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ON CONJUGACY OF SOME SUPPLEMENTS

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1. Introduction

Every group G has a unique maximal normal locally nilpotent subgroup $\Phi(G)$, called the Hirsh-Plotkin radical of G [9]. If G is a group, we define the upper Hirsh-Plotkin series of G to be the ascending series $1 = R_0 \le R_1 \le \dots$ in which $R_{\alpha+1}/R_\alpha = \Phi(G/R_\alpha)$ for each ordinal α and $R_\beta = \cup_{\alpha<\beta} R_\alpha$ for each limit ordinal β. If $R_r = G$ for some natural number r, then G is said to have locally nilpotent length r. $(L\mathrm{N})^r$ denotes the class of groups of locally nilpotent length at most r. The $(L\mathrm{N})^r$-residual of G is defined by

$$(L\mathrm{N})^r(G) = \cap\{N \trianglelefteq G : G/N \in (L\mathrm{N})^r\}.$$

If G is finite, then the locally nilpotent length is called the nilpotent length. And by $G \in \mathrm{N}^r$, we mean G has nilpotent length at most r. Also its residual is defined by

$$\mathrm{N}^r(G) = \cap\ \{N \trianglelefteq G : G/N \in \mathrm{N}^r\}.$$

In 1956, G. Higman showed that for any finite group G, if $\mathrm{N}^r(G)$ is abelian then it is complemented, and any two complements are conjugate. In 1979, in their attempt to generalize this result Losey and Stonehewer proved [8]:

THEOREM 1. *Let G be a finite solvable group. Let two subgroups U and V of G be p-conjugate for every prime p. Suppose that U and V have a nilpotent common normal supplement X in G and that one of the following conditions is satisfied:*

1. *X is abelian;*

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2. *G/X is nilpotent; and*
3. *the Sylow p-subgroups of G have class at most 2 for every prime p.*

Then U and V are conjugate.

In this theorem, the solvability of G is not necessary [4]. This paper is devoted to obtaining the similar results for some locally finite groups. The notations are standard and adapted mainly from [7]. In particular, when π is a set of prime numbers, $O_\pi(G)$ is the largest normal π-subgroup of G. Also if G is a locally finite group, then $\pi(G)$ is the set of prime numbers dividing the orders of elements of G.

2. Theorems

In 1971, A.D. Gardiner, B. Hartley and M.J. Tomkinson[5] introduced a new class of locally finite groups which they denoted by $\mathbf{U}$. Roughly speaking this class was meant to mimic the class of finite solvable groups. Specifically they defined $\mathbf{U}$ to be the largest subgroup-closed class of locally finite groups satisfying the conditions:

1. If $G \in \mathbf{U}$ then G has a finite series

$$1 = G_0 < G_1 < \dots < G_n = G$$

   with locally nilpotent factors.
2. If $G \in \mathbf{U}$ and π is a set of primes then the Sylow π-subgroups of G are conjugate in G.

We prove the following theorem:

THEOREM 2. *Let G be a $\mathbf{U}$-group with $|\pi(G)| < \infty$. Let two subgroups U and V of G be p-conjugate for every prime p. Suppose that U and V have a locally nilpotent common normal supplement X in G and that one of the following conditions is satisfied:*

1. *X is abelian;*
2. *G/X is locally nilpotent; and*
3. *the Sylow p-subgroups of G have class at most 2 for every prime p.*

Then U and V are conjugate.

If p is a prime and every p-subgroup of G satisfies the minimal condition then G is said to satisfy $min - p$, the minimum condition on p-subgroups. The class of locally finite groups with $min - p$ for all primes p has been studied by Šunkov in [12] where he showed that the Sylow p-subgroups of such a group are conjugate. Further work can be found in [1]. We shall let $\mathbf{X}$ denote the class of cou ntable periodic locally solvable groups satisfying $min - p$ for all primes p. We also prove the following theorem:

THEOREM 3. *Let G be an $\mathbf{X}$-group. Let two subgroups U and V of G be p-locally conjugate for every prime p. Suppose that U and V have a locally nilpotent common normal supplement X in G and that one of the following conditions is satisfied:*

1. *X is abelian;*
2. *G/X is locally nilpotent; and*
3. *the Sylow p-subgroups of G have class at most 2 for every prime p.*

Then U and V are locally conjugate.

3. Preliminaries

If G is a Černikov group, we can regard G as a topological space with

$$\{Hx : x \in G, H \leq G\}$$

as a closed sub-base. This topology is called the coset topology of G. The following lemma will be useful for our purpose. Its proof can be found in [2].

LEMMA 1. *Let G and H be Černikov groups with coset topologies. Then the following hold.*

1. *G is a compact, T_1-space.*
2. *If $\sigma : G \to H$ is a homomorphism, then σ is closed and continuous.*
3. *If for any $y \in G$ the maps σ_y, σ'_y are defined by*

$$\sigma_y(x) = xy, \sigma'_y(x) = yx, \quad \text{for all} \quad x \in G,$$

then σ_y and σ'_y are both closed and continuous.

Now we prove another interesting lemma.

LEMMA 2. *Let G be an* **X**-*group. If U and V are locally conjugate Černikov subgroups of G, then U and V are conjugate.*

Proof. Let G^0 be the radicable part of G [7]. Then $U^0 \leq G^0_\sigma$ for some finite set σ of primes, where G^0_σ is the Sylow σ-subgroup of G^0. Hence $C_G(G^0_\sigma) \leq C_G(U^0)$. Also G^0_σ is a characteristic abelian subgroup of G. Hence $G/C_G(G^0_\sigma)$ is finite [7,1.F.3]. Therefore, $|G : C_G(U^0)| < \infty$. Let $\{a_1, ..., a_n\}$ be a transversal of $C_G(U^0)$ in G. On the other hand, we can find finite subgroups F_i such that

$$1 = F_0 < F_1 < F_2 < ... < U^0 = \cup_{i \geq 0} F_i.$$

Thus

$$G \geq C_G(F_1) \geq C_G(F_2) \geq ... \geq C_G(U^0).$$

Since $|G : C_G(U^0)| < \infty$, we can find a subgroup $F \leq U^0$ such that $C_G(F) = C_G(U^0)$. Also there exists a finite subgroup $K \leq U$ such that $U = U^0 K$. Then $< F, K >$ is a finite group. By hypothesis $U^\delta = V$, for some locally inner automorphism δ of G. So, there exists $g \in G$ such that $z^\delta = z^g$ for all $z \in < F, K >$. Now $g = x a_k$ for some $x \in C_G(U^0)$ and some k.

We claim that $z^g = z^\delta$, for all $z \in U^0$. For, if $z \in U^0$, then $< z, F > \leq U^0$. So there exist $y \in C_G(U^0)$ and some $j \in \{1, ..., n\}$ such that $z^\delta = (y a_j)^{-1} z (y a_j)$ and $f^\delta = (y a_j)^{-1} f (y a_j) = a_j^{-1} f a_j$ for all $f \in F$. But $f^\delta = a_k^{-1} f a_k = a_j^{-1} f a_j$. Since $\{a_1, ..., a_n\}$ is a transversal of $C_G(F)$ in G, $a_k = a_j$. Hence

$$z^\delta = (y a_j)^{-1} z (y a_j) = a_j^{-1} z a_j = (x a_j)^{-1} z (x a_j) = z^g$$

for each $z \in U^0$. Since $z^\delta = (x a_j)^{-1} z (x a_j)$ for each $z \in K$, and since $U = U^0 K$, it follows that $V = U^\delta = (x a_j)^{-1} U (x a_j)$, so U and V are conjugate.

4. Reduction Lemmas

In this section, we obtain various reductions that enable us to stren gthen the hypotheses. Throughout this section G is assumed to be a U-group with $|\pi(G)| < \infty$. Moreover, $G = XU = XV$ is assumed to be a counter example to Theorem 2.

LEMMA 3. [12.1] *There are counter examples in which X is a p-group for someprime p.*

Proof. Suppose no counter example exists with $\pi(X) = \{p\}$. We shall prove by induction on $|\pi(X)|$ that G does not exist. By assumption, the induction starts with $|\pi(X)| = 1$. Suppose that Theorem 2 holds for $K = YU = YV$, with $|\pi(Y)| < |\pi(X)|$. Now $X = X_p X_{p'}$, so $G/X_p = (X/X_p)(UX_p/X_p) = (X/X_p)(VX_p)/X_p)$. However $|\pi(X/X_p)| = |\pi(X)| - 1$, and UX_p/X_p and VX_p/X_p are q-conjugate for every prime q. So by the induction hypothesis, there exists $g \in G$ such that $UX_p = (VX_p)^g = V^g X_p$. Replacing V^g by V, we may assume that $UX_p = VX_p \equiv H$. Then $G = HX_{p'}$. Consider the natural isomorphism

$$G/X_{p'} = HX_{p'}/X_{p'} \longrightarrow H/(H \cap X_{p'}).$$

In $G/X_{p'}$, $UX_{p'}/X_{p'}$, and $VX_{p'}/X_{p'}$ are q-conjugate for every prime q and $\pi(X/X_{p'}) = \{p\}$. By assumption, $UX_{p'}/X_{p'}$ and $VX_{p'}/X_{p'}$ are conjugate and hence their images $(H \cap UX_{p'})/(H \cap X_{p'})$ and $(H \cap VX_{p'})/(H \cap X_{p'})$ are conjugate in $H/(H \cap X_{p'})$. However, if $h \in H \cap UX_{p'} = UX_p \cap UX_{p'}$, then $h = u_1 a = u_2 b$, for $u_1, u_2 \in U$, $a \in X_p$, and $b \in X_{p'}$. Hence $ab^{-1} = u_1^{-1} u_2 \in U$. Using the fact that a and b are commuting elements of coprime order, it follows that a, $b \in U$. Hence $h = u_1 a \in U$ and so $U \leq UX_p \cap UX_{p'} = H \cap UX_{p'} \leq U$. Hence $U = H \cap UX_{p'}$, and similarly $V = H \cap VX_{p'}$. So U and V are conjugate. Hence if there are counter examples to the theorem, there are counter examples in which X is a p-group.

The last part of the argument in the proof of Lemma 3 is essentially that occurring in [8].

By Lemma 3 we may suppose G is a counter example with X a p-group for some prime p. From now on, we assume that U and V have a common Sylow p-subgroup U_p. This we may do because of the hypothesis of p-conjugacy, so $U_p = V_p^g$ for some g and we can replace V by V^g.

LEMMA 4. [12.2] *We may assume that $G = \langle U, V \rangle$.*

Proof. Notice that if $H = \langle U, V \rangle$ then by the Dedekind Law $H = (X \cap H)U = (X \cap H)V$. By the assumption on G, $U_p = V_p$. On the other hand, for $q \neq p$, U_q, $V_q \in Syl_q(H)$, where U_q and V_q are Sylow q-subgroups of U and V. Now

1. if X is abelian, then $X \cap H$ is abelian;
2. G/X is locally nilpotent, then $H/(X \cap H)$ is locally nilpotent;
3. if the Sylow q-subgroups of G have class at most 2, then the Sylow q-subgroups of H have class at most 2.

Since G is a counter example, so is H.

We now suppose $G =< U, V >$ is a counter example with X a p-group.

LEMMA 5. [12.3] *We may assume that* $O_p(U) = O_p(V) = 1$ *and hence* $X \cap U = X \cap V = 1$.

Proof. Clearly $\Phi(G) \cap U_p = (\Phi(G) \cap U)_p char(\Phi(G) \cap U) \leq U$. Hence $\Phi(G) \cap U_p \leq O_p(U)$. On the other hand, $XO_p(U)$ is a normal p-subgroup of G, so $O_p(U) \leq \Phi(G) \cap U_p$. Hence $\Phi(G) \cap U_p = O_p(U)$. Similarly, $\Phi(G) \cap U_p = O_p(V)$. So, $O_p(U) = O_p(V)$, and hence, $O_p(U) \trianglelefteq G =< U, V >$. However if G is a counter example to the theorem, so is $G/O_p(U)$ and $O_p(U/O_p(U)) = 1$. Hence we may suppose $O_p(U) = O_p(V) = 1$. Since $X \cap U \leq O_p(U)$, $X \cap U = 1 = X \cap V$.

With all the above assumptions we may further obtain:

LEMMA 6. *We may assume that U is a Černikov group.*

Proof. Since $O_p(U) = 1, \Phi(G) \cap U$ is a p'-group because $(\Phi(G) \cap U)_p = O_p(U) = 1$. However, $\Phi(G) = \Phi(G) \cap XU = X(\Phi(G) \cap U) = \Phi(G) \cap XV = X(\Phi(G) \cap V)$, by the Dedekind Law so, $\Phi(G) \cap U = \Phi(G) \cap V$ is characteristic in G. We have

$$\begin{aligned} G/\Phi(G) \cap U =& (X(\Phi(G) \cap U)/\Phi(G) \cap U) \cdot (U/\Phi(G) \cap U) \\ =& (X(\Phi(G) \cap U)/\Phi(G) \cap U) \cdot (V/\Phi(G) \cap U). \end{aligned}$$

However by Corollary D1 in [6], $G/\Phi(G)$ is Černikov since $|\pi(G)| <\subset nfty$ and $U/\Phi(G) \cap U \approx U\Phi(G)/\Phi(G) \leq G/\Phi(G)$. Thus $U/\Phi(G) \cap U$ and $V/\Phi(G) \cap V$ are Černikov and $G/\Phi(G) \cap U$ is also a counter example to Theorem 2.

In the above proof $O_p(U/\Phi(G) \cap U)$ need not be 1. However if necessary we can apply the argument of Lemma 5 to the group $G/\Phi(G) \cap U$ and obtain a counter example satisfying Lemmas 3 through 6.

LEMMA 7. *[12.4]* $[X, U] = X$.

Proof. For each prime q, pick a Sylow q-subgroup V_q of V. Then $U_q^{ux} = V_q$ for some $U_q \in Syl_q(U)$, $u \in U$ and $x \in X$. Then $V_q \leq [X,U]U$, and hence $V \leq [X,U]U$ since a Černikov group is always generated by a complete set of Sylow q-subgroups. But this means $G = [X,U]U$ since $G =< U, V >$. Hence $X = X \cap [X,U]U = [X,U](X \cap U)$ by the Dedekind Law. However G is a counter example with $X \cap U = 1$. So it follows that $X = [X, U]$.

5. Proof of Theorem 2

In this section $G = XU = XV$ is assumed to be a **U**-group with $|\pi(G)| < \infty$ that is a counter example in which Lemmas 3 through 7 are assumed.

CLAIM 1. *If X is abelian, then Theorem 2 holds.*

Proof. Let G be a counter example in which U has minimal locally nilpotent length. Let the locally nilpotent length of U be $r + 1$. If we let $Q = (L\mathbf{N})^r(U)$, then Q is a p'-group because Q is locally nilpotent and $O_p(U) = 1$. According to a theorem of Tomkinson [14, Thm.2.2], $X = [X, Q] \times C_X(Q)$. If $C_X(Q) = X$, then $Q \trianglelefteq G$. Consider

$$G/Q = (XQ/Q) \cdot (U/Q) = (XQ/Q) \cdot (V/Q).$$

Since U/Q has locally nilpotent length r, the minimal choice of U implies that U and V are conjugate, a contradiction. So $C_X(Q) \neq X$. Then $[X, Q] \neq 1$. Let $A = [X, Q]$. Then $A \trianglelefteq G$ since X is abelian and $Q \trianglelefteq U$. Consider

$$G/A = X/A \cdot UA/A = X/A \cdot VA/A.$$

Since $[X/A, QA/A] = 1$, $C_{X/A}(QA/A) = X/A$. As before UA/A and VA/A are conjugate. Hence there exists $g \in G$ such that $UA = V^g A \equiv K$. It is clear that $A = [A, Q]$. We show that $B \equiv (L\mathbf{N})^{r+1}(K) = A$. Since $K/A \approx U \in (L\mathbf{N})^{r+1}$, $B \leq A$. Now $O_p(U) = 1$ and A is a normal p-subgroup of K, hence $O_p(K) = A$. Also $O_p(K/B) = A/B$ because B is a p-group. However

$$U/(U \cap (L\mathbf{N})^r(K)) \approx U(L\mathbf{N})^r(K)/(L\mathbf{N})^r(K) \in (L\mathbf{N})^r,$$

so $Q \leq (L\mathbf{N})^r(K)$. If we let $L/B = \Phi(K/B)$, then $L/B = A/B \times R/B$ for some p'-group R/B. Now $(K/B)/(L/B) \in (L\mathbf{N})^r$ and hence $K/L \in (L\mathbf{N})^r$. Therefore $Q \leq (L\mathbf{N})^r(K) \leq L$. This implies that QB/B is in L/B and hence $QB/B \leq R/B$. However $[A/B, QB/B] = A/B$. So, by theorem 2.2 in [14]

$$1 = C_{A/B}(QB/B) \geq C_{A/B}(R/B) = A/B.$$

Therefore $A = B$.

Now by Theorem 4.12 in [5], U and V are conjugate, a contradiction. An alternative argument (using cohomology theory) goes as follows. Since $A = [A, Q]$, $C_A(Q) = 1$. Considering A as a U-module, this implies that the fixed point set $A^Q = 1$. Then the inflation-restriction exact sequence of cohomology [13, p.213]

$$0 \to H^1(U/Q, A^Q) \to H^1(U, A) \to H^1(Q, A)^U$$

gives $H^1(U, A) = 0$. Here $H^1(Q, A) = 0$ because $\pi(Q) \cap \pi(A) = \emptyset$. Thus K contains a unique conjugacy class of complements of A and, in particular, U and V are conjugate. But this is a contradiction.

Claim 2. *If G/X is locally nilpotent, then Theorem 2 holds.*

Proof. Since $O_p(G/X) = 1$, U is a p'-group. Therefore U and V are Sylow p'-subgroups of G and hence conjugate. But this is a contradiction.

Claim 3. *If the Sylow q-subgroups of G are of class at most 2 for every prime q, then Theorem 2 holds.*

Proof. This can be proven as in the proof of the Černikov case in [10].

6. Proof of Theorem 3

Throughout this section G is assumed to be an $\mathbf{X}$-group. By Lemma 2,we know that p-locally conjugacy and p-conjugacy are identical in an $\mathbf{X}$-group. First, we prove the following lemma.

LEMMA 8. *Let G be an* **X**-*group. If $G = XU = XV$, where X is a locally nilpotent, normal subgroup of G and the Sylow $\sigma-$ subgroups of U are conjugate in G to the Sylow $\sigma-$ subgroups of V, for each finite set σ of primes, then U and V are locally conjugate.*

Proof. Let $\{X_{p_1}, X_{p_2}, ...\}$, $\{U_{p_1}, U_{p_2}, ...\}$, and $\{V_{p_1}, V_{p_2}, ...\}$ be Sylow generating bases of X, U and V respectively. Then $\{X_{p_1}U_{p_1}, X_{p_2}U_{p_2}, ...\}$ and $\{X_{p_1}V_{p_1}, X_{p_2}V_{p_2}, ...\}$ are Sylow generating bases of G. Let $\sigma = \{p_1, ..., p_k\}$ be given. Let

$$\begin{aligned} U_\sigma &= U_{p_1}...U_{p_k} \in Syl_\sigma(U), \\ V_\sigma &= V_{p_1}...V_{p_k} \in Syl_\sigma(V), \\ S_\sigma &= (X_{p_1}U_{p_1})...(X_{p_k}U_{p_k}) \in Syl_\sigma(G), \end{aligned}$$

and

$$T_\sigma = (X_{p_1}V_{p_1})...(X_{p_k}V_{p_k}) \in Syl_\sigma(G).$$

By assumption, there exists $g \in G$ such that $U_\sigma^g = V_\sigma$. Then $\{U_{p_1}^g, ..., U_{p_k}^g\}$ and $\{V_{p_1}, ..., V_{p_k}\}$ are Sylow bases of the solvable Černikov group V_σ. By a Gol'berg's result [1], there exists $v \in V_\sigma$ such that $U_{p_i}^{gv} = V_{p_i}$ for $i = 1, ..., k$. Clearly $(U_{p_i}X_{p_i})^{gv} = V_{p_i}X_{p_i}$ for $i = 1, ..., k$. Now let Ω_σ be the set of all isomorphisms from S_σ onto T_σ with the following properties:

1. For any $\eta \in \Omega_\sigma$, there exists an inner automorphism of G which coincides with η on S_σ, and
2. $(U_{p_i})^\eta = V_{p_i}$ for all $p_i \in \sigma$.

Then Ω_σ is non-empty. Suppose τ is a finite set of primes and $\sigma \subseteq \tau$. Define a map $\Theta_{\sigma\tau} : \Omega_\tau \to \Omega_\sigma$ by: if $\alpha \in \Omega_\tau$, then $\Theta_{\sigma\tau}(\alpha) = \alpha \mid_{S_\sigma}$. Then

$$\{\Omega_\sigma, \Theta_{\sigma\tau} : \sigma \subseteq \tau \quad \text{are finite sets of primes}\}$$

is an inverse system of sets and mappings. We endow Ω_σ with a suitable topology, to facilitate the use of a theorem from general topology. Suppose $\alpha, \beta \in \Omega_\sigma$. Then there exist $x, y \in G$ so that $\alpha = \phi_x \mid_{S_\sigma}$, $\beta = \phi_y \mid_{S_\sigma}$, where ϕ_x and ϕ_y are inner automorphisms of G induced by x and y, respectively. Thus $\beta^{-1}\alpha = \phi_{xy^{-1}} \mid_{S_\sigma}$ and $\beta^{-1}\alpha$ is a periodic automorphism of the Černikov group S_σ. Moreover, $\beta^{-1}\alpha(U_{p_i}) = U_{p_i}$

for $i = 1, ..., k$. So $xy^{-1} \in \bigcap_{p_i \in \sigma} N_G(U_{p_i})$. Now if $z \in_G (U_{P_i})$ and $\alpha \in \Omega_\sigma$, then $\alpha\phi_{z^{-1}} |_{S_\sigma} \in \Omega_\sigma$. Let $K_\sigma = \bigcap_{p_i \in \sigma} N_G(U_{p_i})/C_G(S_\sigma)$, a Černikov group. Let $\alpha \in \Omega_\sigma$ be fixed. Then the above remarks imply that there is a bijection $g_\sigma : \Omega_\sigma \to K_\sigma$. In fact, if $\beta \in \Omega_\sigma$, then $\beta^{-1}\alpha = \phi_z |_{S_\sigma}$ for some $z \in \bigcap_{p_i \in \sigma} N_G(U_{p_i})$ so define $g_\sigma(\beta) = zC_G(S_\sigma)$. It is easy to show that g_σ is a well-defined bijection. Give K_σ the coset topology and give Ω_σ the topology induced via the map g_σ^{-1}. Then Ω_σ is compact and T_1 by by Lemma 1. We check that if β is another eleme nt of Ω_σ then the topology induced on Ω_σ by K_σ, using β as the fixed element, is the same as the one previously obtained. Let τ_1 be the topology induced when α is the fixed element and let τ_2 be the topology induced when β is the fixed element of Ω_σ. Then there exists $z \in \bigcap_{p_i \in \sigma} N_G(U_{p_i})$ so that
$\alpha = \beta\phi_z |_{S_\sigma}$. Let $\{\delta_i : i \in I\}$ be a τ_1−closed subset of Ω_σ. Then, for certain elements $x_i \in \bigcap_{p_i \in \sigma} N_G(U_{p_i})$, $\delta_i = \alpha\phi_{x_i} |_{S_\sigma}$. Thus $\delta_i = \beta\phi_{x_i z} |_{S_\sigma}$. Since $\{\delta_i : i \in I\}$ is τ_1−closed, the set $\{x_i^{-1}C_G(S_\sigma) : i \in I\}$ is closed in K_σ. Hence the set $\{z^{-1}x_i^{-1}C_G(S_\sigma) : i \in I\}$ is closed by Lemma 1. Hence, by definition, $\{\beta\phi_{x_i z} |_{S_\sigma} : i \in I\} = \{\delta_i : i \in I\}$ is a τ_2−closed set and $\tau_1 \subseteq \tau_2$. It follows by symmetry that $\tau_1 = \tau_2$ so the topologies induced on Ω_σ are the same. Suppose τ is finite set of primes and that $\sigma \subseteq \tau$. We show that $\Theta_{\sigma\tau}$ is a closed continuous map. If $\delta_{\sigma\tau} : K_\tau \to K_\sigma$ is the natural homomorphism, then $\delta_{\sigma\tau}g_\tau = g_\sigma\Theta_{\sigma\tau}$. By Lemma 1, $\delta_{\sigma\tau}$ is closed and continuous as a mapping of Černikov groups with coset topologies. Since g_τ, g_σ are homomorphisms, it follows that $\Theta_{\sigma\tau}$ is closed, continuous mapping. Hence by [11], $\Omega = \varprojlim \Omega_\sigma \neq \emptyset$. Let $(f_\sigma) \in \Omega$. Define $f : G \to G$ as follows: If $x \in G$ and σ is a finite set of primes such that $x \in S_\sigma$, then $f(x) = f_\sigma(x)$. Now it is easy to see that f is a well-defined locally inner automorphism such that $f(U) = V$. So U and V are locally conjugate.

Now we prove Theorem 3.

Proof. Let $\sigma = \{p_1, ..., p_k\}$ be a set of primes. We s how that the Sylow σ−subgroups of U are conjugate in G to the Sylow σ−subgroups of V. Then by Lemma 8, U and V are locally conjugate. Let $\{U_{p_1}, U_{p_2}, ...\}$ and $\{V_{p_1}, V_{p_2}, ...\}$ be Sylow generating bases of U and V, respectively. Then $\{X_{p_1}U_{p_1}, X_{p_2}U_{p_2}, ...\}$ and $\{X_{p_1}V_{p_1}, X_{p_2}V_{p_2}, ...\}$ are Sylow generating bases of G. Since the Sylow generating bases of an **X**-group are locally conjugate[3], by Lemma 2 we may assume that $X_{p_i}U_{p_i} = X_{p_i}V_{p_i}$ for all i. Now $G = (X_{p_1}X_{p_2}...)(U_{p_1}U_{p_2}...) = (X_{p_1}X_{p_2}...)(V_{p_1}V_{p_2}...)$. Also $U_\sigma = U_{p_1}...U_{p_k}$, $V_\sigma = V_{p_1}...V_{p_k}$ are Sylow σ-subgroups of U and V, respectively. For each $i = 1, ..., k$, there exists $u_i \in U$ and $x_i \in X$ such that $(u_i x_i)^{-1}U_{p_i}(u_i x_i) = V_{p_i}$. But $(X_{p_1}...X_{p_k})(U_{p_1}...U_{p_k}) = (X_{p_1}...X_{p_k})(V_{p_1}...V_{p_k})$. Choose $\tau = \{p_1, ..., p_k, p_{k+1}, ..., p_n\}$ such that $\sigma \subseteq \tau$, $x_i \in X_\tau = X_{p_1}...X_{p_n}$, $i = 1, ..., k$ and $u_i \in U_\tau = U_{p_1}...U_{p_n}$, $i = 1, ..., k$. Now consider

$$\begin{aligned} G^* =&(X_{p_1}...X_{p_n})(U_{p_1}...U_{p_k}U_{p_{k+1}}...U_{p_n}) \\ =&(X_{p_1}...X_{p_n}) < V_{p_1}...V_{p_k}, U_{p_{k+1}}...U_{p_n} > . \end{aligned}$$

Then

$$\begin{aligned} G^* =&(X_{p_1}...X_{p_n})(U_{p_1}...U_{p_k}(X_{p_{k+1}}U_{p_{k+1}})...(X_{p_n}U_{p_n})) \\ =&(X_{p_1}...X_{p_n}) < V_{p_1}...V_{p_k}, (X_{p_{k+1}}U_{p_{k+1}})...(X_{p_n}U_{p_n}) > \\ =&(X_{p_1}...X_{p_n}) < V_{p_1}...V_{p_k}, (X_{p_{k+1}}V_{p_{k+1}})...(X_{p_n}V_{p_n}) > \\ =&(X_{p_1}...X_{p_n})(V_{p_1}...V_{p_k}(X_{p_{k+1}}V_{p_{k+1}})...(X_{p_n}V_{p_n})). \end{aligned}$$

Let

$$\begin{aligned} U^* =&U_{p_1}...U_{p_k}(X_{p_{k+1}}U_{p_{k+1}})...(X_{p_n}U_{p_n}), \\ V^* =&V_{p_1}...V_{p_k}(X_{p_{k+1}}V_{p_{k+1}})...(X_{p_n}V_{p_n}). \end{aligned}$$

Then $G^* = X_\tau U^* = X_\tau V^*$, X_τ is a locally nilpotent normal subgroup of G^*, and it is clear that

1. if X is abelian, then X_τ is abelian,
2. if G/X is locally nilpotent, G^*/X_τ is locally nilpotent, and
3. if the Sylow p-subgroups of G have class at most 2, then the Sylow p-subgroups of G^* have class at most 2.

But G^* is a solvable Černikov group. So by Theorem A in [10], U^* and V^* are conjugate since they are p-conjugate for every prime p. Therefore, there exists $g \in G^*$ such that

$$\begin{aligned} &(U_{p_1}...U_{p_k}(X_{p_{k+1}}U_{p_{k+1}})...(X_{p_n}U_{p_n}))^g \\ =&V_{p_1}...V_{p_k}(X_{p_{k+1}}V_{p_{k+1}})...(X_{p_n}V_{p_n}). \end{aligned}$$

So, $(U_{p_1}...U_{p_k})^g$ and $V_{p_1}...V_{p_k}$ are Sylow σ-subgroups of V^*, and hence conjugate. This means that U_σ and V_σ are conjugate.

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