J. Korean Math. Soc. **32** (1995), No. 3, pp. 563–571

MARGOLIS HOMOLOGY AND MORAVA K–THEORY OF CLASSIFYING SPACES FOR FINITE GROUP

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1. Introduction

The recent work of Hopkins, Kuhn and Ravenel [H-K-R] indicates the Morava K-theory, $K(n)^*(-)$, occupy an important and fundamental place in homology theory. In particular $K(n)^*(BG)$ for classifying spaces of finite groups are studied by many authors [H-K-R], [R], [T-Y 1,2] and [Hu].

In this paper, we note that the Margolis homology $H(H^*(BG\,;\mathbb{Z}/p), Q_n)$ relates deeply $K(n)^*(BG)$ if the exponent of G is small. We study $K(n)^*(BG)$ for group $|G| = p^3$ and exponent p, $p \geq 3$. Such $K(n)^*(BG)$ are given by Tezuka-Yagita [T-Y 2] by using BP-theory and $BP^*(BG) \otimes_{BP^*} K(n)^* \simeq K(n)^*(BG)$. However we use here only Atiyah-Hirzebruch spectral sequence for $K(n)^*$ theory. Quite recently Leary decided the multiplicative structure of $H^*(BG\,;\mathbb{Z}/p)$ [Ly 2] by using the cohomology of group $\tilde{G}$ which is the central product of G and S^1. Using this $\tilde{G}$ and results of Ravenel [R] and Hopkins-Kuhn-Ravenel [H-K-R], we know the Atiyah-Hirzebruch spectral sequence completely *e.g.* $E_{4p^n-2}^{*,*} \simeq E_\infty^{*,*}$. In particular we correct some inaccuracy of results in Tezuka-Yagita [T-Y 2]. The case $p = 2$ is studied in [C].

2. The nonabelian p-group of the order p^3

Let G be a nonabelian group of $|G| = p^3$. Then G is one of the following groups for $p \geq 3$;

$$E = < a, b, c \,|\, a^p = b^p = c^p = 1,\ [a,b] = c, [a,c] = [b,c] = 1 >$$

$$M = < a, b \,|\, a^{p^2} = b^p = 1,\ [a,b] = a^p >$$

Received July 5, 1994.
1991 AMS Subject Classification: 55N15.
Key words: Margolis Homology, Morava K-theory.
Supported by TGRC-KOSEF.

when $p=2$, $E \simeq M$ and we denote it by D; the dihedral group, and there is the another group Q; the quoternion group $Q = < a, b \mid a^4 = b^4 = 1, [a,b] = a^2 = b^2 >$.
For each group G, there is a central extension

$$(2.1) \qquad 1 \longrightarrow \mathbb{Z}/p \longrightarrow G \longrightarrow \mathbb{Z}/p \oplus \mathbb{Z}/p \longrightarrow 1$$

which induces the spectral sequence

$$E_2^{*,*} = H^*(B(\mathbb{Z}/p \oplus \mathbb{Z}/p\,;\,\mathbb{Z}/p),\ H^*(B(\mathbb{Z}/p\,;\,\mathbb{Z}/p))) \Rightarrow H^*(BG\,;\,\mathbb{Z}/p).$$

When $p \geq 3$, the above E_2-term is $E_2^{*,*} = S_2 \otimes \Lambda_2 \otimes \mathbb{Z}/p[u] \otimes \Lambda(z)$ with $S_2 = \mathbb{Z}/p[y_1, y_2]$, $\Lambda_2 = \Lambda(x_1, x_2), \mathcal{B}x_i = y_i, \mathcal{B}z = u$. For $p=2$, we see $E_2^{*,*} = S_2' \otimes \mathbb{Z}/2[z]$ with $S_2' = \mathbb{Z}/2[x_1, x_2]$. It is known [Ls], [Q], [T-Y 1] that

$$d_2 z = \begin{cases} x_1x_2 & \text{for } G = E, D \\ x_1x_2 + y_2 & \text{for } G = M \\ x_1x_2 + x_1^2 + x_2^2 & \text{for } G = Q. \end{cases}$$

Then by the Cartan-Serre transgression theorem, the next differential are

$$d_3 u = d_3 \mathcal{B} z = \mathcal{B} d_2 z = \mathcal{B}(x_1x_2) = y_2x_1 - y_1x_2 \ \text{ for } p \geq 3,$$
$$d_3 z^2 = x_1^2x_2 + x_1x_2^2 \ \text{ for } p = 2$$

and by the Kudo's transgression theorem, we know

$$\begin{aligned} d_{2(p-1)+1}(u^{p-1} \otimes d_3 u) &= \mathcal{B}\mathcal{P} d_3 u = \mathcal{B}(y_2^p x_1 - y_1^p x_2) \\ &= y_2^p x_1 - y_1^p x_2 \ \text{ for } p \geq 3. \end{aligned}$$

By using this spectral sequence, we get ;

LEMMA 2.2. *When* $G = D$, $H^*(BG\,;\,\mathbb{Z}/p) \simeq E_3 \simeq S_2/(x_1x_2) \otimes \mathbb{Z}/2[z^2]$.

Proof. Since $d_2 z = x_1x_2$, we get $E_2 \simeq S_2/(x_1x_2) \otimes \mathbb{Z}/2[z^2]$. From $d_3 z^2 = x_1^2x_2 + x_1x_2^2 = 0 \bmod (x_1x_2)$, we know $E_2 \simeq E_\infty$. □

For other cases, the spectral sequence is not easy, and hence we consider the another spectral sequence in the next section.

3. The calculation of $H^*(BG\,;\mathbb{Z}/p)$

First we recall the calculation of $H^*(BG\,;\mathbb{Z}/p)$ by P. Kropholler and I. Leary [Ly 1], [Ly 2]. Let $\tilde{G} = G \times_{<c>} S^1$ be the central product of G and S^1. Then there is the exact sequence

$$1 \longrightarrow S^1 \longrightarrow \tilde{G} \longrightarrow \mathbb{Z}/p \oplus \mathbb{Z}/p \longrightarrow 1$$

which induces the spectral sequence

$$E_2^{*,*} = H^*(B(\mathbb{Z}/p \oplus \mathbb{Z}/p\,;\mathbb{Z}/p),\ H^*(BS^1\,;\mathbb{Z}/p)) \Rightarrow H^*(B\tilde{G}\,;\mathbb{Z}/p)$$

where $E_2^{*,*} = S_2 \otimes \Lambda_2 \otimes \mathbb{Z}/p[u]$ and $d_3 u = y_2 x_1 - y_1 x_2$.
Hence the E_3-term is given by

$$E_3^{*,2j} \simeq \begin{cases} S_2 \otimes \Lambda_2/(d_3 u) & j = 0 \bmod p \\ H(S_2 \otimes \Lambda_2,\ d_3 u) & 1 \le j \le p-2 \bmod p \\ Ker\,(d_3 u) & j = p-1 \bmod p. \end{cases}$$

We first compute the above homology;

LEMMA 3.1. $H(S_2 \otimes \Lambda_2,\ d_3 u) \simeq \mathbb{Z}/p\{x_1 x_2\}$.

Proof. Let $f = a_0 + a_1 x_1 + a_2 x_2 + a_{12} x_1 x_2$ be in $Ker\, d_3 u$. Then

(3.2)
$$\begin{aligned} 0 = (d_3 u) f &= (y_2 x_1 - y_1 x_2)(a_0 + a_1 x_1 + a_2 x_2 + a_{12} x_1 x_2) \\ &= (y_2 x_1 - y_1 x_2) a_0 + (y_2 a_2 + y_1 a_1) x_1 x_2. \end{aligned}$$

Hence $a_0 = 0$, $a_1 = -y_2 a'$ and $a_2 = y_1 a'$, that is, $f = (y_2 x_1 - y_1 x_2) a' + a_{12} x_1 x_2$. Since $\mathrm{Im} B(x,y)$ is expressed as the right hand side formula of (3.2), we get Lemma 3.1. □

We see that $d_r\{x_1 x_2 u^i\} = 0$ for $0 \le i \le p-1$ since $\{x_1 x_2 u^i\}$ is y_1 torsion but $S_2 \otimes \Lambda_2/(d_3 u)$ is not. Hence $E_3^{*,*} \simeq E_{2p-1}^{*,*}$. Recall the Kudo's transgression

$$d_{2p-1}(u^{p-1} \otimes d_3 u) = y_2^p y_1 - y_1^p y_2.$$

Since $d_3 u x_1 = y_1 x_1 x_2$ and $d_{2p-1}(u^{p-1} \otimes y_1 x_1 x_2) = (y_1^p y_2 - y_1 y_2^p) x_1$, we get $d_{2p-1}(u^{p-1} \otimes x_1 x_2) = (y_2^p - y_1^{p-1} y_2) x_1$.

In this paper, let us write $\mathrm{gr}\Lambda = F$ if $F = \oplus_{i=0}^{s} F_i/F_{i+1}$ for some filtration $\Lambda = F_0 \supset F_1 \supset \cdots \supset F_s$. Then we can see;

THEOREM 3.3. [Ly 2] *$grH^*(B\tilde{E}\,;\mathbb{Z}/p) \simeq E_{2p+1}^{*,*} \simeq (A \oplus B) \otimes \mathbb{Z}/p[u^p]$ with $A = S_2 \otimes \Lambda_2/(B(x,y),\ B(x,y^p),\ B(y,y^p))$ and $B = \oplus_{i=0}^{p-2}\mathbb{Z}/p\{x_1 x_2 u^i\}$ where $B(w,v) = w_1v_2 - w_2v_1$.*

Proof. First note $d_{2p-1}(u^{p-1} \otimes x_1x_2) = (y_2^p - y_1^{p-1}y_2)x_1 = B(x,y^p)$ mod$B(x,y)$ and $d_{2p-1}(u^{p-1} \otimes d_3u) = B(y,y^p)$. Hence we get $E_{2p+1}^{*,*} \simeq (A \oplus B) \otimes \mathbb{Z}/p[u^p]$, from Lemma 3.1. Since $d_{2p+1}(u^p) = \mathcal{P}^1 d_{2p}(u) = \mathcal{P}^1 B(x,y) = B(x,y^p) = 0$, we know $E_{2p+1} \simeq E_\infty$ and we get the theorem. □

Next consider the fibering

$$\tilde{G}/G \simeq S^1 \longrightarrow BG \longrightarrow B\tilde{G}.$$

This induces the spectral sequence

$$E_2^{*,*} = H^*(B\tilde{G}\,;\mathbb{Z}/p) \otimes H^*(S^1\,;\mathbb{Z}/p) \Rightarrow H^*(BG\,;\mathbb{Z}/p).$$

Since $E_2^{*,i} = 0$ for $i \geq 2$, we get ;

PROPOSITION 3.4. [Ls] *Let $z \in H(S^1\,;\mathbb{Z}/p)$ be a generator. Then $grH^*(BG\,;\mathbb{Z}/p) \simeq H^*(B\tilde{G}\,;\mathbb{Z}/p)/(d_2z) \oplus (Ker\, d_2z)z$.*

We can see $B \subset Ker\, x_1x_2$ and we get (for detailed products structure, see [Ly 2]) ;

THEOREM 3.5. [Ly 1] *$grH^*(BE\,;\mathbb{Z}/p) \simeq ((A/(x_1x_2) \oplus B) \oplus (A^+ \oplus B)z) \otimes \mathbb{Z}/p[u^p]$, where A^+ is the positive degree parts of A.*

4. Margolis homology of $H^*(BE\,;\mathbb{Z}/p)$

Recall that the Milnor primitive derivation Q_n is defined by $Q_0 = \mathcal{B}$ and $Q_n = \mathcal{P}^{p^n}Q_{n-1} - Q_{n-1}\mathcal{P}^{p^n}$. It is known that Q_n is a derivation and $Q_n^2 = 0$. We consider the homology (Margolis homology) defined by the differential Q_n, $H(H^*(BE\,;\mathbb{Z}/p),\ Q_n)$. Recall $A = E_\infty^{*,0} = S_2 \otimes \Lambda_2/(B(x,y), B(x,y^p), B(y,y^p))$ in Theorem 3.3.

LEMMA 4.1. *Each element $f \in A$ is uniquely expressed as $f = a_0 + a_1x_1 + a_2x_2 + a_{12}x_1x_2$ with $a_0 \in S_2/B(y,y^p)$, $a_1 \in S_2/(B(y,y^p)/y_1)$, $a_2 \in \mathbb{Z}/p[y_2]$ and $a_{12} \in \mathbb{Z}/p$.*

Proof. First note $y_1x_1x_2 = y_2x_1^2 = 0$ mod$B(x,y) = y_2x_1 - y_1x_2$ and similarly $y_2x_1x_2 = 0$. Hence $a_{12} \in S_2/(y_1,y_2) \simeq \mathbb{Z}/p$. Next if

$a_2 = y_1 a_2'$, then $a_2 x_2 = y_1 a_1' x_2 = a_2' y_2 x_1 \mod B(x,y)$. Hence we can express $a_2 x_2$ by $a_1 x_1$. Since $0 = B(x, y^p) - y_1^{p-1} B(x, y^p) = y_2^p x_1 - y_1^p x_2 - y_1^{p-1}(y_2 x_1 - y_1 x_2) = (y_2^p - y_1^{p-1} y_2) x_1 = (B(y, y^p)/y_1) x_1$ in A, we get the Lemma. □

LEMMA 4.2. $H(A, Q_n) \simeq S_2/(B(y, y^p), y_1^{p^n}, y_2^{p^n}) \oplus \mathbb{Z}/p\{x_1 x_2\}$.

Proof. The Q_n operator on $x_1 x_2$ is

$$\begin{aligned} Q_n x_1 x_2 &= y_1^{p^n} x_2 - y_2^{p^n} x_1 = (y_1^{p^n - 1} - y_2^{p^n - 1}) y_2 x_1 \\ &= (y_1^{p-1} - y_2^{p-1})(y_1^{(p^{n-1}+\cdots+1)} + \cdots + y_2^{(p^{n-1}+\cdots+1)}) y_2 x_1 \\ &= 0, \text{ since } (y_1^{p-1} - y_2^{p-1}) y_2 = B(y, y^p)/y_1. \end{aligned}$$

Let f be expressed as Lemma 4.1. Then we have

$$Q_n f = a_1 y_1^{p^n} + a_2 y_2^{p^n}. \tag{4.3}$$

If $f \in Ker\, Q_n$, then (4.3) $\in$ Ideal $B(y, y^p) \subset$ Ideal $(y_1 y_2)$. Since $a_2 \in \mathbb{Z}/p[y_2]$, a_2 must be zero. Hence $a_1 y_1^{p^n} \in$ Ideal $B(y, y^p) = y_1(y_2^p - y_1^{p-1} y_2)$ and so $a_1 \in$ Ideal$(y_2^{p-1} - y_1^{p-1} y_2)$. Hence we can take $a_1 = 0$. Therefore we get

$$Ker\, Q_n = \{f \mid f = a_0 + a_{12} x_1 x_2\}.$$

From (4.3), Image Q_n is expressed as Ideal $(y_1^{p^n}, y_2^{p^n})$ in S_2. Thus we get the Lemma. □

It is well-known [Ls], [T-Y] that we can take Chern Classes C_p, C_i in $H^*(\tilde{E})$ as elements $\{u^p\}$, $\{x_1 x_2 u^i\}$, that is, there is a representation $p : \tilde{E} \to U(n)$ such that $p^* C_i$ represents $\{x_1 x_2 u^i\}$ where $H^*(BU(n)) = \mathbb{Z}/p[C_1, C_2, \cdots]$, $|C_i| = 2i$. Since $H^*(BU(n))$ generated by even dimensional elements, all Q_n are zero and so are $\{u^p\}$, $\{x_1 x_2 u^i\}$. Hence we get the following theorem.

THEOREM 4.4. $H(H^*(B\tilde{E} : \mathbb{Z}/p), Q_n) \simeq H((A \oplus B) \otimes \mathbb{Z}/p[u^p], Q_n) \simeq (H(A, Q_n) \oplus B) \otimes \mathbb{Z}/p[u^p]$.

LEMMA 4.5. $H(A/(x_1x_2),\ Q_n) \simeq S_2/(B(y,y^p), y_1^{p^n}, y_2^{p^n})$.

Proof. Recall that $Q_n(x_1x_2) = 0$ and $y_ix_1x_2 = 0$ for $i = 1, 2$. From Lemma 4.2, we get $H(A/(x_1x_2),\ Q_n) = H(A,\ Q_n)/(x_1x_2)$. □

THEOREM 4.6. $grH(H^*(BE;\mathbb{Z}/p),\ Q_n) \simeq S_2/(B(y,y^p), y_1^{p^n}, y_2^{p^n}) \oplus B) \otimes \mathbb{Z}/p[u^p]/(y_1u^{p^n}, y_2u^{p^n}, (x_1x_2u^i)u^{p^n}) \oplus \mathbb{Z}/p[u^p]\{x_1x_2z\}$.

Proof. Let $C = \mathrm{gr}H^*(BE;\mathbb{Z}/p)$ in Theorem 3.5. Let $F_1 = (A/(x_1 x_2) \oplus B) \otimes \mathbb{Z}/p[u^p]$. Then we have the isomorphisms

$$H(F_1,\ Q_n) \simeq (H(A,\ Q_n)/(x_1x_2) \oplus B) \otimes \mathbb{Z}/p[u^p]$$

$$\begin{aligned} H(C/F_1,\ Q_n) &\simeq H((A^+ \oplus B) \otimes \mathbb{Z}/p[u^p]z,\ Q_n) \\ &\simeq (H(A,\ Q_n)^+ \oplus B) \otimes \mathbb{Z}/p[u^p] \otimes \{z\}. \end{aligned}$$

Next consider the spectral sequence

$$E_1 = H(F_1,\ Q_n) \oplus H(C/F_1,\ Q_n) \Rightarrow H(C,\ Q_n).$$

Here we study $d_1 = Q_n$. For this we consider in the spectral sequence (2.1). In the spectral sequence we can prove that

$$Q_n(y_i \otimes z) = y_iu^{p^n}$$

$$Q_n(x_1x_2u^i \otimes z) = x_1x_2 \otimes u^{p^n+i}.$$

Therefore the E_2-term is computed

$$\begin{aligned} E_2 =&(H(A,\ Q_n)/(x_1x_2) \oplus B) \\ &\otimes \mathbb{Z}/p[u^p]/(y_1u^{p^n}, y_2u^{p^n}, (x_1x_2u^i)u^{p^n}) \oplus \mathbb{Z}/p\{x_1x_2z\}[u^p]. \end{aligned}$$

Hence we have the theorem by this spectral sequence.□

5. Morava K-theory

The Morava K-theory $K(n)^*(-)$ is generalized cohomology theory with the coefficient $K(n)^* = \mathbb{Z}/p[v_n, v_n^{-1}]$, $|v_n| = -2p^n + 2$. We consider the Atiyah-Hirzebruch spectral sequence for Morava K-theory

$$E_2^{*,*} = H^*(X;K(n)^*) \Rightarrow K(n)^*(X).$$

It is known [Hu], [T-Y] that the differential $d_{2p^n-1}(x) = v_n \otimes Q_nx$. Hence we get

$$E_{2p}^{*,*} \simeq K(n)^* \otimes H(H^*(X;\mathbb{Z}/p),\ Q_n).$$

THEOREM 5.1. *$grK(n)^*(B\tilde{E}) \simeq K(n)^* \otimes H(H^*(B\tilde{E};\mathbb{Z}/p),\ Q_n)$.*

Proof. From Lemma 4.2 and Theorem 4.4, $H(H^*(B\tilde{E};\mathbb{Z}/p),\ Q_n)$ is generated by even dimensional elements, hence $E_{2p}^{*,*} \simeq E_\infty^{*,*}$. □

Revenel [R] showed that $\dim_{K(n)^*} K(n)^*(BG)$ is finite for each finite group G. Hopkins-Kuhn-Ravenel [H-K-R] defined $K(n)$-theory Euler character χ_n by

$$(5.2)\qquad \chi_n(G) = \dim_{K(n)^*} K(n)^{\text{even}}(BG) - \dim_{K(n)^*} K(n)^{\text{odd}}(BG).$$

For p-groups G, this Euler character can be described in term of conjugay classes of commuting n-tuples of elements in G.

$$\chi_n(G) = \text{number of } \{(g_1,\cdots,g_n)\} \mid [g_i,g_j]=1,\ g_i\in G\}/G$$

with the conjugate action $g\cdot(g_1,\cdots,g_n)\} \sim (gg_1g^{-1},\cdots,gg_ng^{-1})$. They also showed (Lemma 5.3.6. in [H-K-R]) that χ_n is computed inductively

$$(5.3)\qquad \chi_n(G) = \sum_{<g>} \chi_{n-1}(C_G(g))$$

where $< g >$ runs over conjugate classes in G and $C_G(g) = \{h \in G \mid [h,g]=1\}$ is the centralizer of g in G.

Now we consider $K(n)^*(BE)$. Recall $H(\text{gr}(H^*(BE;\mathbb{Z}/p),\ Q_n)$ in Theorem 4.4. If $d_r\{x_1x_2z\} = 0$ for all r, then $E_{2p}^{*,*} \simeq E_\infty^{*,*}$ hence $\dim_{K(n)^*} K(n)^*(BE)$ is infinite since $c^s \neq 0$. This contradicts the result of Ravenel. Therefore we know

$$(5.4)\quad d_r\{x_1x_2z\} = v_n^k u^{sp} \text{ for some } s \text{ with } 2ps-3-1 = 2(p^n-1)k.$$

LEMMA 5.5. *$\dim_{K(n)^*} K(n)^*(BE) = p^{2n}+p^{2n-1}-3p^{n-1}+s$.*

Proof. From Theorem 4.6, $K(n)^*(BE)$ has $K(n)^*$-basis $\{y_1^ky_2^l, y_2^k, C_s\}\otimes u^{jp}\,(1\le k\le p^n,\ 0\le l<p,\ 1\le s\le p-2,\ 0\le j\le p^{n-1})$, $\{u^{hp}\}$ $(0\le h<s)$. Hence we see

$$\begin{aligned}\dim_{K(n)^*} K(n)^*(BE) &= ((p^n-1)p+(p^n-1)+(p-2))p^{n-1}+s\\ &= p^{2n}+p^{2n-1}-3p^{n-1}+s. \ \square\end{aligned}$$

LEMMA 5.6. $\chi_n(E) = p^{2n} + p^{2n-1} - p^{n-1}$.

Proof. The conjugacy classes of E are $< 1 >$, $< C^k >$, $< a^i b^k c^l \mid 0 \leq l < p >$ and their centralizer are $E, E, \mathbb{Z}/p \oplus \mathbb{Z}/p$, respectively. So from (5.3)

$$\begin{aligned}\chi_n(E) &= p\chi_{n-1}(E) + (p^2-1)\chi_{n-1}(\mathbb{Z}/p \oplus \mathbb{Z}/p) \\ &= p\chi_{n-1}(E) + (p^2-1)p^{2n-2}.\end{aligned}$$

Hence we get this Lemma. □

From Lemma 5.5 and Lemma 5.6, we know $sp = 2p^n$, hence $k = 2$. Therefore $d_{4(p^2-1)+1}\{x_1x_2z\} = v_n^2 u^{2p^n}$. From Theorem 4.6, we know

$$E^{*,*}_{4p^n-2} = K(n)^* \otimes H(H(BE;\mathbb{Z}/p);Q_n) - \mathbb{Z}/p[u^p]\{x_1x_2z\})/(u^{2n}).$$

Moreover, $E^{*,*}_{4p^n-2}$ is generated by even dimensional elements, so $E_{4p^2+3} \simeq E_\infty$. Thus we get ;

THEOREM 5.7. [T-Y] $grK(n)^*(BE) \simeq K(n)^*(S_2/(B(y,\ y^p),\ y_1^{p^n},\ y_2^{p^n})) \oplus \mathbb{Z}/p\{C_2,\cdots,C_{p-1}\} \otimes \mathbb{Z}/p[C_p]/(y_1 C_p^{p^{n-1}},\ y_2 C_p^{p^{n-1}},\ C_i C_p^{p^{n-1}},\ C_p^{2p^{n-1}})$ *with* $C_p = u^p, C_i = \{x_1x_2u^i\}$.

REMARK. There are misstype in [T-Y]. In this paper $y_i C_p$ should be $y_i C_p^{p^{n-1}}$ and $C_i C_p^{p^{n-1}} = 0$ should be added. From [T-Y] we also know the above gr$K(n)^*(BE)$ is exactly $K(n)^*(BE)$ except for the products of $C_i\,(1 \leq i \leq p-1)$.

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