□ 論 文 □

최적 버스 크기의 결정을 위한 해석적 연구

An Analytic Study on Optimal Bus Size

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목 차

요 약

버스의 크기에 따른 운행시격과 승객 대기시간의 Trade-off는 대중교통 운영정책의 중요한 기본개념의 하나로서 여태까지 선진 구미의 여러 교통공학자들에 의해 많은 연구가 수행되어져 왔다. 본 논문에서는 시스템 전체의 최적화(System Optimization)라는 측면에서 차량비를 포함한 버스회사의 총 운행경비와 버스 승객들의 대기시간과 주행시간을 금전으로 환산한 사용자 경비를 합산한 총비용을 최소화 시키는 적정 버스크기(=좌석수)를 산정할 수 있는 모델식을 개발코자 하였다. 이를 위해 우선 수집된 자료들의 회귀분석을 통해 버스 운행경비와 버스크기와의 관계를 규명하였으며 이를 토대로 하여 버스 좌석수를 결정변수(Optimizable Variable)로 하는 총비용에 관한 목적함수식을 도출하였다. 또한 개발된 모형의 적절성을 검증하기 위해 미국 수도 워싱턴 지역에서의 교통자료를 인용하여 사례조사를 하였으며 이를 통해 본 연구에서 도출된 모형식의 실용성을 확인할 수 있었다. 추후 본 연구에서 개발된 수식들은 국내의 버스운행 여건과 실태를 잘 반영할 수 있도록 광범위한 자료조사를 통해 수정되어야 할 것이다.

Ⅰ. INTRODUCTION

Determining the optimal bus size is a particularly important issue in bus operation strategies and of course is a potentially major element of the bus service planning packages.In Europe and North America generally the average size of bus has tended to increase as rising labour costs have encouraged operators to use relatively fewer but larger bus to provide the capacity required to cope with peaks in demand. However some observers have argued that smaller buses can offer advantages despite the relatively higher cost of labour.

For a given service capacity the use of smaller buses would provide a higher service frequency so that passengers would not have to wait so long for a bus,but smaller buses cost more to operate per seat offered and so fares would need to be higher to pay for the improved service. It is this trade−off which will be examined analytically in this paper. In essence the question is how far bus operating costs should be traded against passenger's time cost value in order to get total cost minimization. For the same vehicle capacity supply a public transport route could be served by a fewer large vehicles or more smaller ones. But in general the bus company would prefer to select the former. The most likely reason to perform in this way is simply that they underestimate the user cost namely the value of waiting time costs. So the points made depend critically on the use of realistic time value.

Much interesting works have been done on bus operating strategies but relatively little work has been carried out on bus size optimization problem.

A theoretical study was tried by Webster (5) to see what the effect would be of varying the size of bus in the central London bus fleet and what effect the degree of private car restraint would have on the optimum bus size and on overall travel costs in the area. Hauer(13) posed the problem of which out of all equi−cost fleets to select? A vehicle fleet was described by the number and size of serving vehicles. Navin(8) developed a mathematical model which duplicated Webster's results alongside with observation of long established conventional bus routes. Jansson(2) has also explored the consequences of trading off bus capacity cost against uer waiting time cost. Some more recent studies by Bly and Oldfield(3) have tried to examine the cost and benefits of competition between different sized vehicles operating on the same route. However most approaches above have studied certain idealized problems by analytic methods.

This paper addresses the question of what size of bus will minimize the total sum of vehicle operating cost and passenger's generalized cost of travel. Thus the main objective of this research is to formulate a total cost objective function for bus operation and to find a optimal size which minimize the total cost. The model should also be able to optimize headways and to assign vehicles to various routes in different periods. The primary decision variable is vehicle size and then the optimal headway for each time period can be determined sequentially.

Finally numerical results were computed mainly for the purpose of checking the model formulation and of investigating the sensitivity of optimal bus size with respect to various factors such as time value of passengers and vehicle operating cost coefficients.

Ⅱ. MODEL FORMULATION AND OPTIMIZATION

1. BUS OPERATING COSTS AND VEHICLE SIZE

Deriving the composition of a bus company's total costs from its accounts is somewhat difficult problem. In Table 1 the total costs of 21 Swedish bus companies are presented in accordance with the cost classification used by the Association of Swedish Local Bus Service Operators. The total bus traffic costs can be divided into the two main categories, (1)direct traffic operation costs and (2)overhead costs, is obtained by defining overheads to consist of administration, pensions and all capital costs excluding the capital costs of buses. The remaining costs, direct traffic operation costs, are then about 80% of the total operation costs. This proportion is also obtained in the study of "Costing of Bus Operations" in Bradford. The crew costs make up about one half and the bus capital cost one quarter of the direct traffic operation costs. The remaining quarter of the traffic operation costs comprises fuel, repairs and maintenance, insurance and taxes, and some other minor items.

By the way, according to Oldfield and Bly (3) the total operating costs of buses vary lineary with the passenger carrying capacity (i.e.number of seats) of the vehicle used to a very good approximation. Similar results can be found at Jansson's paper titled "A Simple Bus Line Model for Optimization of Service Frequency and Bus Size"(2). Thus in this paper the average hourly bus operating cost B with S seats was tentatively assumed to have such relationship as shown in following formula.

Table 1 Composition of the total costs of urban bus companies

cost centres	% of total cost
Administration	3.6
Crew Costs	41.8
Repairs & Maintenance	13.0
Buildings	2.4
Insurance & Taxes	3.9
Bus Capital Costs	20.9
Pensions	7.6
Fuel	6.8
Total	100.0

$$B = a + b S \qquad (1)$$

Where

B=total hourly bus operating costs($/veh−hr)

S=number of seats(seats/veh)

a=fixed coefficient in operating cost function

b=variable coefficient in operating cost function

And in order to make sure above relationship shown in formula(1) various bus operation cost data were collected and analyzed in this research. Followings are brief description of investigation.

−Type of buses data collected

(1) Station Wagon(8 seats) (2) Van(12

seats) (3) Small Transit Bus(20 seats) (4) Medium Transit Bus(30 seats) (5) Large Transit Bus(50 seats)

– Cost Categories constituting total bus operation cost

(1) Fuel (2) Oil (3) Tire and Tubes (4) Vehicle Repairs and Maintenance (5) Driver Wages and Fringe Benefits (6) Dispatcher Wages and Fringe Benefits (7) Insurance (8) Maintenance of Dispatching Equipment (9) Driver Exam, Training Licenses and Tags (10) Vehicle Storage Costs (11) General and Administrative Expenses (12) Vehicle Capital Cost (13) Dispatching Equipment Capital Cost

Finally, by least square regression the following cost function is obtained.

$B = 27.88 + .24\ S\ (R^2 = 0.96)$ (2)

in which, B=total hourly bus operating cost ($/veh–hr)

S=number of seats(seats/veh)

Other things being equal, reducing the size of a bus reduces its operating cost much less sharply than the number of seats because of size–independent characteristics of crew cost. Thus the cost per seat–hour operated increases as the size is reduced. This is considerable disadvantages not easily offset by the higher level of service which can be operated. Figure 1 shows how the cost per seat–hr varies with bus size. This illustrates the disadvantage which relatively small vehicle would have to cope with. However the adjustment of holding capacity to need is not a negligible source of improvement in efficiency as the value of time is more seriously considered.

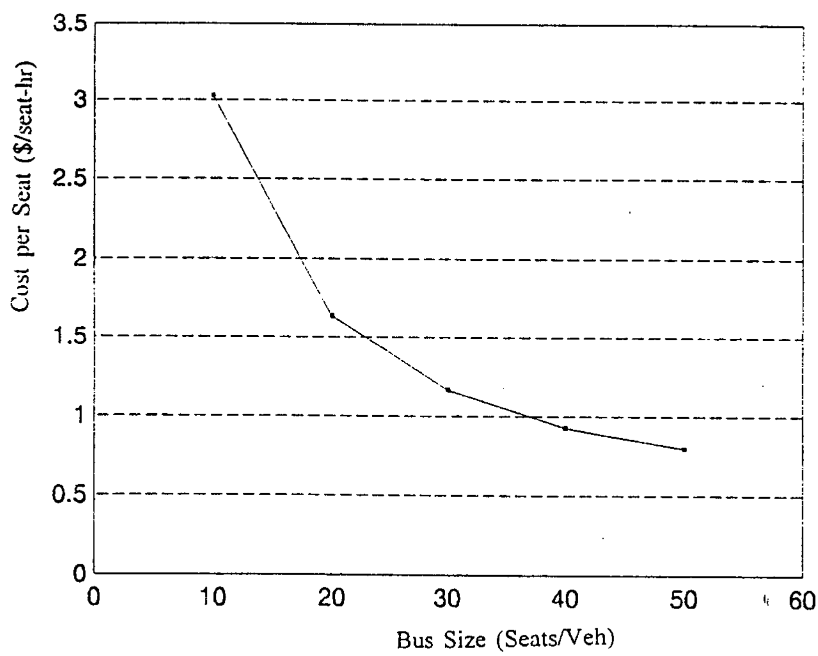

Figure 1. The Relation between Bus Size and Operating Cost per Seat

2. GENERAL FORMULATION

Consider an urban bus line which starts and ends at a given point. It is largely inconsequential for the analysis whether it is a radial, diametral or circular route. The production technique is simply to make bus round trips by a certain number of buses each holding a maximum capacity of S passengers. In formulating the analytic model for conventional bus operating system, the following assumptions are made about the service route.

(1) Conventional fixed route and fixed schedule services are provided on a transit corridor of total length L.
(2) The time required for boarding and alighting the bus is included in average speed V.
(3) The bus load factor is 1.0 at the peak load point.
(4) Bus operating cost per hour increases linearly with bus size.
(5) The total hourly transit demand, Q, is specified with regular discrete distributions alon g the time periods and is invariant with price and service quality.

The total cost objective function includes the operator cost, Co, and the user cost, Cu. The vehicle operating cost parameter, B, includes maintenance and overhead as well as more direct cost of operation(e.g.driver pay, fuel, vehicle depreciation etc.)

The user cost consists of the users' waiting and travel(in vehicle) times multiplied by their respective value of time. The operator cost is the product of the required fleet size, N, and hourly operating cost, B.

$$Co = NB = 2LB/VH \quad (3)$$

in which, L=the length of route in miles; V =the average speed along the route; H=bus headway in hours.

The hourly user cost, C_u ,consists of waiting cost, C_w ,and in-vehicle cost, C_v.

$$C_u = C_w + C_v = (2v_w QH/2) + (2v_v Qd/V) \quad (4)$$

in which, Q=the total hourly boarding passengers along the route of one direction in one time period; v_w =the value of waiting time in dollars per passenger hour; v_v =the value of travel time; d=the average travel distance of boarding passengers in miles.

The hourly total cost, C, is the sum of operator costs(Eq.3) and user costs(Eq.4)

$$C = C_o + C_u = 2LB/VH + v_w QH + 2v_v Qd/V \quad (5)$$

By the way, from above Eq.5 since the bus operating cost B can be formulated as B=a+bS(Eq.1)and with the capacity constraint, H<S/q, the total cost function can be finally converted as following forms.

$$C = \{2Lq(a+bS)/VS\} + v_w SQ/q + 2v_v Qd/V \quad (6)$$

in which, q=the maximum hourly load on the route

The preceding cost function(Eq.6) is our objective function here.

The total cost it represents can be minimized by setting to zero its partial derivative with respect to the optimizable decision variable and solving. In this case the optimized variable is vehicle size, S, and the partial derivative is:

$$\partial C/\partial S = (-2aLq/VS^2) + v_w Q/q = 0 \quad (7)$$

The second derivative of C is:

$$\partial^2 C / \partial S^2 = 4aLq/VS^3 \quad (8)$$

Because all variables in Eq.8 are positive, the second derivative of C with respect to S is positive. Therefore Eq.7 will yield the S value for a minimum rather than maximum total cost. Finally the optimal vehicle S is obtained as following formula.

$$S^{*} = \sqrt{2aLq^{2}/v_{w}VQ} \qquad (9)$$

And the corresponding optimal headway that satisfies demand is:

$$H^{*} = S^{*}/q = \sqrt{2aL/v_{w}VQ} \qquad (10)$$

3. ANALYTIC MODEL FOR MULTIPLE TIME PERIOD

In the preceding section the selected optimal vehicle size and practical optimal headway are only suitable for one specified time period on each route. However there are different passenger volumes in different period as shown in Figure 2.

Often a bus company may use the peak period volume to determine the vehicle size S,but if then,the user cost will be higher in off-peak period because the headway with the large vehicle will be too large. Conversely there will be higher operating cost in peak periods if the small vehicle size based on the off-peak volume is used. Therefore in order to meet time dependent demand the procedure of determining the optimal vehicle size should be modified. For the capacity contraint,H < S/q , the total cost function becomes as following formula.

$$TC = \sum_{t+1}^{m} \{2Lq_{t}\,(a+bS)\,/V_{t}S) + v_{w}\,SQ_{t}/q_{t}+2v_{v}Q_{t}d/V_{t}\} \qquad (11)$$

where

Q_t = the total number of boarding passengers in time period t

q_t = the maximum load on the route in time period t

V_t = the average speed in time period t

m = number of time periods

TC = the total costs for all time periods ($/day)

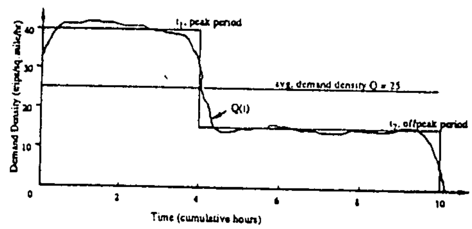

Figure 2. Two Period Distribution of Demand over Time

The vehicle size S that minimizes the total cost TC can be found by setting the derivative of TC with respect to S equal to zero and solving.

$$\partial TC/\partial S = (\sum_{t=1}^{m} - 2aLq_{t}/V_{t}S^{2}) + (\sum_{t=1}^{m} v_{w}Q_{t}/q_{t}) = 0 \qquad (12)$$

Because the second derivative of TC with respect to S is positive the optimal size S* is as following.

$$S^{*} = \sqrt{(\sum_{t=1}^{m} 2aLq_{t}^{2})/(\sum_{t=1}^{m} v_{w}Q_{t}V_{t})} \qquad (13)$$

And the corresponding headway for each time period that satisfies demand is;

$$H_{t} = S^{*}/q_{t}$$

$$= \sqrt{(\sum_{t=1}^{m} 2aLq_{t}^{2})/(\sum_{t=1}^{m} v_{w}Q_{t}V_{t})}\;/\;q_{t} \qquad (14)$$

By the way the optimal vehicle size here is a compromise representing different time periods so the optimal headway for each demand period can be found by setting the derivative of the total cost at time period t,with respect to headway,Ht,equal to zero and solving.

$$C_t = 2L(a+bS)/V_t H_t + v_w Q_t H_t + 2v_v Q_t d/V_t \qquad (15)$$

$$\partial C_t / \partial H_t = -2L(a+bS^* / V_t H_t^2) + v_w Q_t = 0 \qquad (16)$$

The optimal value of H_t is:

$$H_t^* = \sqrt{2L(a+bS^*) / Q_t v_t V_t} \qquad (17)$$

Therfore the practical optimal headway is either the optimal headway Ht from Eq.17 or the maximum headway from Eq.14, whichever is smaller.

$$H_t = \min \{ \sqrt{2L(a+bS^*) / Q_t v_w V_t}, \sqrt{(\sum_{t=1}^{m} 2aLq_t^2)/(\sum_{t=1}^{m} v_w Q_t V_t)} / q_t \} \qquad (18)$$

From the above equations the optimal vehicle size and corresponding headways in each time period can be obtained.

Ⅲ. NUMERICAL RESULTS AND EVALUATION

Numerical results were computed mainly for the purpose of checking the model formulation and of investigating the sensitivity of optimal bus size with respect to various factors such as time value of passengers,vehicle operating cost coefficients,etc.

The following baseline parameter values were selected from the Washington Metropolitan Area Transit Authority(WMATA) for this numerical analysis and they seemed reasonable and representative at the most urban area.

- Transit corridor length: 15.60 miles
- Average local transit speed:
  20.72 mph(peak period)
  28.14 mph(off-peak period)
- Average demand volume:
  326 passengers/hr(peak period)
  128 passengers/hr(off-peak period)
- Fixed transit operating cost: $27.88/veh-hr
- Variable transit operating cost: $0.24/veh.seat.hr
- Value of passenger waiting time: $10/hr
- Value of passenger travel time: $4/hr

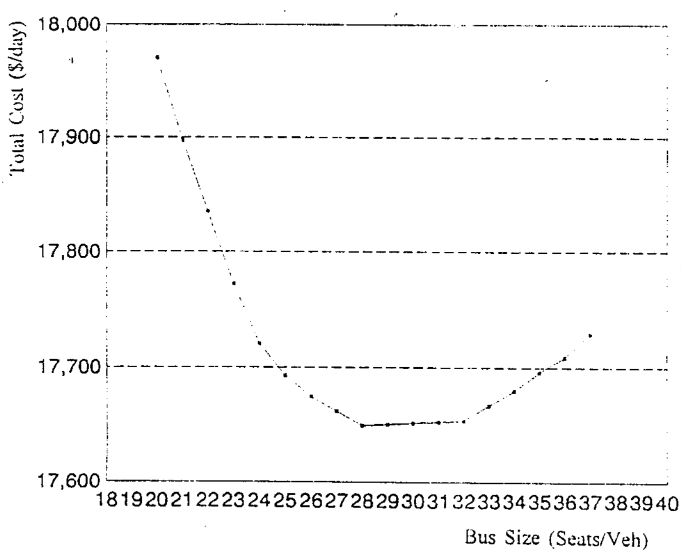

Figure 3. The Relation between Total Cost Function and Vehicle Size

With the above baseline values the relation between the bus size and total cost function is figured out as shown in Figure 3 and the optimal vehicle size is computed to be 28 seats/veh when assumed 4 hours peak and 14 hours off-peak during one day operation. In this figure the total cost function graph is U-shaped and reaches a minimum at a positive value of certain S.

The sensitivity analysis of optimal bus size with respect to other variables or particular parameters gives us good insight into the relations between the factors and shows the relative changes in bus operation if a change occur in another variable. Figure 4 shows how the passenger's value of time influences the optimal vehicle size for bus operation. It shows that if the time value of passenger is larger,the optimal vehicle size should be smaller to reduce the user waitng time. From this figure we noticed that the optimal bus size is very sensitive to the time value of passenger selected and if the value of time was increased by 50% in this numerical example then the optimal size was reduced approximately by 20%.

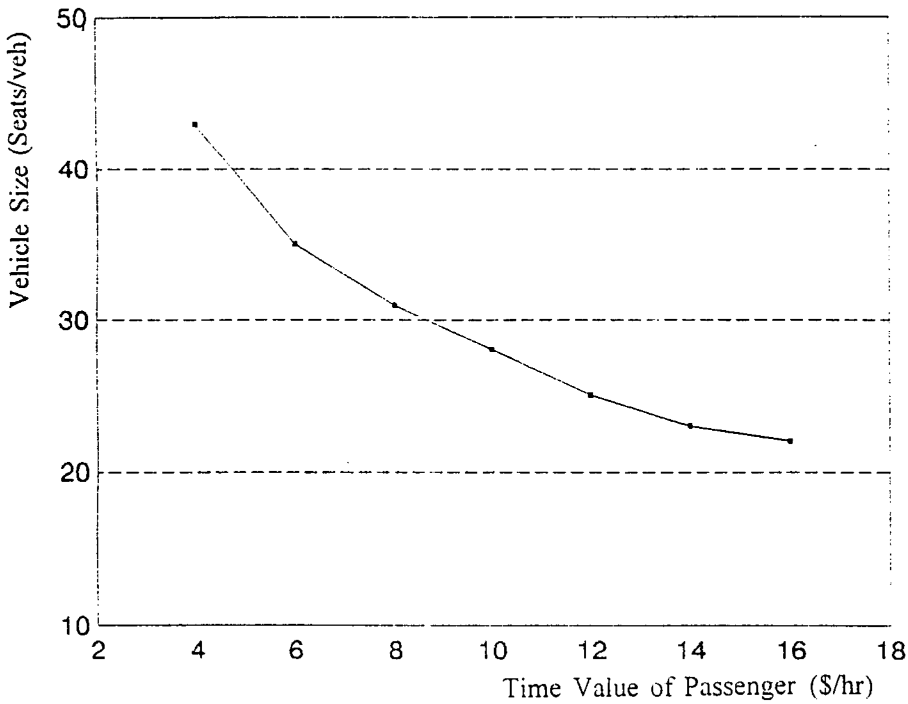

Figure 4. The Relation between Time Value of Passenger and Optimal Vehicle Size

And according to the relations of optimal vehicle size in Eq.9 it is evident that when the length of route increases or the speed of route decreases,the optimal size will increase. It is very interesting to see the relation between the optimal size and the fixed cost coefficient,a,in the vehicle operating cost function.As we know when the value of fixed cost coefficient increases it favors larger vehicles. This can be clearly shown in Figure 5.

Finally,from the relation between the passenger demand level and the optimal vehicle size when the demand increases the optimal size also increases.Actually there are different passenger volume during a day,so the optimal vehicle chosen here was a compromise representing multiple time periods. Therefore if the optimal vehicle size is decided upon the peak demand the user cost is rapidly increased owing to large vehicle size selected and opposite effect will be shown when based upon off−peak demand volume. Table 2 shows clearly the comparison of total cost components of each case.

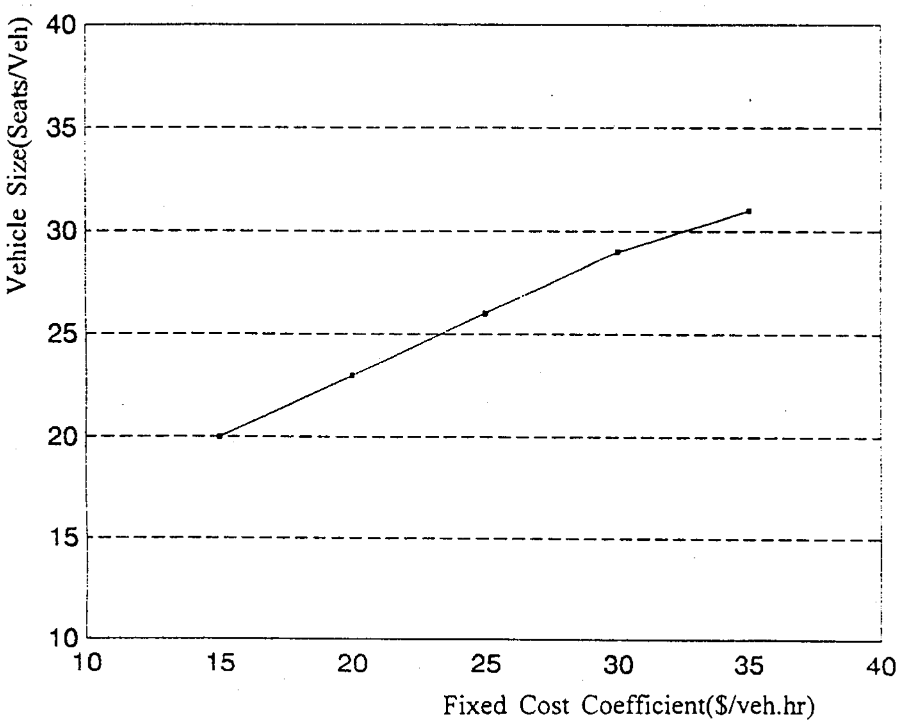

Figure 5. The Relation between Fixed Cost Coefficient and Optimal Vehicle Size

Table 2. Comparision of total cost components

Description	unit	case Ⅰ	case Ⅱ	case Ⅲ
Based Demand Level	pass./hr	peak demand	off−peak demand	compromise demand
Selected Bus Size	seats/veh	32	24	28*
Operating Cost	$/day	5321.84	5791.76	5527.80
Waiting Cost	$/day	4431.84	4029.28	4221.60
In−vehicle Cost	$/day	7900.90	7900.90	7900.90
Total Cost	$/day	17654.58	17721.94	17650.30

Ⅳ. CONCLUSIONS AND RECOMMENDATION

In this paper a mathematical model for optimizing bus size is developed and through the research the following conclusions can be reached.

1. Vehicle size is used as a decision variable in the total cost function of bus operation.The model also be able to optimize headways and to assign vehicles to various routes in different periods.The total cost function is finally expressed as following forms.
   $C = 2Lq(a+bS) + v_w SQ/q + 2v_v Qd/V$
2. To a very good approximation the total operating costs of buses vary linearly with the number of seats of bus used.By least square regression the relation between operation cost and bus size is obtained as following.
   $B = 27.88 + 0.24 S \ (R^2 = 0.96)$
3. The optimal vehicle size for multiple time period can be calculated using following equation.And the optimal size chosen here is a compromise representing different time periods demand.
   $S^* = \sqrt{(\sum_{t=1}^{m} 2aLq_t^2)/(\sum_{t=1}^{m} v_w Q_t V_t)}$
4. The relation between the bus size and total cost function is shown through the numerical results and in this figure the total cost funtion is U−shaped and reaches a minimum at a positive value of S*. The optimal vehicle size represents a trade−off between total operating costs and passenger's waiting time costs.
5. From the sensitivity analysis of optimal bus size with respect to value of time it shows that if the time value of passengers is larger the optimal size should be smaller to reduce the user waiting time. For a given demand if the value of time was increased by 50% then the optimal size was reduced approximately by 20%.
6. It is evident that when the length of route increases or the speed of route decreases the optimal vehicle size will increase.And when the fixed cost coefficient in vehicle operating cost function increase it favors larger vehicles.
7. The optimal vehicle size here is a compromise representing multiple time periods so if the demand volume is far different in peak and off−peak period operating two vehicle size can be considered technically.
8. The analytic model in this paper is developed in view of total bus operation system optimization and the model should be modified in the near future with Korean bus companies'total operation costs.

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