f)^{-1}(\text{ Cl}\lambda)$.

DEFINITION 3.15. A mapping $f : X \to Y$ is said to be fuzzy weakly continuous if for each fuzzy point p in X and each fuzzy open set λ in Y satisfying $f(p) \in \lambda$, there exists a fuzzy open set μ in X such that $p \in \mu$ and $f(\mu) \leq \text{ Cl}\lambda$.

LEMMA 3.16. ([6]) *$f : X \to Y$ is fuzzy weakly continuous if and only if for each fuzzy open set λ in Y, $f^{-1}(\text{ Cl}\lambda) \geq \text{ Cl}f^{-1}(\lambda)$*

THEOREM 3.17. *If $f : X \to Y$ is fuzzy weakly continuous, then f is fuzzy weakly semicontinuous.*

Proof. Let p be a fuzzy point in X and λ a fuzzy open set in X satisfying $f(p) \in \lambda$. Since f is weakly continuous, there exists fuzzy open μ in X such that $p \in \mu$ and $f(\mu) \leq \text{ Cl}\lambda$. Then μ is clearly fuzzy semi-open and so f is weakly semicontinuous.

EXAMPLE 3.18. Let λ and μ be fuzzy sets in the unit interval I defined by $\lambda(x) = 0.3$ and $\mu(x) = 0.4$. Consider fuzzy topologies $\tau_1 I = \{0, 1, \lambda\}$ and $\tau_2 I = \{0, 1, \mu\}$ on I and a mapping $f : (I, \tau_1 I) \to (I, \tau_2 I)$ defined by $f(x) = x$ for each $x \in I$. Then we get

$$\mathrm{sCl} f^{-1}(0) = f^{-1}(\mathrm{Cl}0) = 0, \quad \mathrm{sCl} f^{-1}(1) = f^{-1}(\mathrm{Cl}1) = 1,$$
$$(f^{-1}(\mu))(x) = 0.4, \quad (\mathrm{sCl} f^{-1}(\mu))(x) = 0.4,$$
$$(\mathrm{Cl}\mu)(x) = 0.6 \text{ and } (f^{-1}(\mathrm{Cl}\mu))(x) = 0.6.$$

Thus, $\mathrm{sCl} f^{-1}(\mu) \leq f^{-1}(\mathrm{Cl}\mu)$. By Theorem 2.3, f is fuzzy weakly semicontinuous. But

$$\mathrm{Cl} f^{-1}(\mu)(x) = 0.7 \text{ and so } \mathrm{Cl} f^{-1}(\mu) \not\leq f^{-1}(\mathrm{Cl}\mu).$$

Hence, f is not fuzzy weakly continuous.

THEOREM 3.19. *Let Z be extremely disconnected space. If $f : X \to Y$ is fuzzy weakly semicontinuous and $g : Y \to Z$ is fuzzy weakly continuous, then $g \circ f : X \to Z$ is fuzzy weakly semicontinuous.*

Proof. Let p be a fuzzy point in X and λ a fuzzy open set in Z satisfying $g(f(p)) \lessdot \lambda$. Since g is fuzzy weakly continuous, there exists a fuzzy open set ν in Y such that $f(p) \lessdot \nu \leq g^{-1}(\mathrm{Cl}\lambda)$. Since f is fuzzy weakly semicontinuous, there exist a fuzzy semi-open set μ in X such that $g(f(p)) \lessdot \mu$ and $f(\mu) \leq \mathrm{Cl}\nu$. Since $\mathrm{Cl}\nu \leq \mathrm{Cl} g^{-1}(\mathrm{Cl}\lambda) \leq g^{-1}(\mathrm{Cl}\mathrm{Cl}\lambda) = g^{-1}(\mathrm{Cl}\lambda)$, $g \circ f$ is fuzzy weakly semicontinuous.

THEOREM 3.20. *Let X, X_1 and X_2 be fuzzy topological spaces and $p_i : X_1 \times X_2 \to X_i (i = 1, 2)$ be the projection of $X_1 \times X_2$ onto X_i. Then if $f : X \to X_1 \times X_2$ is fuzzy weakly semicontinuous, $p_i \circ f$ is also fuzzy weakly semicontinuous.*

Proof. Let p be a fuzzy point in X and λ a fuzzy open set in $X_i (i = 1, 2)$ satisfying $(p_i \circ f)(p) \lessdot \lambda$. Since p_i is fuzzy continuous, there exists a fuzzy open set μ in $X_1 \times X_2$ such that $f(p) \lessdot \mu$ and $p_i(\mu) \leq \lambda$. Since f is fuzzy weakly semicontinuous, there exists a fuzzy semi-open set η in X such that $p \lessdot \eta \leq f^{-1}(\mathrm{Cl}\mu)$. Now,

$$\eta \leq f^{-1}(\mathrm{Cl}\mu) \leq f^{-1}(\mathrm{Cl}(p_i^{-1}(\lambda)))$$
$$\leq f^{-1}(p_i^{-1}(\mathrm{Cl}\lambda)) = (p_i \circ f)^{-1}(\mathrm{Cl}\lambda).$$

The proof is complete.

REMARK 3.21. Let λ, μ and η be fuzzy sets in the unit interval I defined by $\lambda(x) = 0.3$, $\mu(x) = \frac{3}{4}x$ and $\eta(x) = 0.4$ for each $x \in I$. Consider $\tau_1 I = \{0, 1, \lambda, \lambda'\}$, $\tau_2 I = \{0, 1, \mu\}$, $\tau_3 I = \{0, 1, \lambda\}$ and $\tau_4 I = \{0, 1, \eta\}$ on I. Let $f : (I, \tau_1 I) \to (I, \tau_2 I)$, $g : (I, \tau_3 I) \to (I, \tau_4 I)$, $h : (I, \tau_4 I) \to (I, \tau_3 I)$ and $k : (I, \tau_1 I) \to (I, \tau_3 I)$ be identity mappings. Then, we get easily the following results; (a) f is fuzzy weakly irresolute and fuzzy weakly continuous, but is not fuzzy semicontinuous. (b) g is fuzzy weakly irresolute and fuzzy semicontinuous, but is not fuzzy weakly continuous. (c) h is not both fuzzy weakly irresolute and fuzzy semicontinuous, but is fuzzy weakly continuous. (d) k is fuzzy semicontinuous, but is not fuzzy weakly irresolute. Thus, these three concepts are independent.

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